I J C T A, 9(2) 2016, pp. 1333-1345

Dynamic Surface Control Based TS-Fuzzy Model for a Class of Uncertain Nonlinear Systems

Uday Pratap Singh[1], Sanjeev Jain[2], Rajeev Kumar Singh[3] Mahesh Parmar[4], Rajni Ranjan Singh Makwana[5] and Jamvant Singh Kumare[6]

ABSTRACT

This paper revisits dynamic surface control based adaptive Takagi-Sugeno (TS) fuzzy model i.e. (DSC-TS), for a problem of uncertain nonlinear dynamical systems with bounded disturbance. Dynamic surface control (DSC) methods with the combination of TS-model are used for control and approximation of uncertain nonlinear systems up to a tolerance limit. DSC has the advantage over integrator backpsteping or multiple sliding controls are that it avoids an explosion of complexity. TS-fuzzy model is constructed by sector nonlinearity which converts the nonlinear model to multiple rule base of the linear model. Using the fuzzy contour integral Lyapunov function a new sufficient condition for the existence of the DSC based TS-fuzzy model filter design are implemented in terms of linear matrix inequalities. The proposed method provides enhancements and produces good results. Two different type examples are given to show the effectiveness of the proposed method.

Keywords: Dynamic surface control, TS-fuzzy model, uncertain nonlinear system, Lyapunov function.

1. INTRODUCTION

Most of the real world problems are uncertain and nonlinear in nature. Approximation of these types of problems are still challenging due to their complexities. Nonlinear problems are more complex when their parameters are uncertain. The designing of such type of intelligent systems are successful when operating point of systems are restricted to a certain region, also such method are failed if operating point of systems are out of range [1,2]. In last few years a number of practical control problems are involved to designing of filter that optimize nonlinear response control, based on differential geometry using feedback linearization is discussed [3]. To control the uncertainties in feedback linearization much focus has been needed to Lyapunov function based deigns such as multiple sliding control or integrator backstepping is developed [4,5]. Drawback of integrator multiple sliding control or backstepping are explosion of terms/complexities are controlled by a robust nonlinear techniques called dynamic surface control [6]. In higher order feedback systems the explosion of terms are arises due to repetition of differentiation of nonlinear functions. DSC can be design such that it can be applied to Lipschitz and non-Lipschitz nonlinear systems [7,8].

A nonlinear function can be estimated to a desired degree of accuracy by using some adaptive systems like neural networks [9-13]. Anadaptive fuzzy control of dynamical or stochastic nonlinear systems with unknown disturbances and adaptive neural network feedback control of uncertain multiple input multiple output (MIMO) systems has been proposed [14-16]. Adaptive dynamic surface controls with uncertainty are approximated by type-2 fuzzy neural network [16]. Robust hybrid intelligent controls for uncertain nonlinear systems with unknown disturbance are discussed in [15,17]. Many control techniques has been proposed for nonlinear dynamical systems such as neural network approach [10-12] fuzzy logic control

[1,3,4,5,6] Madhav Institute of Technology and Science, Gwalior, 474005 (India)
[2] Shri Mata Vaishno Devi University, Katra, Jammu

(FLC) [18-21] hybrid algorithmic approach etc. [22]. Accurate approximations of complex nonlinear dynamical surface control problems are almost impossible via the represented by linear models. FLC is a powerful method of approximation [23-28] and have many applications in industry e.g. ultra sonic motor-actuated servo drives, direct drive motor systems and dc motor system etc [29].

In last few years, nature inspired metaheuristic inspired by nature have been proposed for solution of many optimization problems. These metaheuristic algorithm leads to heavy computational cost and tuning parameters. In [14-16,18] proposed the genetic fuzzy systems with learning ability to solve complex dynamical problems. A type-2 fuzzy neural network with adaptive dynamic surface control [23] approach has been proposed with system uncertainty. An adaptive intelligent system for a class of uncertain nonlinear dynamical systems in strict feedback form has been developed and an adaptive fuzzy backstepping DSC approach has been established for a class of nonlinear MIMO systems [30,31]. To solve explosion of complexity DSC used first order filter and DSC used in power system control, flexible-robot control, ship control and flight control [32-34].

Real systems are nonlinear systems with uncertainties, which exist everywhere. Our goalis the estimation of nonlinear dynamical systems with perturbation. In this paper we have used TS fuzzy inference system is used for dynamical surface control problem. To optimize tracking control error between target and TS model output for DSC in which tracking performance is ideal. Some important contributions of this paper are stated below:

1. In this workwe describeerror tracking of a class of DSC problem with uncertain disturbance and unknown system functions.
2. By employing DSC technology, differential explosion problem is avoided effectively.
3. By introducing compensating signal errors and prediction signal errors is optimizing using TS-model. The performance of tracking error is effectively improved over other existing methods.

Remaining part of the paper organizes as follows. In section 2, problem formulation and some definitions for a class of single input single output (SISO) nonlinear system with perturbation is discussed. In section 3 we give a simple presentation of TS-model. In section 4 the composition of DSC and TS-model control is designed and stability analysis is discussed. The implementation results are discussed for certain benchmark problem in section 5. In section 6, the conclusion of this paper is drawn.

2. DYNAMIC SURFACE CONTROL AND PROBLEM FORMULATION

Dynamic surface control techniqueis used for avoidance of explosion of terms from a nonlinear system control using convex optimization. DSC is an effective design methodology for uncertain nonlinear systems. Mathematical formulation for class of uncertain nonlinear system [26,27] is given below:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1) \\ \dot{x}_2 &= x_3 + f_2(x_2) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{x}_{n-1} &= x_n + f_{n-1}(x_1,\ldots,x_{n-1}) \\ \dot{x}_n &= u + f_n(x_1,\ldots,x_n) \end{aligned}\right\} \tag{1}$$

Where $f(x)$ and $\partial f(x)/\partial x$ continuous bounded function in domain $D \subseteq \Re^n$ s.t. $\left\|\frac{\partial f}{\partial x}\right\| = \|J(x)\| \leq \alpha$ and $f_i : \Re^n \to \Re$ is strict feedback functionwhere f_i only depends on $x_1,\ldots,x_i$ and $\partial f_i(x)/\partial x$ is bounded on convex and compact set $D_i \subseteq D$ and u is input variable. Dynamical systems are modeled using state space framework, using state transition model, which describes the evolution of states over time and a measurement function. Mathematical model of equation are defined as:

$$\left.\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \theta(t)) \\ y(t) &= h(x(t)) \end{aligned}\right\} \tag{2}$$

Where f is state transition function, describe the evolution of state over time, and h is measurement function relating to measurement of states, x is state variable and u is input variable θ and ξ are unknown parameters and y is measurement value. For given state space model x problem arises when output of system are not matched to target. Based on the literature survey an adaptive fuzzy tracking error control [14-16] method for uncertain nonlinear system equation (1),dynamical surface control problem has been proposed.

$$\left.\begin{aligned} \dot{x}(t) &= \Sigma_i\, w_i(\upsilon(t))\left(\mathrm{A}_i\, x(t) + B_i u(t) + a_i\right) \\ y(t) &= \Sigma_i\, w_i(\upsilon(t))\left(\mathrm{C}_i\, x(t) + c_i\right) \end{aligned}\right\} \tag{3}$$

Where $w_i(\upsilon(t)$ are fuzzy membership function and, A_i, B_i and C_i are appropriate matrix and a_i, c_i are constants.

2.1. DSC Design Procedure:

Actual systems are almost uncertain nonlinear and unknown disturbances exist everywhere. Therefore to approximate uncertain nonlinear with disturbances is very meaningful [25]. In this paper we consider DSC to solve the single input single output (SISO) nonlinear system with perturbation is described as follows:

Let $x_1(t) \to y_{1d}(t)$ inpresence of uncertainty, using error $\tilde{x} = x_1 - y_{1d}$, define sliding or errorsurface $E(t) = e(x,t) = 0$, is an invariant set known as sliding mode. Let us define first error surface $E_1 = x_1 - y_{1d}$, where y_{1d} is the target value of variable x_1. Differentiating E_1 we have

$$\dot{E}_1 = x_2 + f_1(x_1) - \dot{y}_{1d} \text{ where } y_{2d}(0) = \tilde{x}_2(0) \tag{4}$$

x_2 is forcing term then outside some boundary layersliding condition satisfied $\tilde{x}_2 = x_2$, where $\tilde{x}_2 = \dot{y}_{1d} - f_1(x_1) - K_1 E_1$. (5)

Next to force $\tilde{x}_2 \to x_2$ and define $E_2 = x_2 - y_{2d}$ where y_{2d} equal to $\tilde{x}_2$, passed through low pass filter. Similarly, forcing $\tilde{x}_3 \to x_3$ and

$$\tilde{x}_3 = \dot{y}_{2d} - f_1(x_1, x_2) - K_2 E_2 \text{, where } y_{3d}(0) = \tilde{x}_3(0) \tag{6}$$

Similar way continuing this process for each consecutive state define the $(i-1)^{\text{th}}$ surface

$$E_{(i-1)} = x_{(i-1)} - y_{(i-1)d} \text{ and } \tilde{x}_i = \dot{y}_{(i-1)d} - f_{(i-1)}(x_1, x_2, \ldots, x_{(i-1)}) - K_{(i-1)} E_{(i-1)} \tag{7}$$

$$y_{id} \text{ is obtained after filtering } \tilde{x}_i \text{ i.e. } \tau_i \dot{y}_{id} + y_{id} = \tilde{x}_i \text{ and } y_{id}(0) = \tilde{x}_i(0) \tag{8}$$

Continuing this procedure we have $E_n = x_n - y_{nd}$

Finally u is given by: $u = \dot{y}_{nd} - f_n(x_1, x_2, \ldots, x_n) - K_n E_n$

or
$$u = \left(\frac{x_n - \tilde{y}_{nd}}{\tau_n} \right) - f_n(x_1, x_2, \ldots, x_n) - K_n E_n \tag{9}$$

where $K's$ are controller gain.

2.2. Error in DSC

After design of DSC, we have to derive error dynamics and stability analysis.DSC can be divided into two phase (1) multiple sliding surface (MSS) (2) Low pass filter (LPF), MSS phase defines the MSS E_i and evaluates forcing values $\tilde{x}_i$ depends on the state information and filtered signal y_{id} [4,5]. After $n$number of iterations input u is evaluated and feed to the system. Using equation (7) error equation of DSC described as follows:

$$\left.\begin{aligned} \dot{E}_1 &= -K_1 E_1 + E_2 + (y_{2d} - \tilde{x}_2) \\ &\vdots \\ \dot{E}_{(n-1)} &= -K_{(n-1)} E_{(n-1)} + E_n + (y_{nd} - \tilde{x}_n) \\ \dot{E}_n &= -K_n E_n \end{aligned}\right\} \tag{10}$$

Let filter error be $\zeta_i = y_{id} - \tilde{x}_i$ for $2 \le i \le n$, define after first order low pass filters, using equation (8) we have

$$\dot{\zeta}_i = \dot{y}_{id} - \dot{\tilde{x}}_i = -(\zeta_i / \tau_i) - \dot{\tilde{x}}_i \tag{11}$$

Differentiating equation (7) we have

$$\begin{aligned} \dot{\tilde{x}}_i &= -\dot{f}_{(i-1)} + \ddot{y}_{(i-1)d} - K_{(i-1)} \dot{E}_{(i-1)} \\ &= -\dot{f}_{(i-1)} - \left(\zeta_{(i-1)} / \tau_{(i-1)} \right) - K_{(i-1)} \dot{E}_{(i-1)} \end{aligned} \tag{12}$$

For $i = 3, \ldots, n$, equation (11) and equation (12) we have

$$\begin{aligned} \dot{\zeta}_2 - K_1 \dot{E}_1 &= -(\zeta_2 / \tau_2) + \dot{f}_1 - \dot{y}_{1d} \\ \dot{\zeta}_3 - \left(\dot{\zeta}_2 / \tau_2 \right) - K_2 \dot{E}_2 &= -(\zeta_3 / \tau_3) + \dot{f}_2 \\ &\vdots \\ \dot{\zeta}_n - \left(\dot{\zeta}_{(n-1)} / \tau_{(n-1)} \right) - K_{(n-1)} \dot{E}_{(n-1)} &= -(\zeta_n / \tau_n) + \dot{f}_{(n-1)} \end{aligned} \tag{13}$$

Using equation (12) and equation (13), over all error of nonlinear problem (1) is represented as

$$T\dot{v} = A_v v + \tilde{B}_w \dot{f} + \tilde{B}_e \ddot{y}_{1d} \tag{14}$$

where
$$v = [E_1 \cdots E_1 \ \zeta_2 \cdots \zeta_n]^T \in \Re^{2n-1}$$
$$\dot{f} = [\dot{f}_1 \cdots \dot{f}_{n-1}] \in \Re^{n-1} \tag{15}$$

Where the system matrices of above equation is given by

$$T=\begin{pmatrix} I_n & 0 \\ -K & T_\zeta \end{pmatrix}, A_v=\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{21} \end{pmatrix}, \tilde{B}_w=\begin{pmatrix} 0 \\ I_{n_w} \end{pmatrix} \text{ and } \tilde{B}_e=\begin{pmatrix} 0 \\ -b_r \end{pmatrix}.$$

andequivalent sub block these matrices are:

$$T_\zeta=\begin{pmatrix} 1 & 0\cdots & 0 & 0 \\ -\frac{1}{\tau_2} & 1\cdots & 0 & 0 \\ 0 & -\frac{1}{\tau_3}\cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0\ \cdots & -\frac{1}{\tau_{n-1}} & 0 \end{pmatrix}, K=\left[diag(K_1,\cdots,K_{n-1})\ 0_{n-1}\right]$$

$$A_{11}=\begin{pmatrix} -K_1 & 1\ \cdots & 0 \\ 0 & -K_2\ \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0\ \cdots & -K_n \end{pmatrix}, A_{12}=\begin{pmatrix} I_{n-1} \\ 0^T_{n-1} \end{pmatrix}, A_{22}=-diag\left(-\frac{1}{\tau_2},\cdots,-\frac{1}{\tau_n}\right)$$

and $b_r=\left[1\quad 0\cdots\quad 0\right]^T$

Now using equation (15) reduced form of Jacobian matrix is defined as:

$$\frac{\partial f}{\partial x}=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & 0 & 0\cdots & 0 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & 0\cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_{n-1}}{\partial x_1} & \frac{\partial f_{n-1}}{\partial x_2} & \frac{\partial f_{n-1}}{\partial x_3}\cdots & \frac{\partial f_{n-1}}{\partial x_{n-1}} \end{pmatrix} \tag{16}$$

Assume that without loss of generality there exist α s.t. $\|J(x)\|\le\alpha$, $\forall\, x$ in $D_i\subseteq D$. Since T_ζ and T are full rank matrix therefore both are invertible.

$$T_\zeta^{-1}=\begin{pmatrix} 1 & 0\cdots & 0 & 0 \\ \frac{1}{\tau_2} & 1\cdots & 0 & 0 \\ \frac{1}{\tau_2\tau_3} & \frac{1}{\tau_3}\cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\tau_2\ldots\tau_{n-1}} & \frac{1}{\tau_3\ldots\tau_{n-1}}\cdots & \frac{1}{\tau_{n-1}} & 1 \end{pmatrix}$$

therefore

$$T^{-1} = \begin{pmatrix} I_n & 0 \\ T_{\varsigma}^{-1}K & T_{\varsigma}^{-1} \end{pmatrix}$$

Now operate T^{-1} on both side of equation (14), we have

$$\dot{v} = T^{-1}A_v v + T^{-1}\tilde{B}_w \dot{f} + T^{-1}\tilde{B}_e \ddot{y}_{1d}$$

$$\dot{v} = A_{cl}v + B_w \dot{f} + B_e \ddot{y}_{1d} \tag{17}$$

where $T^{-1}A_v = A_{cl}$, $T^{-1}\tilde{B}_e = B_e$ and $T^{-1}\tilde{B}_w = B_w$.

Lemma 1: For given a class of nonlinear dynamical system equation (1) DSC error is given by

$$\dot{v} = A_{cl}v + B_w w + B_r r \tag{18}$$

where w is perturbation term affects filter error dynamics $\|w\| \le \alpha \|C_v v\|$ and $\dot{f} = [\dot{f}_1 \cdots \dot{f}_{n-1}]^T$ $= \dfrac{\partial f}{\partial x}\dot{x} = Jx + C_v v + J_1 \dot{x}_{1d}$

$J = \dfrac{\partial f}{\partial x}$ and J_1 is the first column of J and $r = \begin{pmatrix} J_1 \dot{x}_{1d} \\ \ddot{x}_{1d} \end{pmatrix}$, $B_r = \begin{pmatrix} 0 & 0 \\ T_{\varsigma}^{-1} & -T_{\varsigma}^{-1}b_r \end{pmatrix} = [B_w \;\; B_e]$

Using above term equation (18) becomes

$$\dot{v} = T^{-1}(A_{cl}v + \tilde{B}_w J C_v)v + T^{-1}\tilde{B}_r r$$

$$= A(x)v + B_r r \tag{19}$$

where $A(x) = T^{-1}(A_{cl}v + \tilde{B}_w J C_v)$

$$= \begin{bmatrix} I_n & 0 \\ T_{\varsigma}^{-1}K & T_{\varsigma}^{-1} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ Jx\hat{A}_{11} & A_{22} + JxT_{\zeta} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ T_{\varsigma}^{-1}(KA_{11} + J\hat{A}_{11}) & T_{\varsigma}^{-1}(KA_{12} + A_{22} + JT_{\zeta}) \end{bmatrix}$$

Hence given nonlinear system (1) is stable if $\|w\| = \|J(x)\| \le \alpha \|C_v v\|$.

3. TS-FUZZY MODEL

TS-fuzzy model was developed by Takagi-Sugeno consists of fuzzy IF-THEN rules base. The IF-THEN rules consisting two parts (i) antecedents represents a subset of model variables into fuzzy setsand (ii) consequents of each rule are a functional representation[35,36]. The i^{th} rule is described as follows:

IF $\upsilon_1(t)$ is M_i^1 and . . . and $\upsilon_r(t)$ is M_i^r THEN $\dot{x}(t) = A_i\, x(t) + B_i u(t)$, $x(0) = 0$ (20)

Where $i = 1, 2, ..., n$ and M_i^j $(j = 1, 2, ..., r)$ are fuzzy sets and state vector $x(t) \in \Re^n$, input vector $u(t) \in \Re^m$, A_i, B_i are matrices and $\upsilon_1(t), \ldots, \upsilon_r(t)$ are fuzzy variables and r is the number of fuzzy IF-THEN rules. Given pair $[x(t), u(t)]$ the final fuzzy systems are inferred as follows:

$$\dot{x}(t) = \Sigma_i w_i(\upsilon(t))\left(\mathrm{A}_i x(t) + B_i u(t)\right) \tag{21}$$

Where $$\omega_i(\upsilon(t)) = \frac{\bar{\omega}_i(\upsilon(t))}{\sum_{i=1}^n \bar{\omega}_i(\upsilon(t))} \quad \text{and} \quad \bar{\omega}_i(\upsilon(t)) = \prod_{k=1}^r \mathrm{M}_{ik}(\upsilon_k(t))$$

$\mathrm{M}_i^k(\upsilon_k(t))$ membership function gradeof variable $\upsilon_k(t)$ in fuzzy set M_i^k. It is assumed that $\omega_i(\upsilon(t)) \geq 0$ where $i = 1, 2, ..., n$ and $\sum_{i=1}^n \omega_i(\upsilon(t)) = 1$, represents the convexity.

Fuzzy modelsare two type (i) identification based on input-output data (ii) sector nonlinearity based. In this paper we consider sector nonlinearity TS fuzzy model. Equation (21) represents a class of nonlinear systems:

$$\dot{x}(t) = \Sigma_{j=1}^p f_{ij}(x(t))\, x_j(t) + \Sigma_{k=1}^q g_{ij}(x(t))\, u_k(t) \tag{22}$$

Where p and q are number of state variables and inputs, and $x(t) = (x_1(t), \ldots, x_p(t))$ is state vector and $u(t) = (u_1(t), \ldots, u_q(t))$ is input vector $f_{ij}(x(t))$ and $g_{ik}(x(t))$ are function of $x(t)$. TS-fuzzy model based on sector nonlinearity we have to find out maximum and minimum value of $f_{ij}(x(t))$ and $g_{ik}(x(t))$. Let $a_{ij1} = \max_{x(t)}\{f_{ij}(x(t))\}$, $a_{ij2} = \min_{x(t)}\{f_{ij}(x(t))\}$, $b_{ij1} = \max_{x(t)}\{g_{ik}(x(t))\}$ and $b_{ij2} = \min_{x(t)}\{g_{ik}(x(t))\}$. By using a'sand b's, f_{ij} and g_{ik} can be expressed as:

$$f_{ij}(x(t)) = h_{ij1}(x(t))\, a_{ij1} + h_{ij2}(x(t))\, a_{ij2}$$

$$g_{ik}(x(t)) = \upsilon_{ij1}(x(t))\, b_{ij1} + \upsilon_{ij2}(x(t))\, b_{ij2}.$$

Where $h_{ij1} + h_{ij2} = 1$ and $\upsilon_{ij1} + \upsilon_{ij2} = 1$ also these membership functions are defined as:

$$h_{ij1} = \frac{f_{ij}(x(t)) - a_{ij2}}{a_{ij1} - a_{ij2}} \quad \text{and} \quad h_{ij2} = \frac{a_{ij1} - f_{ij}(x(t))}{a_{ij1} - a_{ij2}}$$

$$\upsilon_{ik1} = \frac{g_{ik}(x(t)) - b_{ij2}}{b_{ij1} - b_{ij2}} \quad \text{and} \quad \upsilon_{ik2} = \frac{b_{ij1} - g_{ik}(x(t))}{b_{ij1} - b_{ij2}}.$$

using above term in equation (22) can be written as:

$$\dot{x}(t) = \Sigma_{j=1}^p \sum_{l=1}^2 f_{ijl}(x(t))\, x_j(t) + \Sigma_{k=1}^q \sum_{l=1}^2 g_{ijl}(x(t))\, u_k(t) \tag{23}$$

3.1. Stability of TS-Model

Design andstability analysis are presented in the form of linear matrix inequality (LMI), in LMI where variables are linearly related matrices [25,36]. Let us consider LMI as follows:

$$F(x) = F_0 + \sum_i x_i F_i > 0 \tag{24}$$

$x \in \Re^n$ $F_i = F_i^T : \forall i = 1, \dots, m,$ are symmetric matrix. $S = \{x : x \in \Re^n, F(x) > 0\}$ is known as feasible set and subset of $\Re^n$. A given matrix is congruence if there exists $P = P^T$ and a full rank matrix Q it holds $P > 0 \Rightarrow QPQ^T > 0$.

$$M = M^T \; M = M^T = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}^T \end{pmatrix}$$

M_{11} and M_{22} are square matrices. Then

$$M < 0 \Leftrightarrow \begin{cases} M_{11} < 0 \\ M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0 \end{cases} \Leftrightarrow \begin{cases} M_{22} < 0 \\ M_{11} - M_{12}^T M_{22}^{-1} M_{12} < 0 \end{cases}$$

$$F_i = F_i^T \; \forall i = 1, \dots, p \; x \in \Re^n \; x^T F_i x \geq 0 \; \forall i = 1, \dots, p$$

$$x^T F_i x > 0 \; x \neq 0 \; \tau_i > 0 : \forall i = 1, \dots, p \; F_0 - \sum_{i=1}^{p} \tau_i F_i > 0.$$

Also for given two matrices of X and Y of proper size $Q = Q^T > 0$, the following inequality holds

$$X^T Y + Y^T X \leq X^T Q X + Y^T Q^{-1} Y \tag{25}$$

any control problem is stable if it satisfy equation (26)

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i(\mathrm{z}) \omega_j(\mathrm{z}) \Upsilon_{ij}(\mathrm{z}) < 0: \tag{26}$$

Υ_{ij} is a symmetric matrix and $\omega_i(\mathrm{z}) > 0$ and $\sum_{i=1}^{n} \omega_i(\mathrm{z}) = 1$ and $\omega_i(\mathrm{z})$ and $\omega_j(\mathrm{z})$ satisfy commutative law s.t. $\omega_i(\mathrm{z})\omega_j(\mathrm{z}) = \omega_j(\mathrm{z})\omega_i(\mathrm{z})$, $\Upsilon_{ii} < 0$ and $\Upsilon_{ij} + \Upsilon_{ji} < 0$, $\forall i, j = 1, \dots, n$.

Lemma 2: from equation (26) is satisfied if following condition holds:

$$\Upsilon_{ii} < 0 \text{ and } \frac{2}{n-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0 : i = 1, \dots, m, j = 1, \dots, m \tag{27}$$

Lemma 3: Condition (26) satisfies provided that following conditions holds:

If there exist a matrix $Q_{ii} > 0$, $i = 1, \dots, m$, $\Upsilon_{ij} + \Upsilon_{ji} + Q_{ij} + Q_{ji} < 0$ (28)

$$\begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1m} \\ Q_{21} & Q_{22} & \cdots & Q_{2m} \\ Q_{m1} & Q_{m2} & \cdots & Q_{mm} \end{pmatrix} > 0$$

$$i = 1, \dots, m, \; j = 1, \dots, m.$$

4. QUADRATIC STABILITY

We consider quadratic Lyapunov function for stability of TS fuzzy model, the quadraticLyapunov function is:

$$f(x) = x^T P x, \text{ with } P = P^T > 0 \tag{29}$$

If $f(x)$ is quadratic stable means $f(x)$ is stable but converse need not be true. Therefore condition obtained in equation (29) is only sufficient. If we assume that $u=0$, $\dot{x}=\Sigma_{i=1}^{m}\omega_i(z)A_i x$, is asymptotic stable, if there exist a matrix $P = P^T$ such that following problem is feasible $PA_i + A_i^T P^T < 0$, for $i=1,\ldots,m$

4. SIMULATION RESULTS

In this section we will discuss some benchmark problem and numerical simulation to demonstrate the effectiveness of proposed TS-model. The main objective of proposed model is to design a control law such that output of the system can approximately track a given reference signal with significant error. For simulation of uncertain nonlinearsystem with MATLAB platform with ODE45 function.

Example 1: one-link manipulator with a BDC motor

For validation of proposed TS-model is tested on a popular benchmark problem, known as one-link manipulator actuated by brush dc (BDC) motor [29,30]. Differential equation of BDC motor is given in equation (25)

$$\left.\begin{aligned} D\ddot{q}+B\dot{q}+N\sin(q)&=I+\Delta_I \\ M\dot{I}&=-HI-K_m\,\dot{q}+V \end{aligned}\right\} \tag{30}$$

Where q, $\dot{q}$ and $\ddot{q}$ be the angular position, velocity and acceleration respectively.Whereas I denotes the motor current, Δ_I denotes the current disturbance and V is the input control voltage, values of parameter are given by: *D=1, B=1, M=0.05, H=0.5, N=10,* K_m *=10* and current disturbance Δ_I = *4sin(t)*. In equation (30) first part represent one-link robot and second part represents BDC electrical system. Convert equation (30) into the form of uncertain nonlinear system equation (1). Let $x_1=q,\ x_2=\dot{q},\ x_3=I$ and *u=V* then equation (30) can be expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -10\sin(x_1)-x_2+x_3+4\sin(t) \\ \dot{x}_3 &= -200x_2-10x_3+20u \\ y &= x_1 \end{aligned} \tag{26}$$

The reference signal is $y_r=(\pi/2)\sin(t)(1-\mathrm{e}^{-0.2t^2})$ and $x_1\in[-1,1]$, $x_2\in[-3,3]$ and $x_3\in[-5,5]$. Figure (1) and figure (2), shows the (t,x_1,x_2) space and tracking performance of TS model, where as figure (3) shows the tracking error.

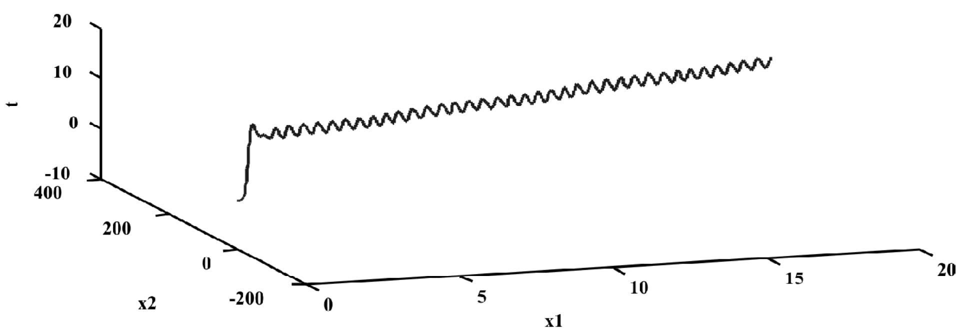

Figure (1): $(t,\ x_1,x_2)$ **space representation**

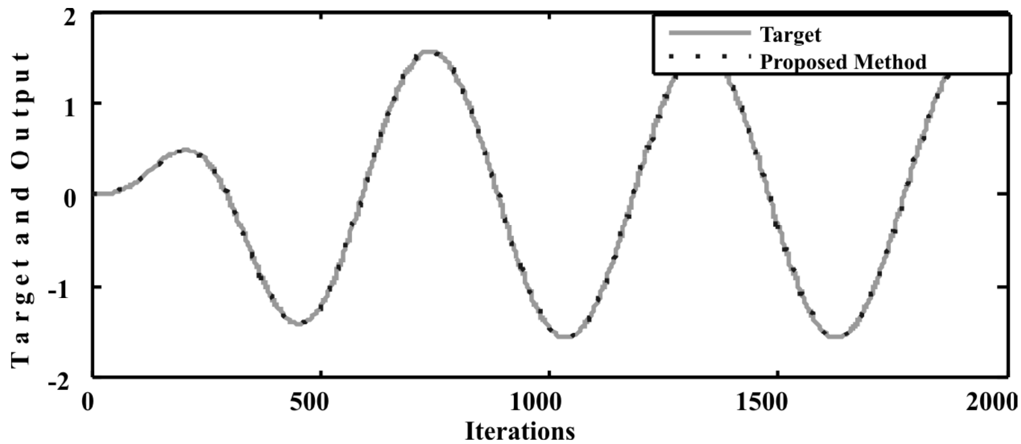

Figure (2): tracking performance of TS model output and reference signal

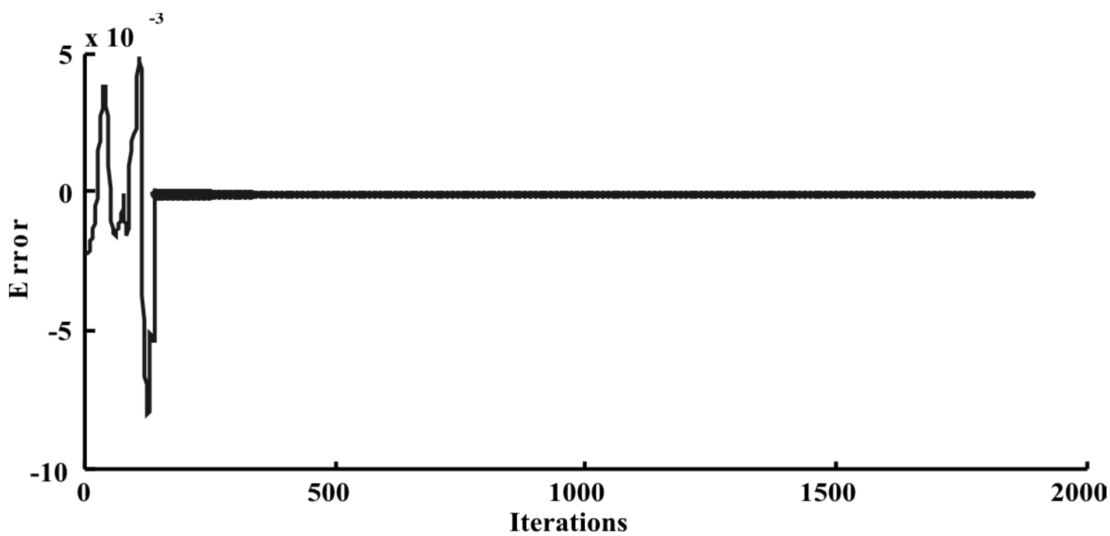

Figure (3): Trajectory of tracking error using proposed method

Example 2: Benchmark problem

In this example we will consider another important benchmark problem i.e. following second order uncertain nonlinear system in the general form

$$\dot{x}_1 = 0.5x_1 + (1 + 0.1x_1^2)x_2 + 0.1(1 - \cos^2(x_1x_2))$$
$$\dot{x}_2 = x_1x_2 + (2 + \cos(x_1))u + 0.2x_2\sin(x_1x_2)$$
$$y = x_1$$

The reference signal $y_r = \sin(t)$ for ODE45 the centers for $x_1 \in [-1,1]$ and $x_2 \in [-3,3]$ are evenly spaced.

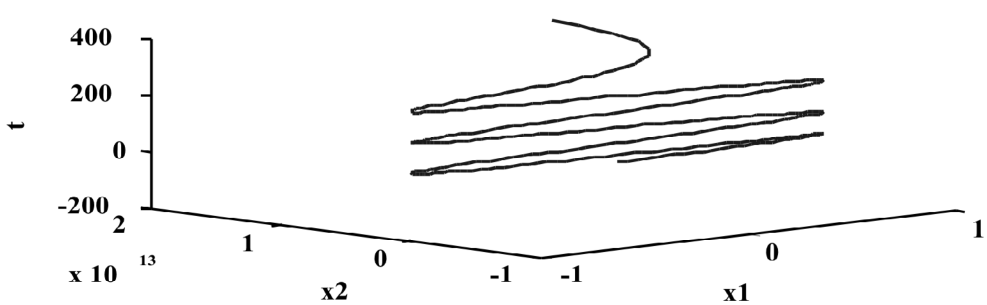

Figure 4: (t, x_1, x_2) **space representation**

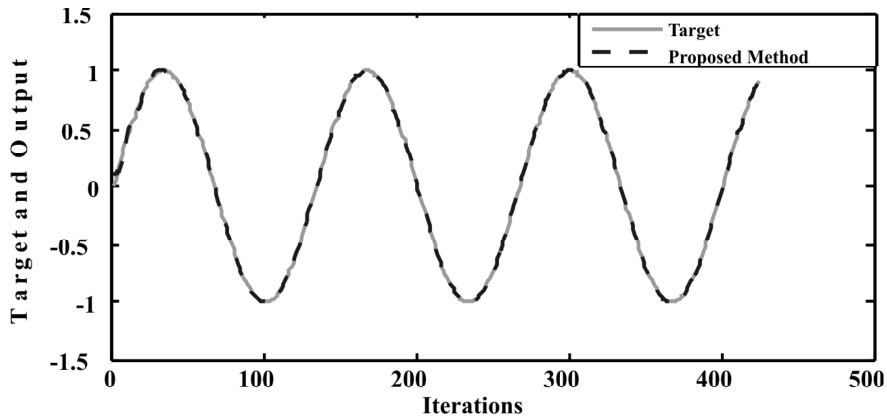

Figure 5: tracking performance of TS model output (dotted line) and reference signal (solid line)

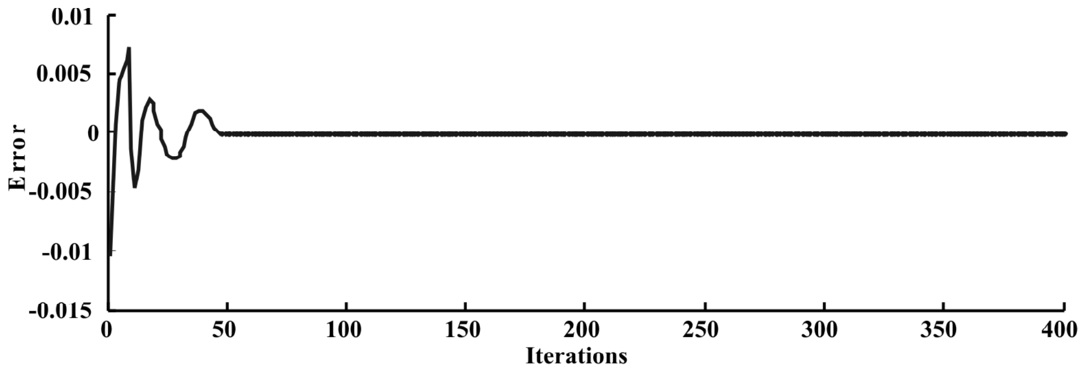

Figure 6: Trajectory of tracking error using proposed method

5. CONCLUSION

This paper has focused the problem of adaptive tracking error control of uncertain nonlinear systems. A systematic design with composition of DSC and TS-fuzzy model are proposed for a class of uncertain nonlinear system. The adaptive TS model based controller for identification of target has been discussed for control systems. The Lyapunov stability condition for multiple sliding surface and low pass filter shows that system is stable. The effectiveness of proposed algorithm has been discussed with some uncertain nonlinear problems. In future research work we will concentrates on stochastic nonlinear model with combination of metaheuristic algorithm and adaptive neural networks.

ACKNOWLEDGMENT

The authors would like to thank the editor and reviewers for their respected suggestions and productive comments which will really help us improve presentation and superiority of paper.

REFERENCES

[1] Y. Liu, C.L. Philip Chen, G. Wen, S. Tong, Adaptive neural output feedback tracking control for a class of uncertain discrete time nonlinear systems, IEEE Trans. Neural Netw., 22 (2011), 1162-1167.

[2] S. Ge., C. Hang, T. Lee, T. Zhang, Stable adaptive neural Network Control, Kluwer, Norwell, MA, USA, 2002.

[3] H. Nijmeijer, A. Vander Schaft, Nonlinear Dynamical Control Systems. Springer, NewYork (1990).

[4] M. Won, J.K. Hedrick, Multiple surface sliding control for a class of nonlinear systems, Int. J. Control 64(4), 693–706 (1996).

[5] C. Wen, Y.C. Soh, Decentralized adaptive control using integrator backstepping, Automatica 33(9), 1719–1724 (1997).

[6] M. Krsti´c, I. Kanellakopoulous, V.P. Kokotovi´c, Nonlinear and Adaptive Control Design Wiley, New York (1995).

[7] D. Swaroop, J.K. Hedrick, P.P. Yip, J.C. Gerdes, Dynamic surface control for a class of nonlinear systems, IEEE Trans. Autom. Control 45(10), 1893–1899 (2000).

[8] B. Song, J.K. Hedrick, Nonlinear observer design for Lipschitz nonlinear systems, In Proceedings of the American Control Conference (2011).

[9] S. Ge, C. Wang, Direct adaptive NN control of a class of nonlinear systems, IEEE Trans. Neural Netw., 13 (2002), 214-221.

[10] Y. Li, S. Qiung, X. Zhuang, O. Kaynak, Robust and adaptive backstepping control for nonlinear system using RBF neural networks, IEEE Trans. Neural Netw., 15 (2004), 693-701.

[11] H. Zhang, F. Yang, S. Liu, Q. Zhang, Stability analysis for neural network with time-varying delay based on quadratic convex combination, IEEE Trans. Neural Netw. Learn Syst., 24 (2013), 513-521.

[12] H. Zhang, Z. Wang, D. Liu, Global asymptotic stability of recurrent neural network with multiple time-varying delays, IEEE Trans. Neural Netw. 19 (2008) 855-873.

[13] H. Zhang, W. Yang, Stability analysis of Markovian jumping stochastic cohen-Grossberg neural network with mixed time delays, IEEE Trans. Neural Netw., 19 (2008), 366-370.

[14] Y. Wang, H. Zhang, Y. Wang, Fuzzy adaptive control of stochastic nonlinear systems with unknown virtual gain function, ActaAutom.Sin. 32 (2006), 170-178.

[15] V. Sakhare, U.P. Singh, S. Jain S, FCPN Approach for uncertain nonlinear dynamical systems with unknown disturbance, International Journal of Fuzzy System, Springer HeidelBerlegs, DOI: 10.1007/s40815-016-0145-5, 2016.

[16] Y. Yang, W. Chan, Adaptive dynamic surface control for uncertain nonlinear systems with interval type-2 fuzzy neural networks, IEEE Trans. Cybern, 44 (2014) 293-304.

[17] D. Wang, J. Huang, Neural network based adaptive dynamic surface control for a class of uncertain nonlinear system in strict feedback form, IEEE Trans. Neural Netw., 16 (2005) 195-202.

[18] E. Sanchez, T. Shibata, L.A. Zadeh, Genetic algorithm and fuzzy logic system, Soft Computing Perspective, World Scientific Singapore, 1997.

[19] C.F. Hsu, Self-organizing adaptive fuzzy neural network for a class of nonlinear system, IEEE Trans. Neural Netw., 18 (4) (2007) 1232-1241.

[20] S. Laboid, T. Guerra, Adaptive fuzzy control of a class of SISO nonaffine nonlinear systems, Fuzzy Set Syst., 158 (2007) 1126-1137.

[21] S.C. Tong, H.X. Li, W. Wang, Observer based adaptive fuzzy control for SISO nonlinear systems, Fuzzy Sets Syst. 148 (3) (2004) 355-376.

[22] M. Johansson, A. Rantzer, Computation of piecewise quadratic Lyapunov functions for hybrid systems. IEEE Trans,Autom Control 43(4), 555–559 (1998).

[23] Y. Yang, W. Chan, Adaptive dynamic surface control for uncertain nonlinear systems with interval type-2 fuzzy neural networks, IEEE Trans. Cybern, 44 (2014) 293-304.

[24] T. Li, D. Wang, G. Feng, S. Tong, A DSC approach to robust adaptive NN tracking control for strict-feedback nonlinear systems, IEEE Trans. Syst. Man. Cybern, Part B Cybern 40 (2010) 915-927.

[25] Song, B., Hedrick, J.K.: Observer-based dynamic surface control for a class of nonlinear systems: an LMI approach, IEEE Trans. Autom. Control 49(11), 1995–2001 (2004).

[26] D. Wang, J. Huang, Neural network based adaptive dynamic surface control for a class of uncertain nonlinear system in strict feedback form, IEEE Trans. Neural Netw., 16 (2005) 195-202.

[27] X. Wu, X. Wu, X. Luo, X. Guan, Dynamic surface control for a class of state constrained nonlinear systems with uncertain time delays, IET Control Theory Appl. 6 (2012) 1948-1957.

[28] S. Tong, Y. Li, G. Feng, T. Li, Observer based adaptive fuzzy backstepping dynamic surface control for a class of MIMO nonlinear systems, IEEE Trans. Syst. Man. Cybern, 41 (2011) 1124-1134.

[29] B. Xu, D. Wang, H. Wang, S. Zhu, Command filter based robust nonlinear control of hypersonic aircraft with magnitude constraints on state and actuators, J. Intell. Robot. Syst. 73 (2014) 219-231.

[30] T.P. Zhang, S.S. Ge, Adaptive dynamic surface control on nonlinear system with unknown deadzone in pure feedback form, Automatica 44(7) 2008, 1895-1903.

[31] S.C. Tong, Y.M. Li, P. Shi, Observer based adaptive fuzzy backstepping output feedback control of uncertain MIMO pure feedback nonlinear systems, IEEE Trans. on fuzzy syst. 20(4) 2012, 771-785.

[32] J.W. Grizzle, G. Abba, F. Plestan, Asymptotically stable walking for biped robots: analysis via systems with impulse effects. IEEE Trans. Autom. Control 46(1), 51-64 (2001).

[33] J.K. Hedrick, M.C. Won, S.B. Choi, Fuel injection control of automotive engines. In: Proceedings of IEEE CDC, Buena Vista, FL (1994).

[34] J. H. Green, J.K. Hedrick, Nonlinear speed control of automotive engines. In: Proceedings of the American Control Conference, San Diego, CA, pp. 2891-2897 (1990).

[35] A. Sala, T.M. Guerra, R. Babuska, Perspectives of fuzzy systems and control, Fuzzy Sets and Systems 156(3), 432-444 (2005).

[36] K. Tanaka, T. Hori, H. Wang, A multiple Lyapunov function approach to stabilization of fuzzy control systems, IEEE Transactions on Fuzzy Systems 11(4), 582-589 (2003).

[37] J. Carroll, D. Dowson, Integrator Backstepping technique for the tracking control of permanent magnet brush dc motor, IEEE Trans. Ind. Appl., 31 (1995), 248-255.

[38] T.P. Zhang, H. Wen, Q. Zhu, Adaptive fuzzy control of nonlinear system in pure feedback form based on input to state stability, IEEE Trans. on fuzzy syst. 18(2) 2010, 1-13.