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OPERATIONS ON GEAR GRAPH WITH TOPOLOGICAL INDICES

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Abstract: *In this article, we studied gear graph for applying different operators viz., derived graph, double graph, line graph, line graph of subdivision graph, line graph of double graph. Also some general form for operation of gear graph established topological indices namely, first and second Zagreb index, harmonic index, Randic index, symmetric division deg index and some relation between certain topological indices.*

Key Words: *Derived graph, double graph, line graph, subdivision graph, gear graph, topological index.*

AMS Subject Classification: *05C05, 05C12, 05C75*

1. INTRODUCTION AND PRELIMINARIES

A systematic study of topological indices is one of the most striking aspects in many branches of mathematics with its applications and various other fields of science and tech- nology. Many different topological indices have been investigated so far, most of the useful topological indices are distance based or degree based. This indices may be used to derive the quantitative structure property relationship (QSPR) or quantitative structure activity rela- tionship (QSAR). Let $G = (V, E)$ be a simple graph with vertex set $V(G) = v_1, v_2, \ldots, v_n$ and edge set $E(G) = e_1, e_2, \ldots, e_n$ and its vertex set be the cardinalities of $V(G)$ and $E(G)$ are called the order and size of G respectively. Here d_u denotes degree of vertex u and $d(u, v)$ represent the length of the shortest path between any two vertices u and v connected with each other. We recall well known definitions of the first and second Zagreb indices, have been introduced more than thirty years ago by I. Gutman and Trinajstic [10]. They defined as,

$$M_1(G) = \sum_{u \in V(G)} d_u^2$$

$$M_2(G) = \sum_{uv \in (G)} d_u . d_v .$$

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Where d_u and d_v are the degree of u and v. These definitions and related work see, [5, 13, 20, 21].

There are many topological indices defined on the basis of the vertex-degree of graph one of the vertex-degree based index namely harmonic index $H(G)$ is first introduced in [4], defined as,

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v}.$$

For more results on harmonic index we refer to the articles [16, 23, 24, 27].

The connectivity index introduced in 1975 by Milan Randic [19], who has shown this index to reflect molecular branching, the randic index was defined as,

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u . d_v}}.$$

The results on the randic index we refer the articles [9, 14].

Among 148 discrete adriatic indices [1, 3] we considered symmetric division deg discrete adriatic index. For collection of recent results on [11], the symmetric division deg index is defined as,

$$SDD(G) = \sum_{uv \in E(G)} \frac{d_u^2 + d_v^2}{d_u . d_v}.$$

In 1981, Bertz introduced the first topological index on the basis of the line graph in [2]. For more details about the applications of line graph in chemistry, we refer the article [6, 7, 8].

The line graph $L(G)$ is the graph whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common.

The subdivision graph $S(G)$ is the graph obtained by replacing each of its edge by a path of length 2 or equivalently, by inserting an additional vertex into each edge of G, we refer the articles [20, 21, 22].

The derived graph $(G)^{\dagger}$ of the graph G is the graph having the same vertex set as G, two vertices of $(G)^{\dagger}$ being adjacent if and only if their distance in G is two, we refer the article [12, 25].

Munarini *et al.* [18], defined the double graph of a simple graph denoted as $D(G)$. The double graph of a simple graph G can be build up taking two distinct copies of the graph G and joining every vertex V in one copy to every vertex W' in the other copy corresponding to a vertex W adjacent to V in the first copy. In this paper we study some distance based topological indices for double graph also we refer the articles [15, 17].

The Gear graph [26] is a wheel graph with a graph vertex added between each pair of adjacent graph vertex of the outer cycle. The gear graph G_n has $2n + 1$ nodes and $3n$ edges for $n \geq 3$.

For a collection of recent results on degree-based topological indices, we refer to the articles [7, 5, 13, 15].

In this paper, we considered gear graph and its derived graph, double graph, line graph, line graph of subdivision graph, line graph of double graph applying to different topological indices and obtained results.

The construction of paper is organized as follows:

Section 1, consists of introduction and essential definitions which is necessary for the main results. section 2, contains topological indices of gear graph of derived graph, section 3, contains topological indices of gear graph of double graph and final section will consists a operations of gear graph.

2. TOPOLOGICAL INDICES OF GEAR GRAPH OF DERIVED GRAPH

In this segment, we concentrated basic results on the derived graph related to topological indices.

Theorem 2.1. The first Zagreb index of a derived graph of gear graph is

$$M_1((G)_n{}^{\dagger}) = 4\pi^2(2n+1) \ for\ n \geq 3.$$

Proof: Let G_n be a gear graph and $(G)^{\dagger}$ be the derived graph of gear graph and 'n' is the vertices of graph G_n. When $n \geq 3$, the graph contains $2n + 1$ vertices of degree $2n$ and total edges present in that graph is $2n^2 + n$ and we obtain the required result.

Theorem 2.2. The second Zagreb index, harmonic index, randic index and symmetric division deg index of gear graph of derived graph is for $n \geq 3$,

$$M_2((G)_n{}^{\dagger}) = 4\pi^3(2n+1)$$

$$H((G)_n{}^{\dagger}) = \frac{2n+1}{2} = R((G)_n{}^{\dagger})$$

$$SDD((G)_n{}^{\dagger}) = 2n(2n+1).$$

Proof: Let G_n be a gear graph and $(G)_n{}^{\dagger}$ be the derived graph of gear graph and 'n' is the vertices of graph G_n. When $n \geq 3$, the edges are of the type $(2n, 2n)$ and the total edges present in that graph is $2n^2 + n$ edges and applying above topological indices we obtain required results.

Lemma 2.1. The inter-relation between the first and second Zagreb index, harmonic index, randic index and symmetric division deg index of gear graph of derived graph is for $n \geq 3$,

$$M_1\,((G)_n^{\dagger}) = 4n^2\,(2n + 1)$$

$$M_2\,((G)_n^{\dagger}) = nM_1\,((G)_n^{\dagger})$$

$$H((G)_n^{\dagger}) = \frac{M_1((G)_n^{\dagger})}{8\pi^2} = R((G)_n^{\dagger})$$

$$SDD(G^{\dagger}) = \frac{M_1((G)_n^{\dagger})}{2n}.$$

3. TOPOLOGICAL INDICES OF GEAR GRAPH OF DOUBLE GRAPH

In this segment, we derived the gear graph of double graph relationship.

Theorem 3.1. The first Zagreb index of a double graph of gear graph is $M_1\,(D(G_n)) = 8n(n + 13)$ for $n \geq 3$.

Proof: Let G_n be a gear graph and $G(D(G_n))$ be the double graph of gear graph and 'n' is the vertices of graph G_n. When $n \geq 3$, the graph contains $2n$ vertices of degree 6, $2n$ vertices of degree 4, 2 vertices of degree $2n$ and total vertices present in the graph is $4n + 2$. While applying to definition of the first Zagreb index we obtain the required result.

Theorem 3.2. The second Zagreb index, harmonic index, randic index and symmetric division degree index of gear graph of double graph for $n \geq 3$.

$$M_2\,(D(G_n)) = 48n(n + 4)$$

$$H(D(G_n)) = \frac{4n(2n+1)}{5(n+3)}$$

$$R(D(Gn)) = \frac{4n}{\sqrt{6}} + \frac{2\sqrt{n}}{\sqrt{3}}$$

$$SDD(D(G_n)) = \frac{52n + 4n^2 + 36}{3}.$$

Proof: Let G_n be a gear graph and G' be a copy of gear graph. Then $D(G_n)$ is a double graph of gear graph and 'n' is the vertices of graph G_n for $n \geq 3$, the total edges present in that graph is $12n$ edges. When $n = 3$ the edges are of the type $(n + 3, n + 1)$ and $(n + 3, n + 3)$, when $n = 4$ the edges are of the type $(n + 2, n)$ and $(n + 2, n + 4)$, when $n = 5$ the edges are of the type $(n + 1, n–1)$ and $(n + 1, n + 5)$, when $n = 6$ the edges are of the type $(n, n – 2)$ and $(n, n + 6)$. Taking into consideration of edge partition and applied to definitions of M_2 $(D(G_n))$, $H(D(G_n))$, $R(D(G_n))$, $SDD(D(G_n))$ and we obtained above the results.

4. OPERATIONS OF GEAR GRAPH

In this segment, we discussed gear graph of line graph, gear graph of line graph of subdivision graph and gear graph of line graph of a double graph.

Theorem 4.1. The first Zagreb index of a line graph of gear graph is

$$M_1 (L(G_n)) = n^3 + 2n^2 + 19n \text{ for } n \geq 3.$$

Proof: Let G_n be a gear graph and $L(G_n)$ be the line graph of gear graph and 'n' is the vertices of graph G_n. When $n \geq 3$, the graph contains $2n$ vertices of degree 3, n vertices of degree 4 and total vertices present in the graph is $3n$, applying to the definition of first Zagreb index we obtain the result.

Theorem 4.2. The second Zagreb index, harmonic index, randic index and symmetric divi sion degree index of gear graph of line graph for $n \geq 3$,

$$M_2 (L(G_n)) = 3(n^3 + 2n^2 + 5n – 2)$$

$$H(L(G_n)) = \frac{2n}{3} + \frac{3(n-2)}{n+1} + \frac{4n}{n+4}$$

$$R(L(G_n)) = \frac{2n}{3} + \frac{3n-6}{n+1} + \frac{2n}{\sqrt{3(n+1)}}$$

$$SDD (L(G_n)) = 4n + 6(n – 2) + 2n\left[\frac{n^2+2n+10}{3(n+1)}\right].$$

Proof: Let G_n be a gear graph and $L(G_n)$ be the line graph of gear graph and 'n' is the vertices of graph G_n. When $n \geq 3$ and the total edges present in that graph is $7n – 6$ edges. When $n = 3$ the edges of type (n, n), $(n + 1, n + 1)$ and $(n, n + 1)$, when $n = 4$ the edges of type $(n – 1, n – 1)$, $(n + 1, n + 1)$ and $(n – 1, n + 1)$ and when $n = 5$ the edges of type $(n – 2, n – 2)$, $(n + 1, n + 1)$ and $(n – 2, n + 1)$ with these cardinalities taking into consideration and applied to definitions of M_2 $(L(G_n))$, $H(L(Gn))$, $R(L(G_n))$, $SDD(L(G_n))$ and we obtained required results.

Theorem 4.3. The first Zagreb index of line graph of subdivision graph of gear graph is $M_1\left(L(S(G_n))\right) = n^3 + 4n^2 + 33n \ \ for\ n \geq 3.$

Proof: Let G_n be a gear graph and $L(S(G_n))$ be the line graph of subdivision graph of gear graph and 'n' is the vertices of graph G_n, the total vertices present in that graph is $\frac{n^2+13}{2}$, when $n = 3$ the graph containing $(n-1, n, n+1)$ degree of vertices, when $n = 4$ the graph containing $(n-2, n-1, n+1)$ degree of vertices, when $n = 5$ the graph containing $(n-3, n-2, n+1)$ degree of vertices and we obtained required result of M_1 $(L(S(G_n)))$.

Theorem 4.4. The second Zagreb index, harmonic index, randic index an symmetric division degree index of line graph of subdivision graph of gear graph is for $n \geq 3$,

$$M_2\,(L(S(G_n))) = 2(2n^3 - 5n^2 + 26n + 15)$$

$$H\left(L(S(G_n))\right) = \frac{12n}{5} + \frac{4n^2 - 14n + 30}{n+3}$$

$$R\left(L(S(G_n))\right) = \frac{6n}{\sqrt{6}} + \frac{2n^2 - 7n + 15}{\sqrt{2n+2}}$$

$$SDD\left(L(S(G_n))\right) = \frac{2n^4 - 3n^3 + 37n^2 + 21n + 75}{2n+2}.$$

Proof: Let G_n be a gear graph and $L(S(G_n))$ be the line graph of subdivision graph of gear graph and 'n' is the vertices of graph G_n, total edges are present in a graph is $2n^2 - n + 15$, When $n = 3$ in $L(S(G_n))$ the edges of type $(n, n-1)$ and $(n + 1, n-1)$, when $n = 4$ in $L(S(G_n))$ the edges of type $(n-1, n-2)$ and $(n+1, n-2)$, when $n = 5$ in $L(S(Gn))$ the edges of type $(n-2, n-3)$ and $(n+1, n-3)$, When $n = 6$ in $L(S(G_n))$ the edges of type $(n-3, n-4)$ and $(n+1, n-4)$ and Taking into consideration of edge partition and applied to definitions of $M_2\left(L(S(G_n))\right)$, $H\left(L(S(G_n))\right)$, $R\left(L(S(G_n))\right)$, $SDD\left(L(S(G_n))\right)$ and we obtained above the results.

Theorem 4.5 The first Zagreb index of line graph of double graph of gear graph is

$$M_1\left(L(D(G_n))\right) = 8n(n^2 + 2n + 19) \;\; for \;\; n \geq 3.$$

Proof: Let G_n be a gear graph and $L(D(G_n))$ be the line graph of double graph of gear graph and 'n' is the vertices of graph G_n total vertices present in that graph is $6n$ and the graph containing 6 and $2n + 2$ degree of vertices. Then applying the first Zagreb index we obtain the above result.

Theorem 4.6. The second Zagreb index, harmonic index, randic index and symmetric division degree index of line graph of double graph of gear graph for $n \geq 3$,

$$M_2\,(L(D(G_n))) = 8n(n^3 + n^2 + 11n\ \ +47)$$

$$H\left(L(D(G_n))\right) = \frac{7n^3 + 53n^2 + 28n}{3(n+4)(n+1)}$$

$$R\left(L(D(G_n))\right) = \frac{4n}{3} + \frac{4n}{\sqrt{3(n+1)}} + \frac{n(n-1)}{n+1}$$

$$SDD\left(L(D(G_n))\right) = \frac{20n^3 + 64n^2 + 116n}{3(n+1)}.$$

Proof: Let G_n be a gear graph and $L(D(G_n))$ be the line graph of double graph of gear graph and 'n' is the vertices of graph G_n total edges are present in a graph is $2n(n + 7)$, when $n = 3$ in $L(D(G_n))$ the edges of type $(n + 3, n + 3)$, $(n + 3, 2n + 2)$ and $(2n + 2, 2n + 2)$, When $n = 4$ in $L(D(G_n))$ the edges of type $(n + 2, n + 2)$, $(n + 2, 2n + 2)$ and $(2n + 2, 2n + 2)$, When $n = 5$ in $L(D(G_n))$ the edges of type $(n + 1, n + 1)$, $(n + 1, 2n + 2)$ and $(2n + 2, 2n + 2)$, When $n = 6$ in $L(D(G_n))$ the edges of type (n, n), $(n, 2n + 2)$ and $(2n + 2, 2n + 2)$ and applying indices we obtained above the results.

CONCLUSION

Here, we discussed the gear graph structural operations for certain degree based topological indices for derived graphs. This type graph components may be utilized in the chemical phenomena.

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