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OPTIMIZATION OF CONSTRAINED MULTI-ITEM FUZZY INVENTORY PROBLEMS USING GENETIC ALGORITHM

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Abstract: This paper proposes the strategy of optimizing constrained multi-item inventory problems under fuzzy environment using a Genetic Algorithm (GA). The GA is used in the sense that it is computationally simple yet powerful in its search for improvement. The typical inventory analysis is sensitive to reasonable errors in the measurement of relevant inventory costs. Therefore the inventory costs are assumed to be vague and imprecise in this paper. The objective of minimizing the total inventory cost and the constraints' goals are also imprecise in nature. The impreciseness in these variables has been represented by fuzzy linear membership functions. Numerical examples have been worked out to highlight the method, and the results are compared with those of corresponding crisp models. Sensitivity analysis has also been presented for one of the examples.

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Keywords: Inventory, Fuzzy variable, Membership function, Genetic Algorithm.

1. INTRODUCTION

One most important difference among various inventory problems is the question of the relevant costs which will enter into the analysis. The resolution of the cost measurement problem depends very much on the kinds of company records available. In practice, some of the costs may be inadequately determined directly from cost accounting records. Therefore the inventory costs may be flexible with some vagueness in their values. These imprecise and vague parameters are treated as fuzzy in nature.

Ever since Zadeh [1965] developed the concept of fuzzy set theory, a few authors have exhibited their interests in topics of fuzzy mathematical programming (Zimmermann, 1985). Trappey, *et al.*, [1988] applied the theory of fuzzy nonlinear programming in manufacturing problems with fuzzy goal and constraints. Yao and Lee [1998] developed an EOQ model by considering order quantity as fuzzy and allowing shortages. Yao and Su [2000] developed fuzzy inventory with backorder for fuzzy total demand based on interval valued fuzzy set. Venkata Subbaiah, *et al.*, [2000] considered EOQ model with shortages in fuzzy environment by adopting the inventory costs as fuzzy variables. In all these cases, single item inventory models are considered in fuzzy environment.

In constrained multi-item inventory problems, it is required to optimize the objective and satisfy a set of constraints on available resources. There are a variety of multi-item models that have been studied in the literature, e.g., Rosenblatt and Rothblum [1990], Anily [1991], Gallego, *et al*., [1996], Page and Paul [1976], Klein, *et al*., [1989], Aggarwal and Park [1993], Gupta and Keung [1990], Afentakis, *et al*., [1984]. These models stem as extensions of the classical EOQ models, and most of the available algorithms use heuristic techniques to solve the problems approximately.

Generally in inventory systems, only linguistic (Vague) statements are used to describe the problem and it may not be possible to define the objective and constraints' goals precisely, for example, at the beginning of a business/production, normally a target/limitation for the objective (profit/total cost) is fixed. But, during the course of action, a retailer/producer is forced to settle down with a higher/lower amount due to an adverse situation. The same may be happened with respect to the constraints' goals. Hence, the objective and constraint goals are imprecise, i.e., they may be within some limits, and may be better described in a fuzzy environment.

In this paper, constrained multi-item lot sizing problems are formulated in fuzzy environment. The fuzzy concept is considered for ordering costs, carrying costs, unit costs, and the limitations on total cost and constraints imposed. The impreciseness in these variables is represented by linear membership functions. A real coded Genetic Algorithm is used to solve the problems because GAs have much more global perspective than many traditional optimization techniques. The methodology is illustrated numerically and the results are compared with those obtained by crisp analysis. The sensitivity analysis with respect to different values of parameters is also discussed.

2. PROBLEM FORMULATION

In a constrained multi-item inventory optimization problem, the purpose is to obtain the optimal quantities of the items that minimize the total cost of inventory and satisfy a set of constraints. The notations assumed are:

n = number of inventory items (or products).

m = number of constraints.

z_i = demand of item $-i$ per unit time (assumed to be known and constant).

c_{ri} = setup cost or order cost of item $-i$ during an order interval.

c_i = Raw material and direct labour cost per unit of item $-i$.

I = Inventory holding charges expressed as decimal fraction of the inventory value/unit time.

q_i = quantity of item $-i$ produced or ordered.

L_j = Limiting value of j^{th} constraint imposed.

2.1. Crisp Formulation

The problem with the above notations can be formulated mathematically as

Minimize total cost:

$$f = \Sigma_{i=1 \rightarrow n} (z_i c_{ri}/q_i + q_i I c_i/2 + z_i c_i) \tag{2.1}$$

Subject to constraints:

$$h_j(\mathbf{x}) - \mathbf{L_j} \leq \mathbf{0} \qquad \text{for} \qquad \mathbf{j = 1, 2, ..., m} \tag{2.2}$$

$$\mathbf{x} = (\mathbf{q_1} \;\; \mathbf{q_2} \;\; \mathbf{...} \;\; \mathbf{q_n})^{\mathbf{T}} \geq \mathbf{0}$$

The different practical constraints that may encounter with respect to multi-item lot sizing problems are constraints on production facilities, storage facilities, time and money.

2.2. Linear Membership Function

A membership function $\mu_{Ai}(x)$, assumed to be linearly increasing over the tolerance interval p_i can be expressed according to Zimmermann [1991] as:

$$\mu_{Ai}(x) = \begin{cases} 1 & \text{for} \quad x < d_i \\ 1 - (x - d_i)/p_i & \text{for} \quad d_i \leq x \leq (d_i + p_i) \\ 0 & \text{for} \quad x > (d_i + p_i) \end{cases} \tag{2.3}$$

where d_i and $(d_i + p_i)$ are the tolerance limits for x.

Introducing a new variable, α, which corresponds essentially to $\mu_{Ai}(x)$, the corresponding fuzzy variable x at the defined aspiration level α is given by:

$$\mu_{Ai}^{-1}(\alpha) = d_i + (1 - \alpha)\, p_i \tag{2.4}$$

Similarly a membership function $\mu_{Bj}(x)$, assumed to be linearly decreasing over the tolerance interval p_j can be expressed as:

$$\mu_{Bj}(x) = \begin{cases} 1 & \text{for} \quad x > d_j \\ 1 - (d_j - x)/p_j & \text{for} \quad (d_j \pm p_j) \leq x \leq d_j \\ 0 & \text{for} \quad x < (d_j \pm p_j) \end{cases} \tag{2.5}$$

Hence, $$\mu_{Bj}^{-1}(\alpha) = d_j - (1 - \alpha)\, p_j$$

2.3. Fuzzy Formulation

The fuzzy set concepts are adopted for ordering costs, holding costs, unit costs, and the limitations on total cost and constraints imposed. The impreciseness in these variables have been expressed by linear membership functions. Considering the nature of the variables, the membership functions are assumed to be non-decreasing for fuzzy inventory costs, and non-increasing for fuzzy total cost. The membership functions of the constraints' goals or limitations are assumed to be non-decreasing or non-increasing depending on their nature of variation. On applying fuzzy non-linear programming approach to the crisp model, the formulation is:

Maximize: α

Subject to:

$$\Sigma_{i=1 \to n} (z_i \, \mu_{cri}^{-1}(\alpha) + q_i \, \mu_I^{-1}(\alpha) \, \mu_{ci}^{-1}(\alpha)/2 + z_i \, \mu_{ci}^{-1}(\alpha)) - \mu_f^{-1}(\alpha) \leq 0 \quad (2.6)$$

That is,

$$\Sigma (z_i(c_{ri} - (1-\alpha)\,p_{cri})/q_i + q_i(I - (1-\alpha)\,p_I)\,(c_i - (1-\alpha)\,p_{ci})/2 + z_i\,(c_i - (1-\alpha)\,p_{ci})) - (f + (1-\alpha)\,p_f) \leq 0 \quad (2.7)$$

and

$$h_j(\mathrm{x}) - \mu_{\mathbf{Lj}}^{-1}(\alpha) \leq \mathbf{0} \quad \text{for} \quad \mathbf{j = 1, 2, ..., m} \quad (2.8)$$

$$\mathbf{x = (q_1 \;\; q_2 \;\; ... \;\; q_n)^T \geq 0}$$

$$0 \leq \alpha \leq 1.$$

3. GENETIC ALGORITHM

GAs are theoretically and empirically proven to provide robust search in complex spaces [Gold berg, 1999]. The decision variables in a GA are represented in binary strings or real code. But the real coded GAs bring the GA a step closer to classical optimization techniques [Deb, 2002]. The initial population of solutions is created by random selection of a set of chromosomes (solutions). Once a chromosome is created, it is necessary to evaluate the solution, particularly in the context of the underlying objective and constraint functions. The evaluation of a solution means calculating the objective function value and constraint violations. After assigning a relative merit to the solutions (called the fitness), the population of solutions is modified to create hopefully a better population. In this process the three main operators, viz., reproduction, crossover, and mutation are used. This completes the generation of the GA. The following algorithm shows the working principle of the GA implemented.

```
Procedure Genetic Algorithm
begin
    t = 0
    Initialize population : t
    Evaluate population : t
    while (not terminate-condition) do
    begin
        t = t + 1
        reproduction
        crossover
        mutation
        evaluate population : t + 1
    end
end
```

The real coded GA used in the present study implements a tournament selection scheme, where two solutions are compared and the best is selected. Crossing over is done by simulated binary crossover (SBX) operator which works with two parent solutions and creates two offspring [Deb and Agrawal, 1995]. Mutation is done by the polynomial mutation operator [Deb and Goyal, 1996]. The exponents used for SBX and mutation are respectively 2 and 100. The GA parameters-crossover probability (p_c), mutation probability (p_m), population size (s_p) and number of generations (n_g)-used in all the simulation runs are $p_c = 0.9$, $p_m = 0.1$, $s_p = 50$, $n_g = 30$.

4. NUMERICAL ILLUSTRATIONS AND DISCUSSION

To illustrate the proposed methodology, three different examples are considered.

Example 1: A multi-item inventory problem with two products 1 and 2 and two constraints on lot sizes with respect to available warehouse space and the machine setup time has the following numerical data:

z_1 = 200 units/month, z_2 = 400 units/month.

c_1 = \$12/unit, c_2 = \$7/unit,

c_{r1} = \$100/lot, c_{r2} = \$25/lot,

I = 0.005/month (0.5 % per month) f = \$5200/month

a_1, a_2 = Cubic feet of space required for storing one unit of products –1 and 2 = 5 ft^3, 35 ft^3 respectively.

t_1, t_2 = Time required per setup for products –1 and 2 = 40 hrs, 10 hrs respectively.

A = Total average available space excluding aisles, etc = 14000 ft^3.

T = Total available time for setups = 14 hrs.

The maximum acceptable violations of c_{r1}, c_{r2}, I, c_1, c_2, f, A and T are: $p_{cr1} = 25$; $p_{cr2} = 12.5$; $p_I = 0.25\%$; $p_{c1} = 5$; $p_{c2} = 2.5$; $p_f = 500$; $p_A = 2500$; $p_T = 2.5$.

The problem formulation in crisp consists of the objective function mentioned in section-2.1 and the following constraints:

Warehouse space restriction:

$$\Sigma_{i=1 \to n} a_i q_i/2 - A \leq 0 \tag{4.1}$$

Setup time restriction:

$$\Sigma_{i=1 \to n} z_i t_i/q_i - T \leq 0 \tag{4.2}$$

$$q_i^l \leq q_i \leq q_i^u$$

Assuming the membership functions of both the constraints' limitations to be non-increasing and applying fuzzy non-linear programming approach to the crisp formulation (section- 2.3), the results of corresponding fuzzy formulation obtained by GA are furnished in Table 1. It also shows the corresponding crisp model results. Table 2 presents the crisp analysis of this problem for fixed values of constraints limitations and different combinations of extreme values of each of the costs (c_{r1}, c_{r2}, I, c_1, c_2). It is observed that the total cost of fuzzy model falls within its range ($5200 to $5700) and among crisp models, the total cost of only 8 out of 32 models fall within the range. If the parametric studies on the crisp model are made, one of these studies will coincide with the optimum values of fuzzy model, but it is a laborious process.Thus fuzzy analysis replaces the laborious and time consuming parametric studies and obtains optimum results easily. The sensitivity of optimum solution with respect to changes in fuzzy variables is shown in Tables 3 to 6.

Table 1
Comparision of Crisp and Fuzzy Model Results

Example	*Model*	*Order quantities (units)*	*f($)*	α
1	Crisp	q_1 = 1022.52; q_2 = 650.85	5276.98	–
	Fuzzy	q_1 = 1060.01; q_2 = 645.18	5215.43	0.9691
2	Crisp	q_1 = 275.89; q_2 = 1415.53; q_3 = 100.09	2.0827 Millions	–
	Fuzzy	q_1 = 330.84; q_2 = 1371.41; q_3 = 159.44	2.0717 Millions	0.9566
3	Crisp	q_1 = 227.58; q_2 = 216.33; q_3 = 183.51	73,536.54	–
	Fuzzy	q_1 = 218.86; q_2 = 209.79; q_3 = 189.41	73,166.60	0.9762

Table 2
Crisp Analysis of Example 1

c_{r1} (\$)	c_{r2} (\$)	I (%)	c_1(\$)	c_2(\$)	q_1 (units)	q_2 (units)	f(\$)
100	25.0	0.50	12	7.0	1013.31	655.24	5276.86
100	25.0	0.25	12	7.0	1078.34	645.88	5255.85
100	25.0	0.50	12	4.5	1015.49	654.66	4272.79
100	25.0	0.25	12	4.5	1061.24	648.37	4253.83
100	25.0	0.50	7	7.0	1041.05	651.20	4264.18
100	25.0	0.25	7	7.0	1332.31	609.66	4248.41
100	25.0	0.50	7	4.5	1015.49	654.66	3260.11
100	25.0	0.25	7	4.5	1312.01	612.56	3246.49
100	12.5	0.50	12	7.0	1015.49	654.66	5269.25
100	12.5	0.25	12	7.0	1137.24	637.53	5248.06
100	12.5	0.50	12	4.5	1015.49	654.66	4265.16
100	12.5	0.25	12	4.5	1120.31	639.95	4246.06
100	12.5	0.50	7	7.0	1118.11	584.42	4256.23
100	12.5	0.25	7	7.0	1444.94	593.56	4240.10
100	12.5	0.50	7	4.5	1067.07	647.56	3252.42
100	12.5	0.25	7	4.5	1416.43	597.63	3238.24
75	25.0	0.50	12	7.0	1015.49	654.66	5271.96
75	25.0	0.25	12	7.0	1015.49	654.66	5251.00
75	25.0	0.50	12	4.5	1015.49	654.66	4272.79
75	25.0	0.25	12	4.5	1015.49	654.66	4248.96
75	25.0	0.50	7	7.0	1015.49	654.66	4259.27
75	25.0	0.25	7	7.0	1167.99	633.12	4244.39
75	25.0	0.50	7	4.5	1015.49	654.66	3255.18
75	25.0	0.25	7	4.5	1134.62	637.86	3242.41
75	12.5	0.50	12	7.0	1015.49	654.66	5264.33
75	12.5	0.25	12	7.0	1015.49	654.66	5343.36
75	12.5	0.50	12	4.5	1015.49	654.66	4260.23
75	12.5	0.25	12	4.5	1015.49	654.66	4241.32
75	12.5	0.50	7	7.0	1043.36	631.67	4251.61
75	12.5	0.25	7	7.0	1264.43	619.36	4236.41
75	12.5	0.50	7	4.5	1015.49	654.66	3247.54
75	12.5	0.25	7	4.5	1239.75	622.87	3234.47

Table 3
Sensitivity Analysis of p_I, p_{cr1}, p_{cr2}, p_{c1}, p_{c2} of Example 1

$p_I(\%)$	$I(\%)$	q_1	q_2	f	A	T	α
0.00	0.5000	1060.01	645.18	5215.43	14077.12	14.077	0.96915
0.15	0.4954	1060.01	645.18	5215.43	14077.12	14.077	0.96915
0.25	0.4923	1060.01	645.18	5215.43	14077.12	14.077	0.96915
0.30	0.4907	1060.01	645.18	5215.43	14077.12	14.077	0.96915
0.40	0.4877	1151.85	633.64	5215.40	14077.00	14.077	0.9692
p_{cr1}	c_{r1}	q_1	q_2	f	A	T	α
00	100.00	1060.01	645.18	5215.43	14077.12	14.077	0.96915
25	99.22	1060.01	645.18	5215.43	14077.12	14.077	0.96915
30	99.07	1060.01	645.18	5215.43	14077.12	14.077	0.96915
50	98.45	1060.01	645.18	5215.43	14077.12	14.077	0.96915
70	97.84	1151.85	633.64	5215.40	14077.00	14.077	0.9692
p_{cr2}	c_{r2}	q_1	q_2	f	A	T	α
0.0	25.00	1060.01	645.18	5215.43	14077.12	14.077	0.96915
5.0	24.84	1060.01	645.18	5215.43	14077.12	14.077	0.96915
15.0	24.53	1060.01	645.18	5215.43	14077.12	14.077	0.96915
20.0	24.38	1060.01	645.18	5215.43	14077.12	14.077	0.96915
25.0	24.23	1151.85	633.64	5215.40	14077.00	14.077	0.9692
p_{c1}	c_1	q_1	q_2	f	A	T	α
0	12.00	1096.39	625.90	5225.40	14127.00	14.127	0.9492
1	11.95	1040.24	652.62	5222.25	14111.25	14.111	0.9555
2	11.91	1040.13	655.44	5220.20	14101.00	14.101	0.9596
3	11.89	1057.42	651.24	5218.15	14090.75	14.090	0.9637
4	11.86	1192.47	627.10	5216.95	14084.75	14.084	0.9661
5	11.84	1060.01	645.18	5215.42	14077.12	14.077	0.96915
6	11.83	1052.41	651.20	5214.07	14070.37	14.070	0.97185
7	11.81	1014.65	652.66	5213.20	14066.00	14.066	0.9736
p_{c2}	c_2	q_1	q_2	f	A	T	α
0.0	7.00	1080.40	640.64	5225.25	14126.25	14.120	0.9495
0.5	6.97	1040.24	653.00	5222.15	14110.75	14.110	0.9557
1.0	6.96	1048.79	653.56	5219.90	14099.50	14.100	0.9602
1.5	6.94	1057.42	651.24	5218.15	14090.75	14.090	0.9637
2.0	6.93	1063.20	624.26	5216.55	14082.75	14.080	0.9669
2.5	6.92	1060.01	645.18	5215.42	14077.12	14.077	0.96915
3.0	6.91	1052.41	651.20	5214.10	14070.50	14.070	0.9718
4.0	6.90	1070.89	650.16	5212.35	14061.75	14.060	0.9753

Table 4
Sensitivity Analysis of p_f of Example 1

p_f	q_1	q_2	f	A	T	α
000	1200.72	551.40	5200.00	14098.50	14.098	0.9606
200	1057.42	651.24	5207.26	14090.75	14.090	0.9637
400	1063.20	618.93	5212.76	14079.75	14.079	0.9681
500	1060.01	645.18	5215.45	14077.12	14.077	0.96915
1000	1131.67	637.77	5225.70	14064.25	14.060	0.9743
1500	1136.22	623.21	5233.60	14056.00	14.056	0.9776
2000	1215.30	569.43	5240.60	14050.75	14.050	0.9797
2500	1218.49	562.29	5244.25	14044.25	14.044	0.9823

Table 5
Sensitivity Analysis of p_A of Example 1

p_A	A	q_1	q_2	f	T	α
000	14000.00	1021.56	643.98	5215.20	14.076	0.9696
1000	14031.10	1108.68	610.43	5215.55	14.078	0.9689
2500	14077.12	1060.01	645.18	5215.42	14.077	0.96915
3000	14092.10	1090.72	646.84	5215.35	14.076	0.9693
5000	14156.00	1160.62	589.74	5215.60	14.078	0.9688
6000	14198.60	1308.92	592.47	5216.55	14.082	0.9669
7000	14221.20	1110.37	627.83	5215.80	14.079	0.9684

Table 6
Sensitivity Analysis of p_T of Example 1

p_T	T	q_1	q_2	f	A	α
0.0	14.00	1215.04	626.88	5215.75	14078.75	0.9685
1.0	14.03	1149.72	610.98	5215.70	14078.50	0.9686
2.5	14.07	1060.01	645.18	5215.42	14077.12	0.96915
3.0	14.09	1127.67	626.89	5215.40	14077.00	0.9692
5.0	14.15	1069.86	599.36	5215.55	14077.75	0.9689
6.0	14.18	1072.61	627.65	5215.45	14077.25	0.9691
7.0	14.21	1058.50	608.58	5215.70	14078.50	0.9686

It is evident from Table 4 that, when p_f increases from 0 to 2500, α increases, A and T decreases, and q_1 and q_2 varies respectively in between 1057 and 1218, and 551 and 651 with respect to a variation of f from 5200 to 5244. It is observed from Table 3 that, α, f, A and T remains almost invariant with respect to increase in values of p_I, p_{cr1} , and p_{cr2}; and the values of q_1 and q_2 changes respectively from 1060 to 1151, and from 645 to 633 with respect to the corresponding variations of I from 0.5% to 0.4877 %, c_{r1} from 100 to 99.84, and c_{r2} from 25 to 24.23.

When p_{c1} increases from 0 to 7 (Table 3), α increases, f, A and T decreases; and q_1 and q_2 varies respectively in between 1014 and 1192, and 625 and 655 with respect to a variation of c_1 from 12 to 11.81. And when p_{c2} increases from 0 to 4 (Table 3), α increases, f, A and T decreases; and q_1 and q_2 varies respectively in between 1040 and 1080, and 624 and 653 with respect to a variation of c_2 from 7 to 6.9. Table 5 reveals that, when p_A increases from 0 to 7000, α varies in between 0.9696 and 0.9669, f is almost invariant, A increases, and T varies in between 14.076 and 14.082. The values of q_1 and q_2 varies respectively in between 1021 and 1308, and 589 and 646 with respect to a variation of A from 14000 to 14221.2. It is also clear from Table 6 that, when p_T increases from 0 to 7, α varies in between 0.9685 to 0.9692, f is almost invariant, T increases, and A varies in between 14077 to 14078.75. The values of q_1 and q_2 varies respectively in between 1215 and 1058, and 599 and 645 with respect to a variation of T from 14 to 14.21.

Therefore it is to be noted that, the total cost (f), the limitations on available space (A) and setup time (T) are moderately sensitive to the variations in unit costs and violations of total cost value. The changes in all the remaining fuzzy variables do not produce much variation in total cost.

Example 2: Consider a company maintaining 3 inventory items with the following data:

Z_1 = 3,600 units/year; z_2 = 24,000 units/year; z_3 = 600 units/year;

c_1 = \$ 200/unit; c_2 = \$ 50/unit; c_3 = \$ 250/unit;

c_r = \$ 100/order; I = 12% per year; f = \$ 2.05×10^6/year;

TO = Maximum total number of orders/year = 36;

AI = Maximum average inventory investment = \$ 1,00,000.

The maximum acceptable violations of c_r, I, c_1, c_2, c_3, f, TO and AI are: p_{cr} = 10; p_I = 2%; $p_{c1} = p_{c2} = p_{c3}$ = 10; $p_f = 0.5 \times 10^6$; p_{TO} = 6; p_{AI} = 20,000.

The objective function mentioned in section-2.1 along with the following constraints will constitute the problem formulation in crisp.

Orders restriction.

$$\Sigma_{i=1 \to n} z_i / q_i - TO \leq 0 \tag{4.3}$$

Average inventory restriction.

$$\Sigma_{i=1 \to n} q_i c_i / 2 - AI \leq 0 \tag{4.4}$$

$$q_i^l \leq q_i \leq q_i^u$$

Assuming the membership function of orders limitation to be non-decreasing and that of average inventory limitation to be non-increasing, and applying fuzzy non-linear programming approach to the crisp formulation, the results of corresponding fuzzy formulation obtained by GA are furnished in Table 1.

Example 3: Consider the data of a machine shop producing 3 products in lots:

$Z_1 = 500$ units/year; $z_2 = 400$ units/year; $z_3 = 600$ units/year;

$c_1 = \$\ 30$/unit; $c_2 = \$\ 20$/unit; $c_3 = \$\ 70$/unit;

$c_{r1} = \$\ 800$/lot; $c_{r2} = \$\ 600$/lot; $c_{r3} = \$\ 1000$/lot;

$a_1 = 5$ Sq.m; $a_2 = 4$ Sq.m; $a_3 = 10$ Sq.m;

$I = 20\%$ per annum; $f = \$\ 73{,}000$/year;

A = Maximum available storage area = 4000 Sq.m;

AI = Maximum average inventory investment = $ 12,000.

The maximum acceptable violations of c_{r1}, c_{r2}, c_{r3}, I, c_1, c_2, c_3, f, A and AI are: $p_{cr1} = p_{cr2} = p_{cr3} = 50$; $p_I = 5\%$; $p_{c1} = p_{c2} = p_{c3} = 10$; $p_f = 7000$; $p_A = 300$; $p_{AI} = 2000$.

The problem formulation in crisp includes the objective function of section-2.1 and the following constraints.

Warehouse space restriction:

$$\Sigma_{i=1 \to n} a_i q_i - A \le 0 \tag{4.5}$$

Average inventory restriction:

$$\Sigma_{i=1 \to n} q_i c_i/2 - AI \le 0 \tag{4.6}$$

$$q_i^l \le q_i \le q_i^u$$

Assuming the membership functions of constraints' limitations to be non-increasing, and applying fuzzy nonlinear programming approach to the crisp formulation, the results of corresponding fuzzy formulation obtained by GA are furnished in Table 1.

5. CONCLUSIONS

In this paper constrained multi-item inventory problems are formulated in fuzzy environment, and are solved by using a real coded GA. The fuzzification process provides a better approximation of real phenomena. The GA finds the optimum quantities of the inventory items that can yield global minimum of the objective function. This approach has been illustrated with 3 different examples with a sensitivity analysis of one of them.

It is observed from the sensitivity analysis (of example 1) that the selection of violations for fuzzy variables is not so critical because even 20% change in violations leads to less percentage change in optimum ordering quantities and total inventory costs (Tables 3 to 6). Here only linear membership functions are considered to represent the nature of variations of fuzzy variables. The membership functions like parabolic, exponential, hyperbolic, etc. can also be considered.

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