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The Effect of a Light Boson Interaction on the Bound States of a Hydrogenic Atom

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ABSTRACT: The diagonalization of the Hamiltonian for a hydrogenic atom is used to gauge the strength of a possible light boson interaction beyond the Standard Model. It is found that a spin-independent interaction for a light boson rest mass energy between 1.0 μeV and 1.0 keV with a fairly strong electron-nucleon coupling constant produces a change in the ground state energies of U^{91+} of around 0.6 μeV.

The Standard Model of particle physics has been extended to incorporate new interactions through the exchange of low mass bosons, known as WISPs (WeaklyInteracting Slim Particles)[1]. The types of light boson interactions include 8 that are parity-invariant and 8 that are parity-violating [2].

In this paper we examine a WISP light boson interaction that is spin independent. By diagonalization of the Hamiltonian, we find that the light boson interaction produces a change in the ground state energies of U^{91+} by around 0.6 μeV. Unless otherwise stated, we will use atomic units.

The radial Schrodinger equation for a Hydrogenic atom is given by:

$$H(r)P_{nl}(r) = E_{nl}P_{nl}(r), \tag{1}$$

where

$$H(r) = -\frac{1}{2}\frac{d^2}{dr^2} + \frac{l(l+1)}{2r^2} + V(r) + W(r) \tag{2}$$

The electromagnetic interaction is given by:

$$V(r) = -\frac{Zg_e^2}{r}, \tag{3}$$

where Z is the number of protons and the coupling constant $g_e = 1$. The light boson interaction is given by:

$$W(r) = -\frac{Ag_w^2 e^{-r/\lambda}}{r}, \tag{4}$$

where A is the number of nucleons, g_w is the coupling constant, and λ is the range of the light boson. Our choice of the coupling constant is $g_w = 1.0 \times 10^{-6}$. The range of the light boson, λ, is related to its rest mass energy as given by:

$$\lambda = \frac{1.97\times10^{-5}\,(eV\,cm)}{mc^2(eV)} \tag{5}$$

The bound state energies, E_{nl}, of Eq. (1) are obtained by diagonalization of the Hamiltonian, $H(r)$, of Eq. (2). For a $N = 180$ point lattice with a radial mesh spacing of $\Delta r = 0.0050$, we diagonalized the Hamiltonian for U^{91+} with $Z = 92$ and $A = 238$. We used the LAPACK[3] subroutine DSTEQR for a real symmetric tridiagonal matrix. The effect of the light boson interaction is found by taking the difference in bound state energies setting $W(r) = 0$ and then using $W(r)$ of Eq.(4) with four choices for the light boson rest mass. The changes in the energies for the $1s$, $2p$, and $3d$ states of U^{91+} are found in Table 1. As seen the largest change is found for the $1s$ ground state of U^{91+}.

As a check on the effect of the light boson interaction, we repeated the diagonalization of the Hamiltonian with and without $W(r)$ of Eq.(4) using a $N = 360$ point lattice with a radial mesh spacing of $\Delta r = 0.0025$. The changes in the energies found in Table 2 are very similar to those found in Table 1, with the largest change for the $1s$ ground state of U^{91+}.

We note that when the coupling constant g_w is reduced by a factor of 10, the changes in the energies are reduced by a factor of 100, in keeping with g_w^2 in Eq.(4). We also note that the changes in the energies are almost independent of the boson rest mass energy from 1.0 μeV to 1.0 keV. Only when the boson rest mass energy approaches 1.0 MeV and $\lambda = 1.97 \times 10^{-11}$ cm, or 0.0037 in atomic units, is there a reduction in the strength of the light boson interaction. Of course the radius of the first Bohr orbit for U^{91+} is 1/92 = 0.0109 in atomic units.

The radial Dirac equation for a hydrogenic atom is given by:

$$\begin{aligned} H_{11}(r)\,P_{n\kappa}(r) + H_{12}(r)\,Q_{nk}(r) &= E_{nk}Q_{nk}(r) \\ H_{21}(r)P_{n\kappa}(r) + H_{22}(r)Q_{n\kappa}(r) &= E_{n\kappa}P_{n\kappa}(r), \end{aligned} \tag{6}$$

where

$$\begin{aligned} H_{11}(r) &= c\left(\frac{d}{dr}+\frac{\kappa}{r}\right) \\ H_{12}(r) &= V(r)+W(r)-2c^2 \\ H_{21}(r) &= V(r)+W(r) \\ H_{22}(r) &= -c\left(\frac{d}{dr}-\frac{\kappa}{r}\right). \end{aligned} \tag{7}$$

The bound state energies, $E_{n\kappa}$, of Eq.(6) are obtained by diagonalization of the Hamiltonian, $H_{ij}(r)$, of Eq.(7). For a $N = 360$ point lattice with a radial mesh spacing of $\Delta r = 0.0025$, we diagonalized the Hamiltonian for U^{91+} with $Z = 92$ and $A = 238$. We used the LAPACK[3] subroutine DSYEV for a real symmetric matrix.

The effect of the light boson interaction is again found by taking the difference in bound state energies setting $W(r) = 0$ and then using $W(r)$ of Eq.(4) with four choices for the light boson rest mass. The changes in the energies for the $1s(\kappa = -1)$ state of U^{91+} are found in Table 3.

In summary, in support of studies to determine the effect of a spin-independent WISP interaction, we carried out calculations using the Schrodinger and Dirac equations for bound state energies of U^{91+}. For a fairly strong electron-nucleon coupling constant and for a light boson rest mass energy between 1.0 μeV and 1.0 keV, we found changes in the ground state energies of U^{91+} are around 0.6 μeV.

Table 1
Changes in the Bound State Energies for U^{91+} for a N = 180 point lattice with Δr = 0.0050 using the Schrodinger equation

Boson Rest Mass Energy	*Bound State*	*Change in Energy*
1.0×10^{-6} eV	1s	-5.4×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.6×10^{-8} eV
1.0×10^{-3} eV	1s	-5.4×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.6×10^{-8} eV
1.0 eV	1s	-5.4×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.7×10^{-8} eV
$1.0 \times 10^{+3}$ eV	1s	-5.4×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.5×10^{-8} eV

Table 2
Changes in the Bound State Energies for U^{91+} for a N = 360 point lattice with Δr = 0.0025 using the Schrodinger equation

Boson Rest Mass Energy	*Bound State*	*Change in Energy*
1.0×10^{-6} eV	1s	-5.8×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.8×10^{-8} eV
1.0×10^{-3} eV	1s	-5.8×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.8×10^{-8} eV
1.0 eV	1s	-5.8×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.8×10^{-8} eV
$1.0 \times 10^{+3}$ eV	1s	-5.8×10^{-7} eV
	2p	-1.5×10^{-7} eV
	3d	-6.6×10^{-8} eV

Table 3
Changes in the Bound State Energies for U^{91+} for a N = 360 point lattice with Δr = 0.0025 using the Dirac equation

Boson Rest Mass Energy	*Bound State*	*Change in Energy*
1.0×10^{-6} eV	1s(-1)	-6.1×10^{-7} eV
1.0×10^{-3} eV	1s(-1)	-6.1×10^{-7} eV
1.0 eV	1s(-1)	-6.2×10^{-7} eV
$1.0 \times 10^{+3}$ eV	1s(-1)	-6.1×10^{-7} eV

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References

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