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Impact assessment of algorithms of generation of orthogonal directional diagrams on the characteristics of the radio direction finding

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Abstract : The paper discusses the methods of determining the coordinates of MUSIC and Capon radio emission having the property of superresolution, which suggests that the two signal sources located at the distance of less than the Rayleigh limit can be shown as individual peaks on the graph of the space direction-finding relief. A problem of computing resources consumption in this kind of spectral estimation algorithms arises, which can be solved using the methods of building the so-called orthogonal directional patterns, reducing the computational complexity and the dimensionality of the antenna array. For this purpose, there is a formation of the multibeam directional pattern (3D channel) after which a signal preprocessing is performed by means of these 3D channels focused in a certain area. Numerical positional accuracy estimates of the radio-frequency sources by the classical MUSIC and Capon methods, and also after preliminary transformation by orthogonal directional patterns (BS-MUSIC and BS-Capon) depending on the relation signal/noise, numbers of beams and time of averaging of a correlation matrix are given. A large number of methods of formation of the 3D channels with mutual orthogonality, from which it is possible to distinguish as the most known discrete Fourier transformation, spherical sequences and Taylor expansion, are presented. It should be noted that so far a comparative study of accuracy characteristics of such methods (BS-MUSIC BS-Capon) has not been made. It is found that the accuracy of BS-MUSIC and BS-Capon methods increases with the increase in the number of 3D channels from four to five. Thus, accuracy for a method of receiving the channels by method of discrete of the spherical sequences is the highest for the majority of interfering cases considered.
Keywords : Superresolution, digital antenna array, radio direction finding, orthogonal directional patterns.

1 INTRODUCTION

The development of high-precision radio-engineering data-transmission systems with fast response and strong bandwidth to operate within high interference signaling environment under conditions of the limited frequencies range is relevant.

One of the resources increasing the flow capacity of the information data transmission systems is the use of the degrees of freedom associated with the adaptive pattern steering. Thus, a priori information about the location of subscribers and interfering sources is often required. The accuracy of this information affects the correctness of installation, of zeros and maxima of the directional pattern of the antenna arrays, and, therefore, the signal/noise ratio for the signals received by a subscriber and base station. Currently, methods of angular superresolution based on correlation processing of the signals from the outputs of the antenna array are becoming more popular. Among these methods it is first necessary to highlight the MUSIC and Capon algorithms, suitable for antenna arrays of any configuration, that provide high resolution of the signals and which are most promising in the field of superresolution (Berezovskiy *et al.*, 2011; Zhang, & Wang, 2013).

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Adaptive beam forming (BF) antenna arrays (AA) can significantly improve the ratio of signal power of the subscriber to the interference power and noise (SINR). One way for beam formation is based on an assessment of the angular coordinates of radio-frequency sources (RFS). Such algorithms require that the output signals from every antenna components (AC) are available digitally. In many applied tasks, the number of initial processing modules of the received signals and analog-to-digital converters (AD converter) can reach large values. Antenna arrays consisting of several dozen of elements are not uncommon (Krim, & Viberg, 1996). Thus, the order of the computational complexity of methods of the spectral analysis arrays of N antenna components is $O(N^3)$. After applying the processing in space of rays, the expenses involved in CPU time are significantly reduced (Steinwandt *et al.*, 2013). Therefore, the ways of reducing the observation vector length with minimal loss represent a great interest (Huang *et al.*, 2010).

1. Basic assumptions

Let us assume that M signals arrive on the AA from arbitrary directions $\{\theta_m\}_{m=1}^{M}$ and are described by the expression (Krim, & Viberg, 1996):

$$S_m(t) = b_m(t)\exp(j2\pi f_m t),$$

where $b_m(t)$ – amplitude of m-th signal, f_m – carrier frequency of m-th signal, t – time. For a linear AA, the signal with azimuth θ_m on the n-th antenna component acquires phase shift $-2\pi D\lambda^{-1} n \sin(\theta_m)$, and it is possible to receive the direction vector of a linear lattice (Godara, 1997):

$$\vec{a}(\theta_m) = [1\ \exp\{j[-2\pi D\lambda^{-1}\sin(\theta_m)]\}\ \ldots\ \exp\{j[-2\pi D\lambda^{-1}(N-1)\sin(\theta_m)]\}]^T,$$

where $n = 1...N$, $k_m = 2p/\lambda_m$, λ_m – wavelength of m-th signal, D– interelement spacing. For AA, a complex vector of the output signals of the antenna components is described by the expression (Krim, & Viberg, 1996):

$$\vec{x}(t) = A.\vec{s}(t) + \vec{n}(t),$$

where $\vec{x}(t)^T$ – N-measuring vector, describing the output signals of each AA antenna component, $\vec{s}(t)$ – M-measuring vector of the signals; $\vec{n}(t)$ – vector of noise; A – N × M matrix of the direction vectors. Then, the space correlation matrix can be written as (Godara, 1997):

$$R_{xx} = AR_{ss}A^H + \sigma^2 I = E_s\Lambda_s E_s^H + E_n\Lambda_n E_s^H,$$

where, $R_{ss} = E[\vec{s}(t)\vec{s}^H(t)]$, $\lambda_1 > \lambda_2 > \ldots > \lambda_M > \lambda_{M+1} = \ldots = \lambda_N = \sigma^2$ and $\vec{e}_1, \vec{e}_2, \ldots, \vec{e}_N$ are the eigenvalues respectively, and eigenvectors of a matrix R, E_s, E_n are matrices of signal and noise subspaces, Λ_s, Λ_n are diagonal matrices of eigenvalues of signal and noise subspaces.

2. METHODS OF PRETREATMENT

On numerous occasions the number of antenna elements N has great value and then creates N_{BS} of the directional patterns for preprocessing of the received signal by each AE. Then, further assessment and transformation are carried out (Figure 1) (Godara, 1997).

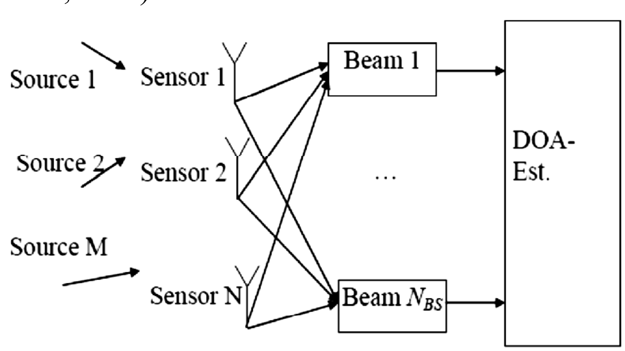

Fig. 1. Block diagram of the preprocessing.

In the form of matrix linear transformations, the diagram from the Figure 1 can be represented as (Sidorenko, & Berezovskiy, 2013; Van Trees, 2002):

$$\vec{z}(k) = T_{BS}^{H}\vec{x}(k)$$

where $\vec{z}(k)$ – R-measuring vector, $T_{BS} = \left[\vec{t}(\psi_1) \;\; \vec{t}(\psi_2) \;\; \dots \;\; \vec{t}(\psi_R)\right]$ $N_{BS} \times N$ transformation matrix, consisting of N_{BS} of weight vectors by means of which a set is generated from N_{BS} directional patterns, focused in the directions of $\psi_i, i = 1,2,\dots N_{BS}$, provided that $M < N_{BS} << N$. The transformation is usually done in the analog part, greatly reducing the required number of AD converters, and it has a number of advantages (Van Trees, 2002; Lee, & Wengrovitz, 1990):

1. Computational complexity is reduced, and, therefore, the statistical stability of the spectral reflectance assessments is achieved.
2. It is possible to suppress powerful RFS before applying the methods for parameter estimation of the signals within the sector.
3. Noise in the scan sector becomes white with unknown variance.

In addition, T_{BS} matrix columns are orthogonal to each other, *i.e.*

$$T_{BS}^{H}T_{BS} = I \tag{1}$$

In some applications, condition (1) is not satisfied and then it is necessary to carry out preliminarily transformation to bring T_{no} to the orthonormal matrix:

$$T_{bs} = T_{no}\left[T_{no}^{H}T_{no}\right]^{-0.5} \tag{2}$$

The orthogonal directional pattern means that each beam is focused to its own radio-frequency source and zeroes are formed in the directions of other RFS. Thus, if there is only one signal, only one element of the vector $\vec{z}(k)$ contains the information about this signal (Nilsen, & Hafizovic, 2009). In the case if the T_{BS} matrix columns are selected incorrectly, there will be a decrease in the characteristics of the methods of the spectral analysis. The observation vector in the antenna array (Sidorenko, & Berezovskiy, 2013):

$$\vec{z}(t) = T_{BS}^{H}A(\theta)\vec{s}(t) + T_{BS}^{H}\vec{n}(t)$$

Spatial covariance matrix taking into account T_{BS} is converted into the following form (Amini, & Georgiou, 2005):

$$R_{zz} = E[\vec{z}(t)\vec{z}^{H}(t)] = T_{BS}^{H}R_{xx}T_{BS} == T_{BS}^{H}ASA^{H}T_{BS} + \sigma^2 T_{BS}^{H}T_{BS} \tag{3}$$

Then, decomposition on the eigenvectors and values (Van Trees, 2002):

$$R_{zz} = E_{BS-s}\Lambda_{BS-s}E_{BS-s}^{H} + E_{BS-n}\Lambda_{BS-n}E_{BS-n}^{H} \tag{4}$$

where E_{BS-s} – $N_{Bs} \times M$ matrix of vectors of the signal subspace, E_{BS-n} – $N_{Bs} \times (N_{Bs} - M)$ matrix of the vectors of the noise subspace consisting of eigenvectors corresponding to (N_{Bs}–M) of the smallest eigenvalues, Λ_{BS-s}, Λ_{BS-n}, – diagonal matrix of the eigenvalues. Matrix of linear transformation T_{BS} displays the linear space of the total dimension in subspace of lesser dimension. Designing of such a matrix obeys to certain criteria.

2.1. Discrete Fourier transform (DFT)

The most common transformation matrix T_{BS} has columns that consist of beam forming factors with maxima when spaced on 2π/N. Matrix row T_{BS}^{H} are then equal to:

$$T_{BS}^{H} = \frac{1}{N}e^{j\left(\frac{N-1}{2}\right)m\frac{2\pi}{N}}\left[1 \quad e^{jm\frac{2\pi}{N}} \quad \dots \quad e^{j(N-1)m\frac{2\pi}{N}}\right] \tag{5}$$

where *m* is the value that identifies the number of the beam in the scan sector.

Together with DFT method, it is possible to use transformations for level decreasing of side petals, such as Hamming windows.

2.2 Rays of the Taylor series

For the case of two signal sources, a matrix of rays is as follows:

$$T_{no} = \left[\vec{a}(\psi_1) \quad \vec{a}(\psi_2) \quad \vec{a}(\psi_{\text{mid}})\right]$$

where $\vec{a}(\psi_i)$ is a direction vector corresponding to true angular coordinates of RFS ψ_1 and ψ_2, $\psi_{\text{mid}} = \frac{(\psi_1 + \psi_2)}{2}$ – a midpoint. However, it is impossible to construct T according to (6), because the true values ψ_1 and ψ_2 are unknown for certain and it is necessary to estimate them.

However, the matrix (6) can be approximated using the Taylor expansion of the vectors $\vec{a}$ in the neighborhood zone that contains useful signals, ψ_{mid}:

$$\vec{a}(\psi_i) = \vec{a}(\psi_{\text{mid}}) + \vec{a}^{(1)}(\psi_{\text{mid}})(\psi_i - \psi_{\text{mid}}) + \vec{a}^{(2)}(\psi_{\text{mid}})\frac{(\psi_i - \psi_{\text{mid}})^2}{2!} + \ldots$$

$$\vec{a}^{(1)}(\psi_{\text{mid}}) = \frac{\partial^k \vec{a}^{(1)}(\psi_{\text{mid}})}{\partial^k \psi_{\text{mid}}}$$

where $i = 1,2,\ldots m$ and k is an order of the derivative direction vector relative to angle ψ_{mid}. Then

$$T_{\text{no}} = \left[\vec{a}(\psi_{\text{mid}}) \quad \vec{a}^{(1)}(\psi_{\text{mid}}) \quad \vec{a}^{(2)}(\psi_{\text{mid}}) \quad \ldots \quad \vec{a}^{(m)}(\psi_{\text{mid}})\right] \qquad (7)$$

After receiving the matrix (7), it is necessary to use a transform (2) to bring T_{no} to orthonormal sight T_{BS}.

2.3. Discrete convex spherical sequences

Let us define vectors $\vec{t}_i, i = 1,2\ldots M$ as the columns of the matrix of the orthogonal directional patterns T_{BS}. Let the ratio of the energy of *i*-th beam in the scanning area $[-\psi_0, \psi_0]$ to the energy of *i*-th BF as a whole be determined by the ratio:

$$\alpha_i = \frac{\int_{-\psi_0}^{\psi_0} \left|\vec{t}_i \vec{a}(\psi)\right|^2 d\psi}{\int_{-\pi}^{\pi} \left|\vec{t}_i \vec{a}(\psi)\right|^2 d\psi}, i = 1,2\ldots M$$

The numerator α_i looks like: $\alpha_{iN} = \vec{t}_i A_{\text{DPSS}} \vec{t}_i^{\,H}$,

where $A_{\text{DPSS}} = \int_{-\psi_0}^{\psi_0} \vec{a}(\psi)\vec{a}^{H}(\psi)\, d\psi$. For AA linear array, the *mn*-th element of the matrix A_{DPSS} shall be determined respectively as:

$$A_{\text{DPSS}}^{mn} = \frac{2\psi_0 \sin(m-n)}{(m-n)}, m \neq n$$

$$A_{\text{DPSS}}^{mn} = 2\psi_0, \; m = n$$

The denominator of the matrix α_i looks like:

$$\alpha_{iD} = 2\pi \vec{t}_i^{\,H} \vec{t}_i$$

Then
$$\alpha_i = \frac{\vec{t}_i A_{DPSS} \vec{t}_i^{H}}{2\pi \vec{t}_i^{H} \vec{t}}, i = 1, 2 \ldots N_{se}$$

It is necessary to maximize $\alpha_i, i = 1, 2 \ldots N_{se}$ provided that there is mutually orthogonal $\vec{t}_i$, which is equivalent to finding the eigenvectors of matrix A_{DPSS}, corresponding to M of the largest eigenvalues. Thus, we have the equality (Forsterand, & Vezzosi, 1987):

$$2\pi\lambda \vec{t}_i = A_{DPSS} \vec{t}_i$$

Where
$$\sum_{n=1}^{N} \frac{\psi_0 \sin(m-n)}{(m-n)} t_n = \pi\lambda t_m, m = 1, 2, \ldots, N$$

For each of M the largest eigenvalue, we get a sequence that determines a column vector $\vec{t}_i$. The number of important eigenvalues and, consequently, BF is determined by:

$$N_{se} = \frac{\psi_0}{\pi} N + 1$$

Then, the value of the scan sector ψ_0 for this method determines the number of orthogonal beams N_{se} as the eigenvectors of the matrix A_{DPSS}, which actually form the columns T_{bs}.

For clarity, Figure 2 presents 4 orthogonal BF for window coverings (20°; 20°) obtained by the DFT method (Figure 2, *a*), by Taylor expansion (Figure 2, *b*) and discrete spherical sequences (Figure 2, *c*) for linear antenna array composed of 10 AE with a half-wave distance between them.

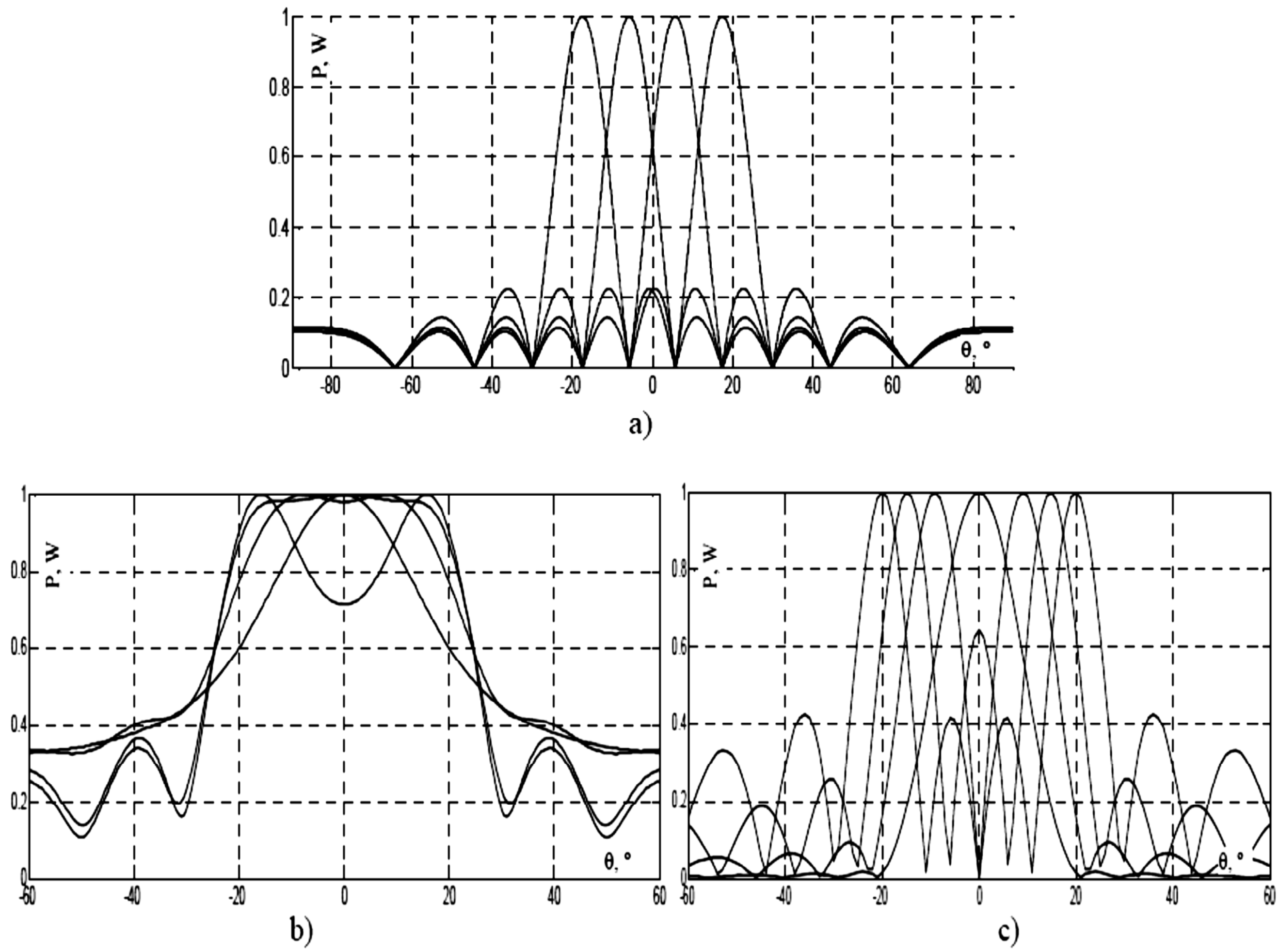

Fig. 2. Set of 4-*x* orthogonal BF for (*a*) DFT (*b*) Taylor series (*c*) spherical sequences.

BF for Taylor rays (Figure 2, b) are obtained by building a matrix (8) for the central angle $\psi_{mid} = 0°$. Then, T_{no} is converted to the orthogonal form by the expression (3). In Figure 1, c, BF are taken as 4 eigenvectors of matrix A_{DPSS}. Figure 2 shows that the method of Taylor rays gives the highest side-lobe level.

3. ASSESSMENT OF ANGULAR COORDINATES OF RFS

3.1. Capon and BS-Capon method

The problem is formulated as follows : It is necessary to find the weight vector $\vec{w}$ that minimizes average output power of AA assuming that for a particular angle of arrival è the transmission coefficient of the lattice is fixed and equals, for example, to unit:

$$\min_{w} \left\langle \left| \vec{w}^{H} \vec{x} \right|^{2} \right\rangle = \min P(\vec{w}) \text{ where } \vec{w}^{H} \vec{a}(\theta) = 1$$

In order to solve it, it is necessary to make the Lagrange functional in the form of :

$$\Phi(\vec{w}) = \left\langle \left| \vec{w}^{H} \vec{x} \right|^{2} \right\rangle - \chi \left(\vec{w}^{H} \vec{a}(\theta) - 1 \right),$$

where χ is the indefinite Lagrangian multiplier; $\left\langle \left| \vec{w}^{H} \vec{x} \right|^{2} \right\rangle = \vec{w}^{H} R \vec{w}$. The gradient of this functionality will be equated to zero, and we get equality, which minimizes the average output power according to the criterion of Capon (Capon, 1997):

$$\vec{w} = \frac{R^{-1} \vec{a}(\theta)}{\vec{a}^{H}(\theta) R^{-1} \vec{a}(\theta)}$$

Thus, for the method Capon the permission function is as follows (Capon, 1997) :

$$P_{Capon}(\theta) = \frac{1}{\vec{a}^{H}(\theta) R^{-1} \vec{a}(\theta)}$$

This method can resolve the correlated signals. However, its accuracy is generally lower than that of the actual structural methods such as MUSIC.

Assuming that a vector $\vec{a}$ and columns of the matrix $\mathbf{R}$ undergo mapping into the same subspace formed by the columns of the matrix T_{bs}, we get that the Capon method, modified for the use with orthogonal BF (the so-called Beamspace Capon or BS-Capon), is reduced to the calculation of the function:

$$P_{BS\text{-}Capon}(\theta) = \frac{1}{\vec{a}^{H}(\theta) T_{Bs} R_{zz}^{-1} T_{Bs}^{H} \vec{a}(\theta)}$$

3.2. MUSIC and BS-MUSIC method

The property of orthogonality of the noise subspace and the signal vectors (Schmidt, 1986) is used when implementing MUSIC method :

$$E_{n}^{H} \vec{a}(\theta_{m}) = 0, \theta_{m} \in \{\theta_{1}, \theta_{2}, ..., \theta_{M}\} \quad (8)$$

And then the coordinates of the signals will correspond to the maximum of function (Schmidt, 1986):

$$P_{MUSIC}(\theta) = \frac{1}{\vec{a}^{H}(\theta) E_{n} E_{n}^{H} \vec{a}(\theta)}.$$

MUSIC method has an infinitely high resolution in the absence of noise and amplitude-phase mismatch of AA channels. The method is unstable, if signals are correlated (Nechaev, & Makarov, 2010; Nechaev *et al.*, 2011).

According to the formulas (3) and (4), it can be concluded that :

$$\Re\{E_{BS-s}\} = \Re\{T^{H} A(\theta)\},$$

where $\Re\{M\}$ denotes the space formed by the columns of the matrix of a random matrix M. Then $\Re\{E_{BS-s}\}$ can be used as signal subspace that is orthogonal to the noise formed with $\Re\{E_{BS-n}\}$, *i.e.* $\Re\{E_{BS-s}\}\perp\Re\{E_{BS-n}\}\Rightarrow\Re\{T^{H}A(\theta)\}\perp\Re\{E_{BS-n}\}$. Therefore, like (8) it is possible to write down expressions for determination of coordinates of RFS after transformation by orthogonal BF (Stoica, & Nehorai, 1991):

$$P_{BS\text{-}MUSIC}(\theta) = \frac{1}{\vec{a}^{H}(\theta)T_{Bs}E_{BS-n}E_{BS-n}^{H}T_{Bs}^{H}\vec{a}(\theta)} \tag{9}$$

where T_{Bs} is selected with a specified number of rays focused to a certain region of space and formed by one of the methods described above. The expression (9) bears the name of Beamspace MUSIC (BS-MUSIC).

4. STUDY

4.1. Assessment of the accuracy of determining the angular coordinates of RFS

The well-known properties of BS-MUSIC method, concerning the accuracy of determining the angular coordinates, are the following: the root-mean-square deviation of the estimates of the azimuth coordinates of RFS for any matrix T_{BS} cannot be less than that of the classical method of MUSIC (Xu, & Buckley, 1993; Li, & Lie, 1994). For the proof, it is possible to give the expression for dispersion of the MUSIC method (Stoica, & Nehorai, 1991):

$$\text{var}_{MUSIC}(\theta) = \frac{\sigma^2}{2N}\left[\frac{R_{ss}+\sigma^2R_{ss}^{-1}\left(A^{H}A\right)^{-1}R_{ss}^{-1}}{D^{H}\left(I-A\left(A^{H}A\right)^{-1}A^{H}\right)D}\right] \tag{10}$$

where $D=\left[\vec{d}(\theta_1),\ldots,\vec{d}(\theta_M)\right]$, $\vec{d}(\theta_m)=\frac{\partial}{\partial\theta_m}\vec{a}(\theta_m), m=1\ldots M$. The expression (10) can be distributed to BS-MUSIC (Stoica, & Nehorai, 1991):

$$\text{var}_{BS\text{-}MUSIC}(\theta) = \frac{\sigma^2}{2N}\left[\frac{R_{ss}+\sigma^2R_{ss}^{-1}\left(A^{H}QA\right)^{-1}R_{ss}^{-1}}{D^{H}Q\left(I-A\left(A^{H}A\right)^{-1}A^{H}\right)D}\right]$$

where $Q=T_{BS}T_{BS}^{H}$; $\text{var}_{MUSIC}(\theta)\le\text{var}_{BS-MUSIC}(\theta)$.

In addition, in (Stoica, & Nehorai, 1991) it is shown that if there are two matrices T_{BS1} (with dimension of $N\times N_{BS1}$) and T_{BS2} (with dimension of $N\times N_{BS2}$), where $N_{BS1}\ge N_{BS2}$, then the root-mean-square deviations of the assessments of angular coordinates of RFS obtained by using T_{BS2} are not less than the assessments T_{BS1}.

The expression for obtaining T_{BS} can be used to compare the characteristics of the classical MUSIC method with BS-MUSIC. In particular, the comparison with the respective Cramer-Rao bound is of some interest. The analytical expression of the Cramer-Rao bound for the stochastic case looks like:

$$CRB = \frac{\sigma^2}{2N}\left[\text{Re}\left\{\left(D^{H}P_{A}^{\perp}D\right)\otimes\left(R_{ss}A^{H}R_{xx}^{-1}AR_{ss}\right)\right\}\right]^{-1},$$

where $P_{A}^{\perp}=I-A\left(A^{H}A\right)^{-1}A^{H}$ is an orthogonal projection matrix of the noise subspace (Stoica, & Nehorai, 1991).

The lower variance bound of RFS coordinates to preprocessing by orthogonal BF looks like (Stoica, & Nehorai, 1991; Anderson, & Nehorai, 1995):

$$CRB_{BS} = \frac{\sigma^2}{2N}\left[\text{Re}\left\{\left(D^{H}T_{BS}P_{BS-A}^{\perp}T_{BS}^{H}D\right)\otimes\left(R_{ss}A^{H}T_{BS}R_{xx}^{-1}T_{BS}^{H}AR_{ss}\right)\right\}\right]^{-1},$$

where $P_{A}^{\perp}=I-T_{BS}^{H}A\left(A^{H}QA\right)^{-1}A^{H}T_{BS}$.

The expressions for the bounds CRB and CRB_{BS} correspond to the general case and include the assumption of uncorrelated signals. It is known that MUSIC method is an implementation of the maximum likelihood method and reaches the Cramer-Rao bound with a sufficiently large number of measuring references N, the meaning of RSP and provision that R_{SS} is diagonal (Nechaev *et al.*, 2009). In real situations, only the limited number of temporary pictures of N is available and the signals have correlation factor, other than zero, in consequence of which the dispersion of the estimates of azimuthal coordinates of RFS increases (Sun, & Yang, 2004). However, these factors are not investigated for BS-MUSIC and BS-Capon methods (Sun, & Yang, 2004; Tidd, 2011; Stoica, & Nehorai, 1991; Xu, & Buckley, 1993; Li, & Lie, 1994).

4.2. Estimating the number of countdowns on the accuracy of the radio direction finding

Modeling of the linear density-tapered array, consisting of N = 10 AE, located at the distance 0.5 ё, was carried out. Then, the technique of formation of orthogonal BF with the number of beams of $N_{BS} < N$ in various noise situation is applied to this AA. The scan sector is (–20°; 20°); two RFS of equal power with the corner point coordinates –50 and 190, respectively, were modeled. The number of tests varied from 10 to 200. The assessment of a root-mean-square deviation (RMSD) of an assessment of a bearing from its true value is given.

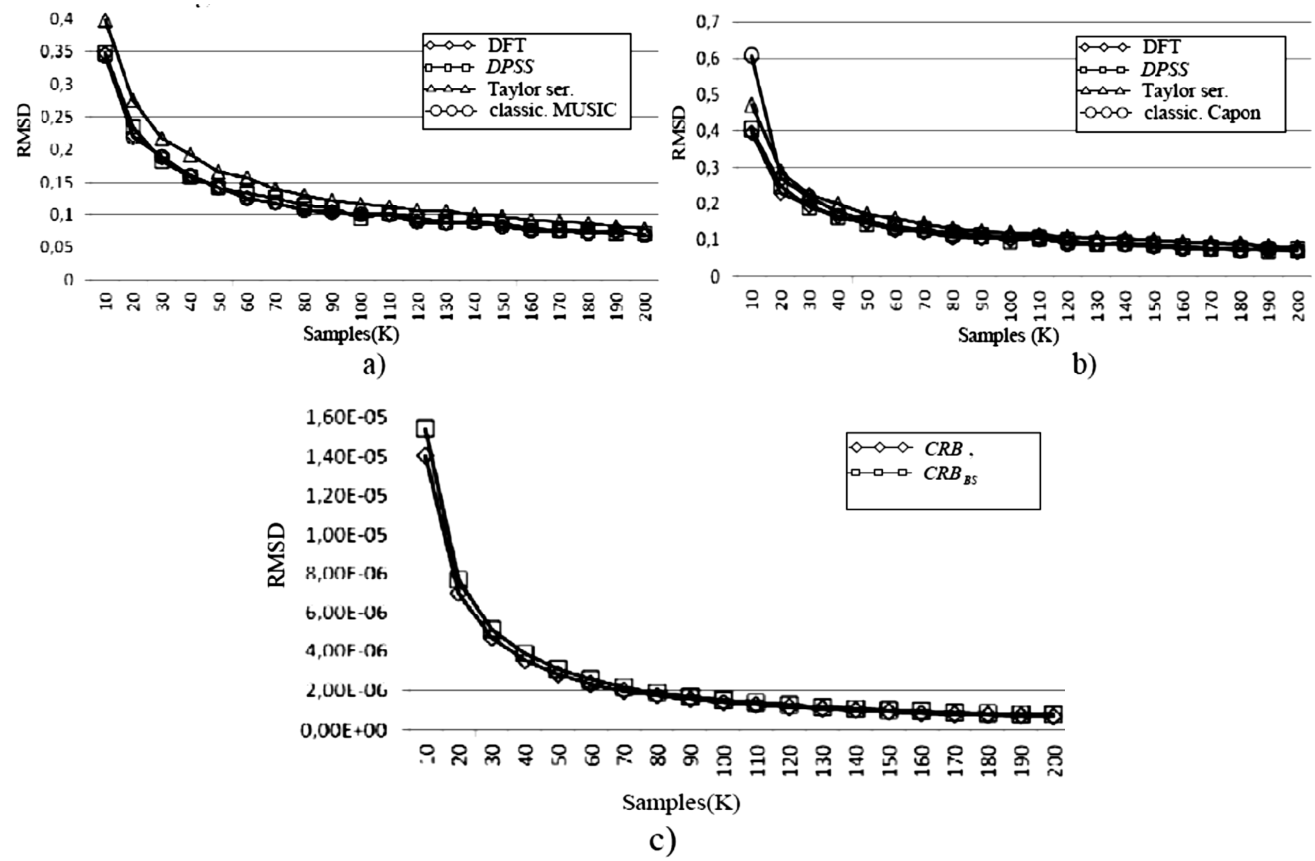

Fig. 3. Dependence of RMSD on the scale numbering (*a*) BS-MUSIC (*b*) BS-Capon (*c*) CRB.

Figure 3 shows the relationship between RMSD estimates of angles of arrival of the radio signal by BS-MUSIC, BS-Capon methods and by time of averaging K for various methods of obtaining T_{BS} matrix. Besides, the values of the lower bound of RMSD according to the Cramér–Rao method are given. The dependencies shown in Figure 3 allow drawing some conclusions. RMSD is inversely proportional to the scale numbering of averaging K and is consistent with existing results (Nechaev *et al.*, 2009). The form of graphs of Figures 3 (*a*) and (*b*) is the same, but asymptotically BS-Capon and BS-MUSIC will reach the Cramer-Rao bounds with a sufficiently large value of K, which is unattainable in real applications, when transforming the matrix R_{ss} to diagonal form is almost impossible with a stochastic model. RMSD of BS-Capon and BS-MUSIC methods are approaching their acceptable values when the number K is more than 100 measuring references.

4.3 Assessment of the effect of the number of orthogonal BF for bearing accuracy

Terms of study : Linear density-tapered array with interelement spacing equal to 0.5 λ, the number of AE is 10, the number of orthogonal directional patterns of N_{BS} amounted to 4 and 5 to assess the impact on the accuracy of estimates of the angular coordinates of RFS, signal/noise ratio (SNR) varies from -20 to 0 dB. In the first case, two signals are with coordinates –5° and 18° that fall within the scan sector (–20°; 20°), the second is added by another signal with an azimuth of 50°, which is not included in the window of interest.

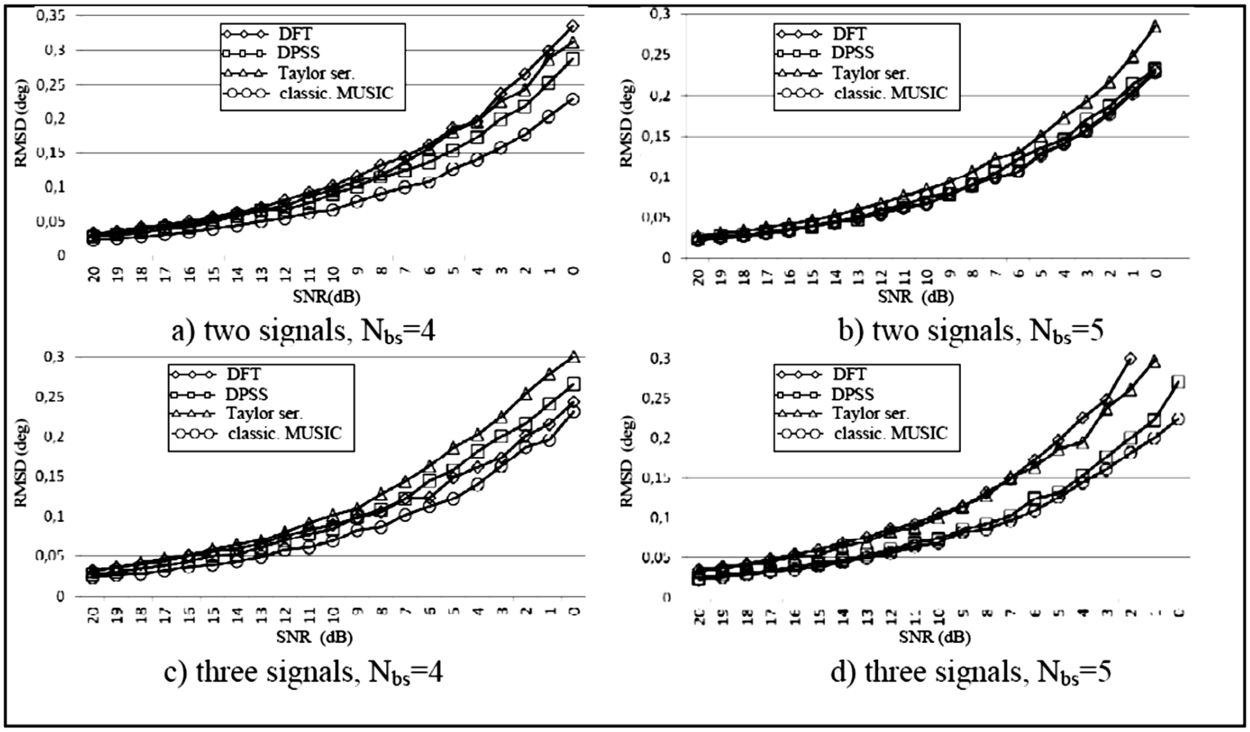

Fig. 4. BS-MUSIC.

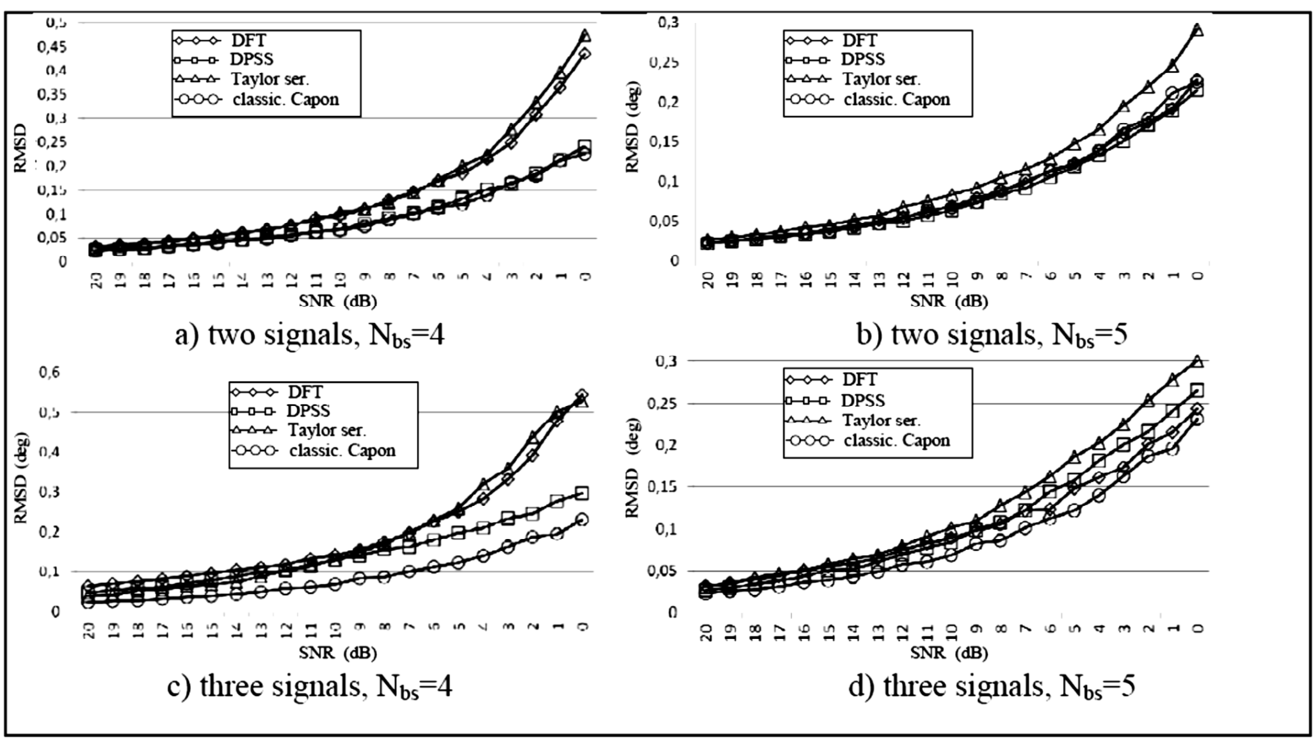

Fig. 5. BS-Capon.

Figures 4-5 show that for the concerned cases RMSD of BS-MUSIC and BS-Capon methods are decreasing with an increase of N_{BS} from 4 up to 5. For two signals in the sector of interest, when N_{BS} = 5, RMSD of BS-MUSIC method is faintly different from MUSIC regardless of the method of receiving T_{BS}. For the two signals, DPSS allows reducing RMSD of BS-Capon method almost to the values of classical Capon. For three signals, RMSD of BS-MUSIC method at N_{BS} = 4 is higher than that of MUSIC. For the three signals, all considered methods to get T_{BS} together with BS-Capon have RMSD values close to the classical method of Capon. Thus, the accuracy of BS-MUSIC and BS-Capon for the DPSS method is the highest for the majority of cases, which is consistent with Figure 2 since the side-lobe level is low, there is high population density of useful sector, and the maximum of one BF coincides with zero of others, preventing ambiguity in $\vec{z}(t)$. Taken as a whole, BS-Capon accuracy is higher than that of BS-MUSIC.

4.4. Research of the resolution capability

Figure 6 reflects the dependence of resolution upon RSPN, which is achieved using these methods after orthogonal transformations of BS-Capon and BS-MUSIC; the value of resolution was adopted as the angle between the directions at RFS, at which in 98% of implementations it is possible to distinguish two maxima on the direction-finding relief corresponding to the directions at RFS. Formation of T_{BS} was conducted using DFT, because, as it seen from Figures 4 and 5, other methods of forming orthogonal BF are in general similar in accuracy of the direction-finding algorithms BS-MUSIC and BS-Capon.

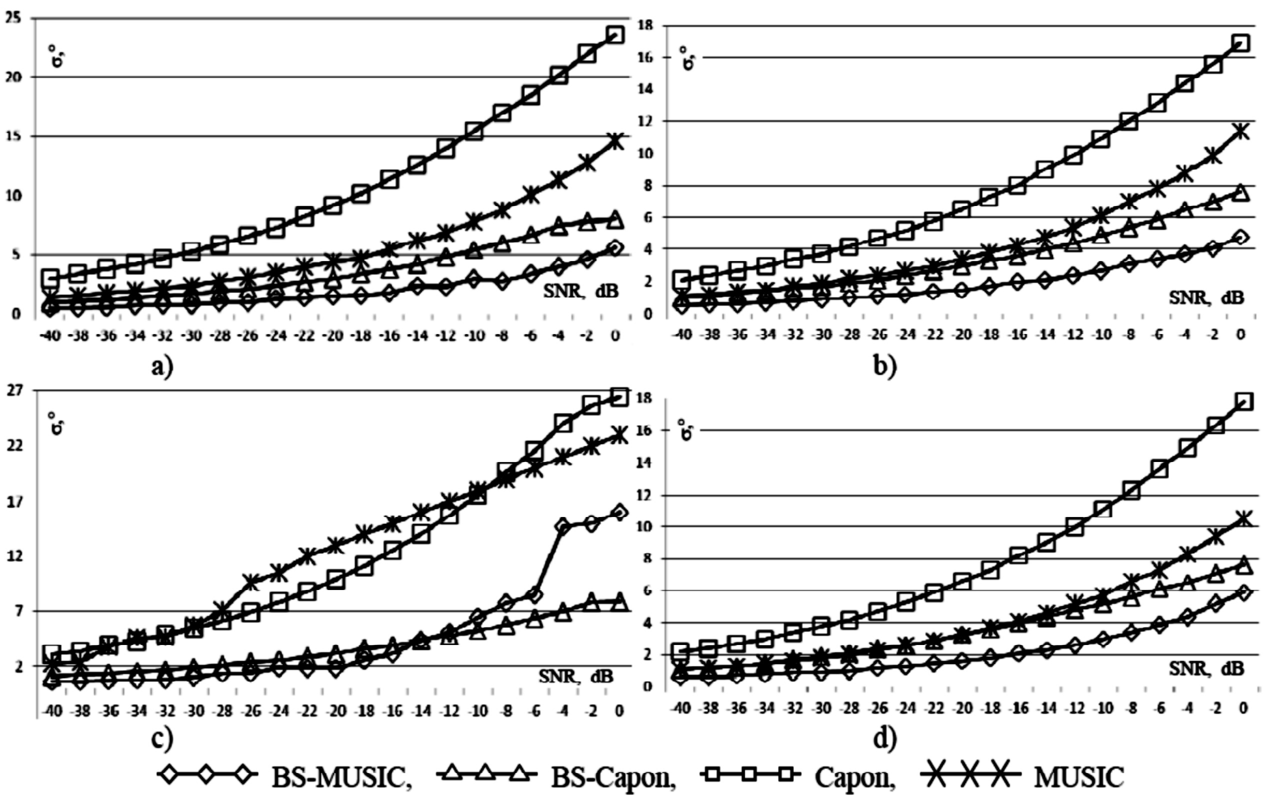

Fig. 6. Dependence of the resolution capability upon SNR at N_{BS}= 4 (*a*, *c*) and N_{BS}= 5 (*b*, *d*) at N = 10 for *a*, *b*) two and *c*, *d*) three RFS.

From Figure 6, it can be seen that at the same noise conditions and the number of rays T_{BS} equal to number of AE of the classical MUSIC and Capon methods the resolution of BS-MUSIC and BS-Capon BS-MUSIC is much higher. Also, it should be noted that at the number of orthogonal rays equal to 5 BS-Capon has a resolution lower than BS-MUSIC. The given results can be confirmed with direction-finding reliefs from Figure 7 in which BS-MUSIC and BS-Capon methods have more distinct and sharp peaks than classical methods of superresolution at identical quantity of N = 5 and N_{BS} = 5 elements and two RFS with azimuths –5° and 10°, SNR = 5dB.

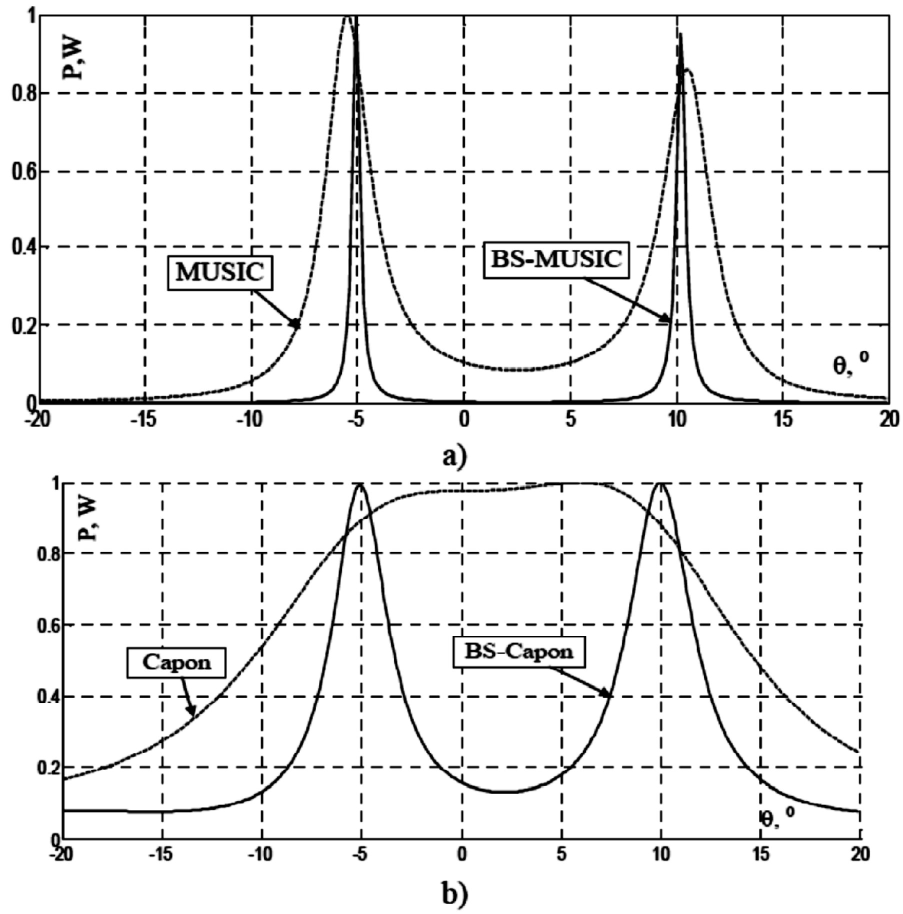

Fig. 7. Direction-finding relief of the methods (*a*) MUSIC and BS-MUSIC and (*b*) *Capon* and *BS-Capon*.

4.5. Evaluation of energy efficiency depending on the window width of scanning

As can be seen from Figure 2, the forms of the orthogonal directional patterns obtained by various methods are very different. Moreover, the difference is noticeable both in the main lobe and side lobes level. For this reason, it is necessary to assess such parameters as the dependence of the window width of the overview depending on the quantity of the directional patterns.

Fig. 8. The window width, depending on the number of orthogonal BF.

For an assessment of the window width formed by a set of orthogonal BF, the modeling of linear density-tapered array, consisting of 64 antenna elements, the distance between which is 0.5 λ, is carried out. The width was estimated as the difference between the extreme points of the entire set of BF according to level -3 dB from the maximum.

In addition to the above methods, the calculation was made for BF former, the weight vectors of which are just $\vec{a}$, shifted to 2π/N and converted to an orthogonal form using the expression (2). Then, smoothing was carried out by Taylor's window to reduce the level of the side-lobe level to -30db.

Figures 8-9 show the calculation data (in Figure and further below "DFT" – discrete Fourier transform, "Taylor Window" – method of reducing SLL to -30 dB "Var. Taylor" – a method of BF forming by a decomposition of the direction vector in a Taylor series, "DPSS" – formation of BF by the spherical sequences).

From Figure 8, it can be seen that all methods show similar window width of the review along the extreme points. The divergence starts when the number of beams is over 34. Receiving the orthogonal directional patterns according to Fourier transformation method gives a wider broad vision window.

To assess the shape and level of the main and side lobes, such parameter was calculated as the energy efficiency (11), *i.e.* the ratio of power inside the overview window that is generated by the set of orthogonal BF to total capacity of all BF in azimuth $[-n; n]$:

$$E = \frac{\sum_{i=1}^{Nbs} \int_{-\psi_0}^{\psi_0} \vec{t}_i^{\,H}(\psi)\vec{a}(\psi)d\psi}{\sum_{i=1}^{Nbs} \int_{-\pi}^{\pi} \vec{t}_i^{\,H}(\psi)\vec{a}(\psi)d\psi} \qquad (11)$$

The values $-\psi_0$, ψ_0 were determined according to the level –3 dB of extreme BF.

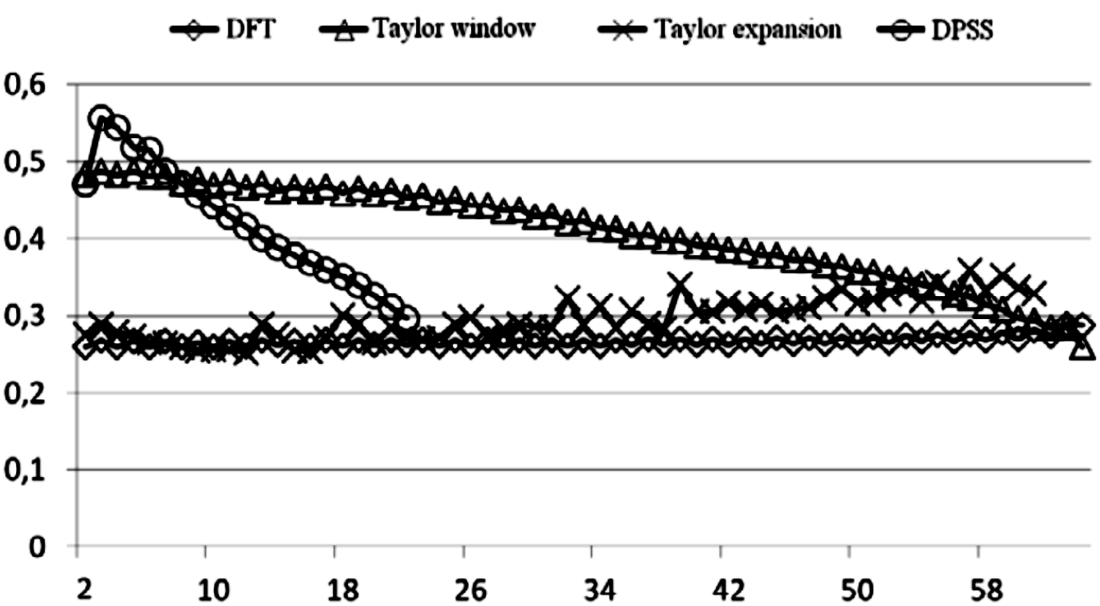

Fig. 9. Energy efficiency in the overview window of orthogonal BF.

From Figure 9, it can be seen that the best efficiency and stability are shown by a smoothing method of Taylor's window. There is a concentration of power inside the window of interest due to the significant reduction of the side-lobe level. Furthermore, it can be seen that for small values of orthogonal beams, *i.e.* $N_{bs} = 4$ and 5, in which the research was conducted, the high efficiency is shown by DPSS spherical sequences method, as is evidenced by the diagrams of Figure 2, the shape of which is symmetric in nature.

5. CONCLUSION

The possibility of using the methods of formation of orthogonal directional patterns for the problem of the assessment of RFS angular coordinates for a linear equidistant antenna array is considered. The application of this methodology to the problem of direction finding with superresolution by MUSIC and BS-MUSIC methods is analyzed using numerical modeling under different values of signal to noise ratio, scale numbering, space correlation matrix and rays of the orthogonal directional patterns. The assessment of the influence of different methods of receiving a matrix of T_{BS} on the direction finding accuracy by means of BS-MUSIC is carried out.

It is found that with the increase of the number of orthogonal beams RMSD reduces. The accuracy of BS-MUSIC for the matrix T_{BS}, obtained according to DPSS method, is the highest and practically coincides with MUSIC both for the case of the presence of signals only in the coverage area and by ingestion of an additional interfering signal into the jamming area for $N_{BS} = 5$. Besides, good indicators of the method DPSS are confirmed by an assessment of the power efficiency. However, the computational complexity is reduced 8 times, according to the criterion of $O(N^3)$. In addition, with the increase in the number of measuring references of averaging up to K = 100, RMSD of BS-MUSIC method is approaching its acceptable value.

By increasing the number of signals from two to three, where one of them should fall on the zeros of the directional pattern, the standard deviation of the methods of the superresolution of BS-MUSIC and BS-Capon increases. When the signals are out of the scan sectors by the orthogonal BF, the robustness and accuracy can be increased by algorithms for convex optimization (Hassanien *et al.*, 2006).

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