International Journal of Control Theory and Applications

ISSN : 0974-5572

Volume 9 • Number 45 • 2016

Automatic Loop Shaping of Robust QFT Controller for Permanent Magnet Stepper Motor using Flower Pollination Algorithm

Nitish Katal[a] and Shiv Narayan[a]

[a]*Electrical Engineering Department, PEC University of Technology, Chandigarh, India.*

E-mail: nitishkatal@gmail.com

Abstract: Automatic design of robust control systems for uncertain systems has gained prominence in recent years and much emphasis is being given on performance robustness. In this paper, loop-shaping step for designing the quantitative feedback theory (QFT) based controllers has been automated for the robust position control of an uncertain permanent magnet stepper motor (PMSM) using flower pollination algorithm (FPA). Long and continuous operation of PMSM leads to spawning of parametric uncertainties in the system and this makes it strenuous for the control system to exert quality control over time; this drastically impacts process safety and quality of products.The proposed method offers flexibility to the designer in designing low-order fixed structure controllers. The QFT design problem has been posed as an optimization problem and FPA has been used to minimize the cost function such that the objectives of robust stability and disturbance rejection are satisfied. The simulation results clearly show that the QFT controller obtained from automated loop shaping using flower pollination algorithm offers robust stability, disturbance rejection and proper reference tracking over a range of PMSM's parametric uncertainty.

Keywords: *Quantitative Feedback Theory; Automatic Loop Shaping; Permanent Magnet Stepper Motor; Flower Pollination Algorithm; Robust Control.*

1. INTRODUCTION

Quantitative Feedback Theory (QFT) has been introduced in 1960's by Issac Horowitz and is based on the Bode's famous gain-phase integrals for designing controllers. QFT controller offers a robust response over a range of plant parametric uncertainty. Loop shaping on Nichols charts is the prime step for synthesising the QFT controller, such that the designed controller satisfies pre-defined performance criteria [1]. Generally the loop shaping of QFT controllers is done manually and the automatic design of QFT controllers is still an open problem [2].

QFT has been successfully applied to several diverse applications ranging form, missile trajectory control [3],[4] to electrical systems [5], [6], [7], waste treatment plants [8] to aerospace applications [9], [10] and even some applications in civil engineering [11]. But most of the work, focuses on the manual loop shaping of the open loop transmission $L_0(jw)$ on the Nichols chart and still there is no assurance that the obtained controller is an optimal one. Several efforts have been made to address the automatic loop shaping of the QFT controllers. Initially, very high order controllers were obtained by Gera & Horowitz [12] who introduced a semi-automatic loop shaping procedure. Some other researchers used linear programming [13], global mixed integer non-linear programming [14] etc. to automate the loop shaping procedure. But these approaches were based on the rational and unrealistic approximations.

In another attempt to automate the synthesis of PSV Nataraj *et. al*[15], [16] expressed the QFT design requirements as quadratic inequalities and used interval constraint satisfaction technique to solve it. Nature inspired algorithms has been widely used in control system design. In QFT, the main aim is to design a controller such that the predefined robustness and performance specifications are met. In QFT controller synthesis, soft computing algorithms like genetic algorithm [17], evolutionary algorithm [18], [19], particle swarm optimization [20] has been used. But the synthesis of the controller is still dependent on the generation and use of templates and bounds.

In this paper, the loop shaping of the QFT controller has been automated using flower pollination algorithm. The designed controller has been implemented for the robust position control of a permanent magnet stepper motor (PMSM) with parametric uncertainties. PMSM is an electromechanical actuator and finds application in applications like robotics, process control, etc. Inherently, PMSM exhibit nonlinear and uncertain behavior. As the dynamics of motor are time variant, so it becomes essential for the controller to exert robust behavior. In this paper, the QFT controller design problem has been posed as an optimization problem and solved using flower pollination algorithm. This eliminates the need of templates and bounds for designing the QFT controller. As per the results obtained in this paper, the system offers a robust response over a range of PMSM's parametric uncertainty.

2. MATHEMATICAL MODEL OF PERMANENT MAGNET STEPPER MOTOR

Physical modeling approach has been used to develop the mathematical model of the system as in [21] and the system has been divided into two sub systems with known behavior a) Electrical subsystem and b) Mechanical subsystem. The PMSM is subject to uncertainties because of the continuous operation and inherited nonlinearities. The schematic representation of the 2 phase PMSM is shown in Figure 1. In figure, each phase is denoted by A & B, there are $2N_r$ magnetic poles in both stator and rotor and uniformly winded at lintervals. The transfer function for the 2 phase PMSM is given by Equation 1 as:

$$G(s) = \frac{\frac{r}{L} w_{np}^2}{s^3 + \left(\frac{r}{L_p} + \frac{D}{J}\right)s^2 + \left(\frac{rD}{L_p J} + w_{np}^2\left(1 + k_p\right)\right)s + \left(\frac{r}{L_p}\right)w_{np}^2} \tag{1}$$

where, $L_p = L - M,$

$$w^2_{np} = \frac{2n_r^2 n\phi_m I_0 \cos\left(\frac{N_r\lambda}{2}\right)}{J}$$

$$k_p = \frac{n\phi_m \sin^2\left(\frac{N_r\lambda}{2}\right)}{L_p I_0 \cos\left(\frac{N_r\lambda}{2}\right)}$$

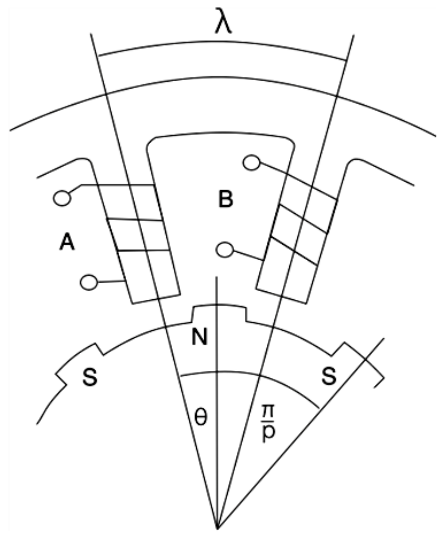

Figure 1: Representation of permanent 2-phase permanent magnet stepper motor

The nominal values of the various parameters of the PMSM along with their uncertainties are given in Table 1.

Table 1
PMSM Variable's Nominal & their Range

Parameter	*Nominal Value*	*Range*
Stator Resistance (r) *ohm*	33	[29.7, 36.3]
Self Inductance (L) *mH*	5.4	[4.86, 5.94]
Mutual Inductance (M) *mH*	0.4	[0.36, 0.44]
Rotor Inertia (J) $g.cm^2$	0.16	-
Number of Rotor Teeth (N_r)	6	-
Viscous Friction (D) $(N.m.s/rad)*10^{-5}$	1.35	[1.215, 1.485]
Tooth Pitch (λ) *rad*	π/12	-
Stationary Current (I_o) Amp	0.15	-
Flux Linkage $(n\phi_M Tm^2)*10^{-3}$	1.2	[1.08, 1.32]

3. QUANTITATIVE FEEDBACK THEORY

Performance robustness of a control system must be assured, so that the designed system does handle the uncertainties. Uncertainties' in the system can be due to several reasons, non-linearity of the components, parametric uncertainties, unmodeled dynamics, etc. [21]. Several established robust control theories like H_2, H¥, LQR, m- synthesis are addressing these issues. But these theories still ignore the fact that, the model used in the controller synthesis is just an approximation of the real system [22]. Thus it becomes essential to model uncertainties into the plant while synthesising the controller.

Issac Horowitz in 1960's introduced a frequency domain controller design technique of QFT based on Bode's famous gain-phase integrals. QFT signifies on shaping the feedback such that a set of predefined objectives of robust stability, sensitivity, tracking performance etc. are satisfied. QFT has a 2-degree of freedom controller structure. A controller K(s) minimizes the effect of closed loop uncertainties and a pre-filter for shaping the response of the system. The configuration of QFT control structure is shown as:

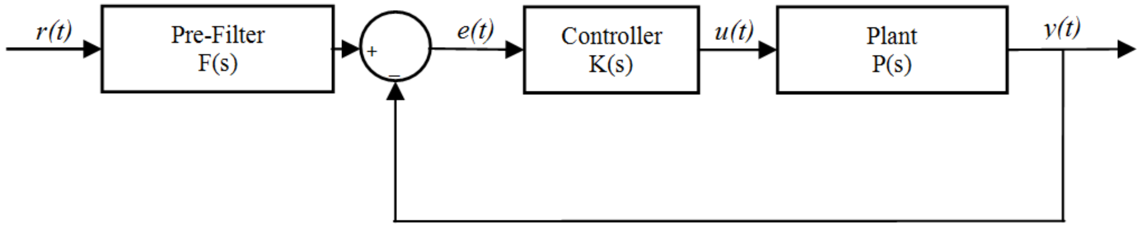

Figure 2: Schematic representation of 2 DoF QFT Configuration

To give insights about the uncertainty in the system, templates are generated. The controller K(s) is designed by shaping the open loop transmission $L_0(s)$ on the Nichols charts. The open loop transmission is shaped such that all the bounds at all the design frequencies are satisfied. Bounds act as guidelines for shaping the open loop transmission such that pre-defined designed specifications are satisfied.

4. FLOWER POLLINATION ALGORITHM

Flower pollination algorithm (FLA) is a meta-heuristic algorithm introduced by Xin She Yang in 2012 [23] FLA mimics the natural phenomenon of pollination, which occurs in flowering plants. In pollination, the pollens are transferred via pollinators like wind, insects, animals etc. and are the medium of reproduction in flowering plants. Pollination happens in 2 ways: *a*) Self Pollination, *b*) Cross Pollination. Flower pollination algorithm can be divided into 4 steps:

1. Cross-pollination is regarded as global pollination. Pollinators perform Lévy flights to transmit the pollens.
2. Self-pollination is regarded as local pollination.
3. The probability of reproduction depends directly upon the perpetuity of the flower involved.
4. Switching probability, $p \in [0, 1]$, influences the global and local pollination.

Pollinators perform Lévy flightsover long distances and flower fidelity governs the pollination and reproduction of the best flower. Mathematically given as:

$$x_i^{t+1} = x_i^t + \mathrm{L}\left(x_i^t - g_*\right) \tag{2}$$

where, x^t_i is the i^{th} pollen at iteration t and g_* is the current best solution at the present iteration.

Step size L signifies the strength of the pollination and is given as:

$$\mathrm{L} \sim \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}}, \left(s >> s_0 > 0\right) \tag{3}$$

where, $\Gamma(\lambda)$ is the standard gamma function, and the Lévy distribution is valid for large flights with $s > 0$.

In local pollination, the flower constancy is given mathematically as:

$$x_i^{t+1} = x_i^t + \varepsilon\left(x_j^t - x_k^t\right) \tag{4}$$

where, x^t_j and x^t_k are pollens form different flowers but of same species. If x^t_j and x^t_k are selected from the same population, this will be a random local search and ε is chosen as uniform distribution *i.e.* $\varepsilon \in [0, 1]$.

5. AUTOMATED LOOP SHAPING OF QFT CONTROLLER FOR PMSM USING FLA

Manual loop shaping of QFT controllers is a difficult task and still there is no guarantee that an optimal controller has been obtained. Automatic loop shaping (ALS) of the QFT controller aims at designing a controller automatically such that pre-specified performance and robustness objectives are met and offers the flexibility to the designer to pre-specify the controller structure. In this paper, two design objectives of robust stability T*(jw)* and sensitivity S*(jw)*are considered. Mathematically robust stability is given as in Eq.5 and Eq. 6 gives sensitivity.

$$T(jW) = \frac{L(j\omega)}{1+L(j\omega)} \tag{5}$$

where,

$$L(jW) = K(jW)\, G_0(jW)$$

$$S(jW) = \frac{1}{1+L(j\omega)} \tag{6}$$

The cost function Jconsidered for the optimization using flower pollination algorithm has been posed as aggregate of function objective function given by Eq. 7. The proposed cost function considers the minimization of the peak amplitude of robust stability and sensitivity at each design frequency.

$$J = \min(|T(jW)| + |W_P \times S(jW)|) \tag{7}$$

where, W_p is the weighting function and has been chosen to reduce the sensitivity over a range of 0-10 rad/sec. Given as:

$$W = \frac{0.5s+10}{s+0.1} \tag{8}$$

6. RESULTS AND SIMULATION

In this paper, for the optimal design of the QFT based controllerK(s), a fixed order controller (PID) has been chosen. The nominal plant transfer function given by $G_0(s)$ as in Eq. 10 has been used to shape the $L_0(s)$ such that the desired criteria is satisfied by minimizing the objective J.

$$K(s) = K_P + \frac{K_I}{s} + K_D \cdot s \tag{9}$$

$$G_0(s) = \frac{350}{s^3 + 15.04 \cdot s^2 + 177.8 \cdot s + 378} \tag{10}$$

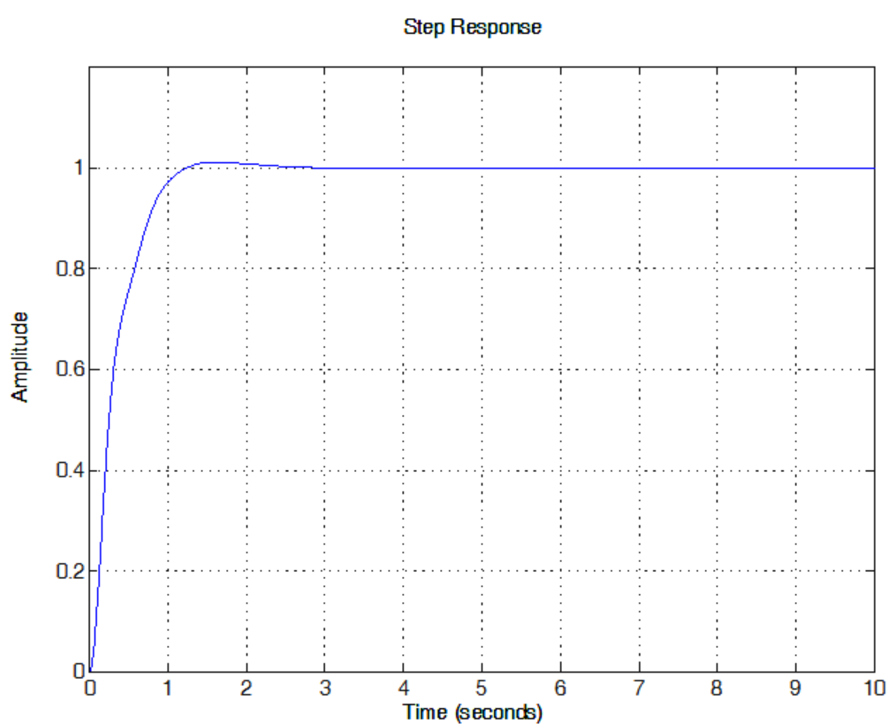

Figure 3: Closed loop step response of the PMSM (nominal case) with FPA tuned QFT controller

The minimization of J using flower pollination algorithm, shapes the $L_0(s)$ such that an optimal QFT controller is obtained. The frequency range chosen for the design purpose as $w = [0.4, 0.8, 1.2, 1.7, 2.1, 10, 25, 50, 100, 200]$. The automatic synthesis of the QFT controller has been carried out using MATLAB and aims at finding the optimal values of $[K_P, K_I, K_D]$ such that the desired criteria is met. The flower pollination algorithm parameters considered in the design process is given in Table 2.

Eq. 11 gives the optimal QFT controller obtained from the tuning using flower pollination algorithm.

$$K(S) \quad = \quad 1.1854 + \frac{3.254}{s} + 0.093 \cdot s \tag{11}$$

To check the response of the system, the controller given in Eq. 1 has been used in closed loop configuration. Figure 3 shows the step response of the closed loop response of the system for the nominal plant case and the magnitude plot for the frequency response of the closed loop system is in Figure 4. Figure 5 shows the plot for $L_0(s)$ on the Nichols chart.

Table 2
FLA Parameters Considered in QFT Design Process

Flower Pollination Algorithm Parameters	*Values*
Population Size	20
Probability Switch	0.8
Iterations	100
β	1.5
Lévy Filght's Step Size	0.01

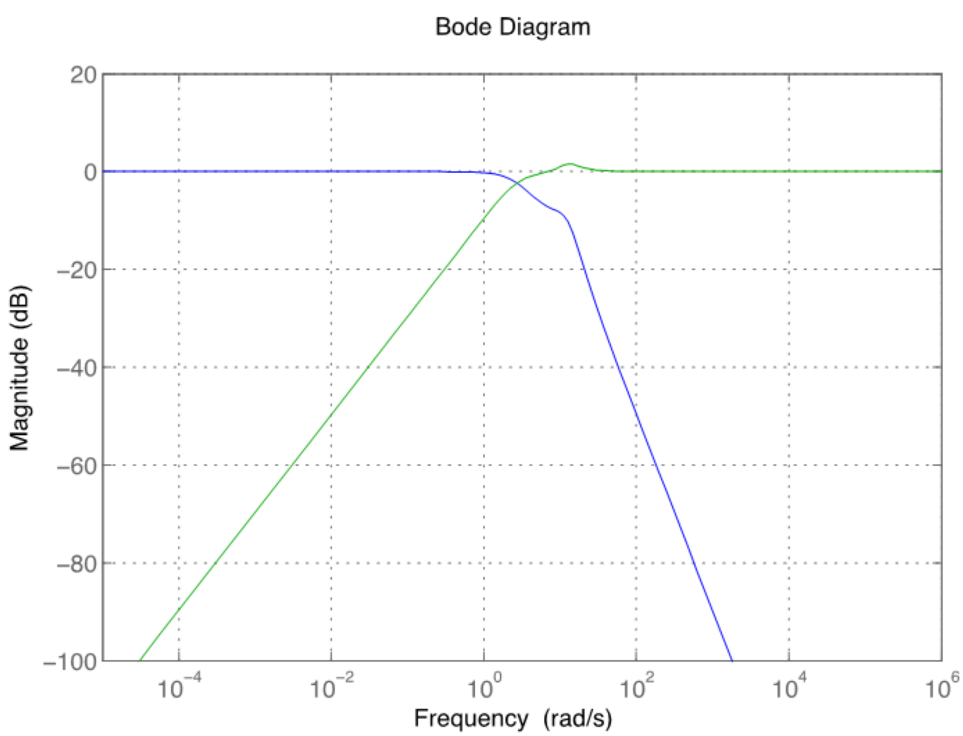

Figure 4: Frequency response of the PMSM (nominal case) with FPA tuned QFT controller

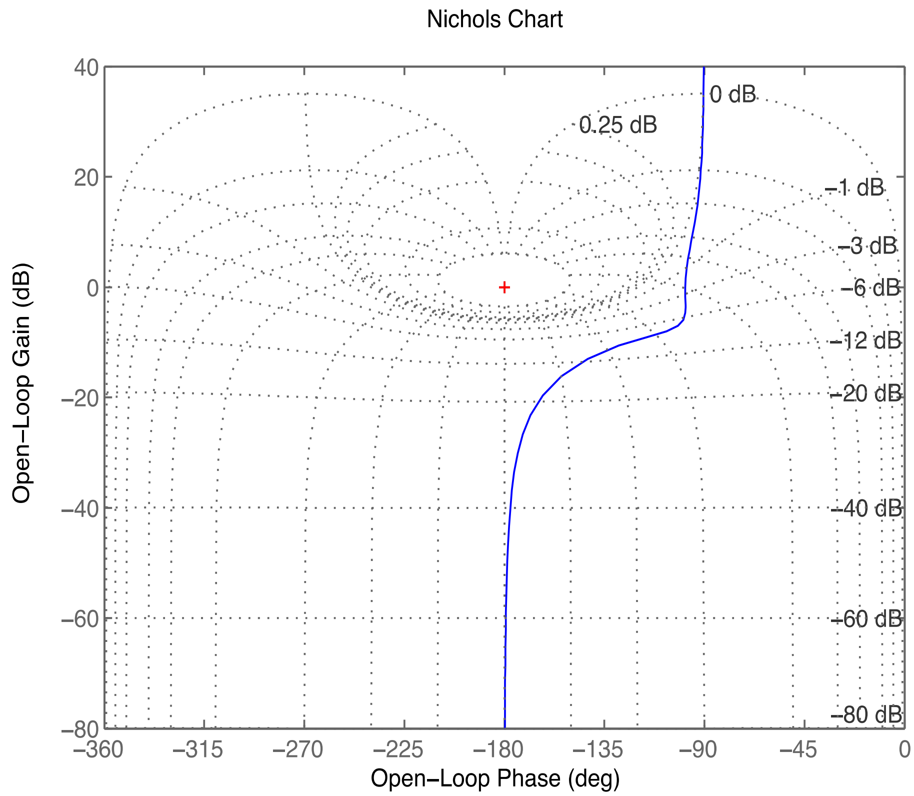

Figure 5: Nichols plot for the open loop transmission $L_0(jw)$ the PMSM (nominal case) with TFPA tuned QFT controller

In table 3, various time domain characteristics of the closed loop PMSM for nominal and worst case response are given.

Table 3
Time domain indices of closed loop system for nominal & worst-case plant scenario

Time Domain Performance Index	*Values*	*Worst Case Response*
Rise Time	0.6737 Sec.	0.6709 Sec (max)
Settling Time	1.049 Sec.	2.0266 Sec (max)
Overshoot Percentage	1.0114 %	3.634 % (max)

7. DESIGN VALIDATION

The designed controller must offer a robust response over a range of plant parametric uncertainties. So, here we have considered parametric uncertainties in the model of the permanent magnet stepper motor. A ±10% variation of parameters as given in Table 1 has been used to develop the uncertain model of the PMSM. Figure 6 shows the step response of the closed loop system with parametric uncertainties and the frequency domain response of the compensated system with parametric uncertainties is shown in Figure 7. From both the figures, it can be seen that the designed controllers offers a stable and robust response both in time and frequency domain and that too in a very tight envelope. Thus with the continuous operation of the PMSM over a longer duration of time, as the parameters will vary, there will be almost negligible effect on the output of the system.

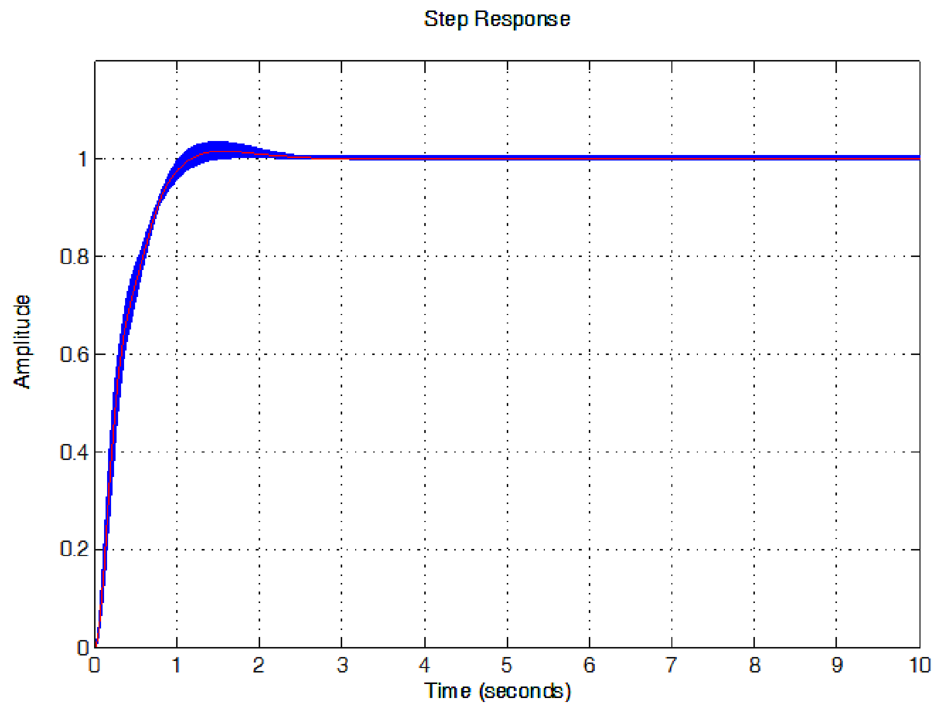

Figure 6: Closed loop step response of the uncertain PMSM (worst case with ±10% uncertainty of nominal value) with FPA tuned QFT controller

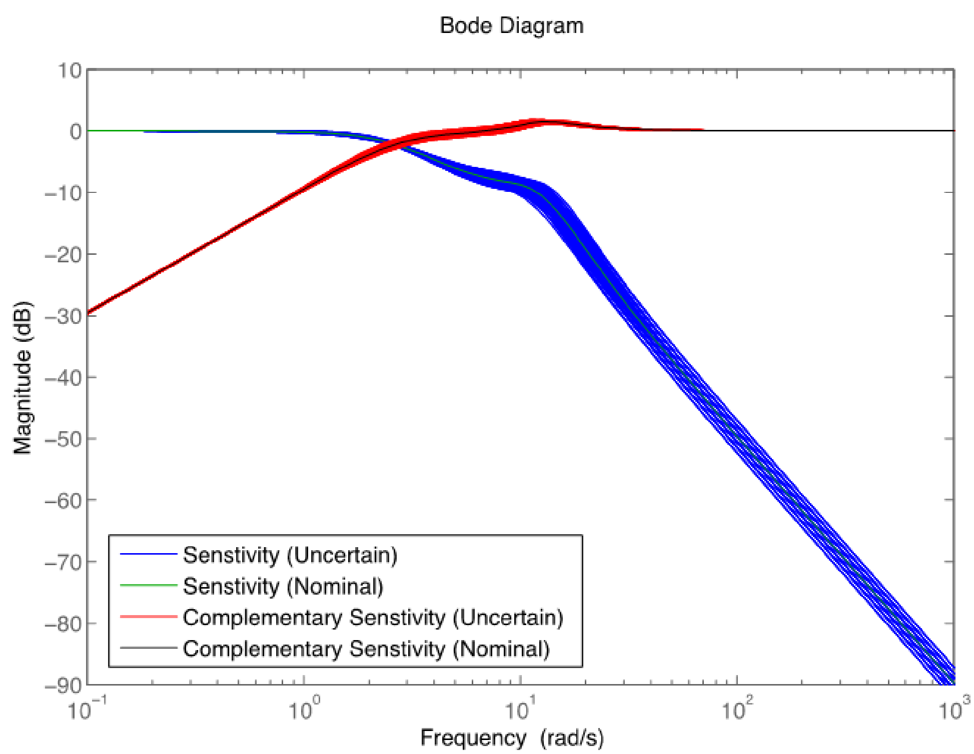

Figure 7: Frequency response of the uncertain PMSM (worst case with ±10% uncertainty of nominal value) with FPA tuned QFT controller

8. CONCLUSIONS

Parametric uncertainties arise in the permanent magnet stepper motor when subjected to long and continuous operation. So for ensuring the process safety and the quality of goods, the quality control must be exerted. QFT tackles the parametric uncertainties by modeling them in the plant used for controller design process. In this paper, the automatic synthesis the QFT controller has been considered for a 2 phase PMSM using flower pollination algorithm. A ±10% parametric uncertainty of the motor parameters has been considered. From the results obtained in this paper, the designed controller offers a robust response over a range of plant's parametric uncertainties and that too in a very tight envelope.

REFERENCES

[1] Horowitz, I.M.: Quantitative Feedback Design Theory (QFT). QFT Publications, Boulder (1993).

[2] S.P. Bhattacharyya, H. Chapellat, L.H. Keel: Robust control: The parametric approach. Prentice Hall, 1st edn. (1995)

[3] Li, C., Ye, Z., Wang, Y., & Liu, L. (2010). Design of attitude decoupling control system for BTT missile using quantitative feedback theory. *The 2010 International Conference on Modelling, Identification and Control (ICMIC)*, 819-824. IEEE.

[4] Zhen, S., Fei, W., &Xiu-zhi, W. (2011). Design of missile lateral channel controller based on QFT. *IEEE Conference on Computational and Information Sciences*. 917-920. IEEE. DOI: 10.1109/ICCIS.2011.128.

[5] Saxena, A.R.; Veerachary, M.. (2010). QFT based robust controller design for fourth-order boost dc-dc switching power converter. IEEE *Joint International Conference on Power Electronics, Drives and Energy Systems (PEDES-2010) Power India, 2010.* 1-6. DOI: 10.1109/PEDES.2010.5712522,

[6] Khodabakhshian, A., &Hemmati, R. (2012). Robust decentralized multi-machine power system stabilizer design using quantitative feedback theory. *International Journal of Electrical Power & Energy Systems*, *41*(1), 112-119. DOI: 10.1016/j.ijepes.2012.03.023.

[7] Alavi, S. M., &Saif, M. (2012). A QFT-based decentralised design approach for integrated fault detection and control. *IEEE Transactions on Control Systems Technology,*,*20*(5), 1366-1375. DOI: 10.1109/TCST.2011.2162646

[8] Barbu, M., &Caraman, S. (2011). *QFT Robust Control of Wastewater Treatment Processes.* INTECH Open Access Publisher.

[9] Xu, D., He, K., Wei, Y., & Zhao, J. (2012). Research on QFT controller design for LOS stabilization system of opto-electronic load for UAV. *10th World Congress onIntelligent Control and Automation (WCICA-2012)*. 1982-1986. IEEE. DOI: 10.1109/WCICA.2012.6358201

[10] Ke, L., Zhengzhong, W., &Qiyou, C. (2014). Design of flight control system for tiltrotor conversion using QFT. *26th The ChineseControl and Decision Conference (CCDC-2014)*. 3073-3076. IEEE. DOI: 10.1109/CCDC.2014.6852702

[11] Amini, F., &Katebi, J. (2008) Application of a robust QFT linear control in civil engineering. *Proceedings of the 14th World Conference on Earthquake Engineering*

[12] Gera, A., & Horowitz, I. (1980). Optimization of the loop transfer function. *International Journal of Control*, 31(2), 389-398. DOI: 10.1080/00207178008961049

[13] Ballance, D. J., &Gawthrop, P. J. (1991). Control systems design via a quantitative feedback theory approach. *International Conference on Control* (Control'91)., 476-480. IET.

[14] Nandakumar, R.; Halikias, G.D.; Zolotas, A.. (2007). Robust control design of a hydraulic actuator using the QFT method. *European Control Conference (ECC-2007).* 2908,2915.

[15] Patil, M. D., & Nataraj, P. S. V. (2012). Automated synthesis of multivariable QFT controller using interval constraint satisfaction technique. *Journal of Process Control,* 22(4), 751-765. DOI:10.1016/j.jprocont.2012.02.006

[16] Jeyasenthil, R., & Nataraj, P. S. V. (2013). Automatic Loop Shaping in QFT Using Hybrid Optimization and Consistency Technique. *Dynamics and Control of Process Systems 10(1).* 427-432. DOI: 10.3182/20131218-3-IN-2045.00160

[17] Chen, W. H., Ballance, D. J., Feng, W., & Li, Y. (1999). Genetic algorithm enabled computer-automated design of QFT control systems. *IEEE International Symposium on Computer Aided Control System Design,* 1999. 492-497). IEEE. DOI: 10.1109/CACSD.1999.808697.

[18] Kim, M. S., & Chung, C. S. (2005). Automatic loop-shaping of QFT controllers using GAs and evolutionary computation. *AI 2005: Advances in Artificial Intelligence.* 1096-1100. Springer Berlin Heidelberg. DOI: 10.1007/3-540-45561-2_18.

[19] Molins, C., & Garcia-Sanz, M. (2009, July). Automatic loop-shaping of QFT robust controllers. *IEEE National Aerospace & Electronics Conference (NAECON-2009)*. 103-110. IEEE. DOI: 10.1109/NAECON.2009.5426643

[20] BiplabSatpati, ChiranjibKoley&SubhashisDatta (2014) Robust PID controller design using particle swarm optimization-enabled automated quantitative feedback theory approach for a first-order lag system with minimal dead time. *Systems Science & Control Engineering: An Open Access Journal*, 2:1, 502-511, DOI: 10.1080/21642583.2014.912570

[21] Ali, Hazem I. "Robust QFT controller design for positioning a permanent magnet stepper motors." *International Journal of Computer and Electrical Engineering* 1.1 (2009): 1793-8198.

[22] Horowitz, I., 1963, Synthesis of Feedback Systems, Academic Press, New York, 1963.

[23] Yang, Xin-She. "Flower pollination algorithm for global optimization." *Unconventional computation and natural computation*. Springer Berlin Heidelberg, 2012. 240-249.