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FLOW OF VISCOELASTIC FLUID THROUGH TRIANGULAR CHANNEL

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ABSTRACT: In this paper numerical solution of viscoelastic liquid through equilateral triangle of side 1 has been studied under the influence of (1) Exponential decreasing pressure gradient, (2) Periodic pressure gradient, (3) Constant pressure gradient One side is taken as axis and a line perpendicular to it at one edge as y-axis. With $h = \frac{1}{2}$ $k = \frac{\sqrt{3}}{12}$ triangular region is descritized giving 25 internal nodal points. Five point finite difference formula [11] is employed. The velocity distribution along the lines $y = \frac{\sqrt{3}}{12}$, $y = \frac{2\sqrt{3}}{12}$, $y = \frac{3\sqrt{3}}{12}$ are investigated at $t = 10^4$ and 10^2 and $w = 10^{-4}$ etc for different values of parameter B and C.

It is observed that maximum velocity is obtained on the line $y = \frac{2\sqrt{3}}{12}$ in all cases of pressure gradients and also velocity found to increase from $x = 0.80$ to 0.85 on the line $y = \frac{\sqrt{3}}{12}$ and decreases there afterward in all cases of pressure gradient. Velocity is parabolic on line $y = \frac{2\sqrt{3}}{12}$, $y = \frac{3\sqrt{3}}{12}$ in all cases of pressure gradient. Velocity is maximum at $x = 0.5$ in all cases of pressure gradient.

Key words: Viscosity, Shear stress, Finite difference methods, Stress tensor, Non-Newtonian fluid, Kinematic viscosity.

1. INTRODUCTION

Non-Newtonian liquids such as blood, thick oils, pastes, paints, colloid solutions are highly viscous. Their behavior cannot be explained by the classical hydrodynamic stress-rate strain relations. Generalizing the stress-rate of strain relations of classical hydrodynamics, the rhelogical behavior of the Non-Newtonian liquids have been studied by Rivlin [8], Rivlin and Reiner [9]. Langloise and Rivlin [4] have studied slow steady state flow of viscoelastic fluids through non-circular tubes. Rivlin [10] has discussed some exact solutions of viscoelastic fluids. Dutta [1] has obtained the solutions for viscoelastic Maxwell fluids through a circular annulus. Jones and Walters [2, 3] have discussed the oscillatory motion of viscoelastic liquid. Elsayed F., [12] *et al*. Have studied peristaltic viscoelastic fluid motion in a tube. Daniel D. Joseph [14] have studied viscous and visoelastic potential flow. Paulo J. Olivera [13] have studied a symmetric flows of viscoelastic fluids in symmetric planner expansion geometries. Nand Lal Singh [6] studied unsteady flow of a viscoelastic fluid between two parallel planes under periodic pressure gradient. In view of the considerable interest being evinced at present in the field, it was considered worthwhile to study the flow of viscoelastic liquid specified by three constants λ_1, λ_2, γ_0 . In a channel with equilateral triangular channel.

2. EQUATIONS OF MOTION

The equations of motion together with stress-rate of strain relations of viscoelastic liquids characterized by three material constants a viscosity coefficient and two relaxation times under the approximation of small rates of strain are given by

$$\tau^{ij} = -pg^{ij} + \tau'^{ij} \tag{1}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\tau'^{ij} = 2\eta_0\left(1+\lambda_2 \frac{\partial}{\partial t}\right)e^{ij} \tag{2}$$

$$e_{ij} = (V_{i,j} + V_{j,i})/2 \tag{3}$$

$$\rho\left(\frac{\partial V^i}{\partial t} + V^i, jV^j\right) = \tau^{ij}, j \tag{4}$$

$$V^i, i = 0\,. \tag{5}$$

Operating by $(1+\lambda_1 \frac{\partial}{\partial t})$ on equation (4) and using equations (1) and (2) we have

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial V^i}{\partial t} + V^i, jV^j\right) = -\frac{1}{\rho}\left(1+\lambda_1 \frac{\partial}{\partial t}\right)$$

$$\times \rho g^{ij}, j + 2\gamma_0\left(1+\lambda_2 \frac{\partial}{\partial t}\right)e^{ij}, j\,. \tag{6}$$

Where τ^{ij} and τ'^{ij} denote stress and deviatoric stress tensors, V^i the components of velocity, g^{ij} are contravarient components of metric tensor, e_{ij} the strain rate of deformation, P the pressure, ρ the density, γ_0 kinematic viscosity the coefficients η_0, λ_1 and λ_2 are material constants, subject to conditions (such as $\eta_0 > 0$, $\lambda_1 \geq \lambda_2 \geq 0$) dictated by thermodynamic principles.

It was pointed out by Oldroyd [7] that for a liquid at rest any small sheerstress decays at e^{-t/λ_1} and in a liquid element free from stress, any small rate of strain decays as at e^{-t/λ_2} Michael C. Williams and R. Byron [5] has shown that the ratio of material constants (λ_2/λ_1) varies from 1/9 to 2/3

3. FORMULATIONS OF PROBLEM

We shall investigate the flow of viscoelastic liquid described by (1) to (5) equations through a pipe whose cross section is equilateral triangle of side 1 under the influence of

1. Exponentially decreasing pressure gradient ($\varphi(t) = \alpha e^{-m^2 t}$)
2. Periodic pressure gradient ($\phi(t) = \alpha \cos wt$)
3. Constant pressure gradient ($\phi(t) = \alpha$)

The axis of the cylinder is taken as Z-axis

$$V_1 = 0,\ V_2 = 0,\ V_3 = V_z\,(x, y, t)\ \text{ with boundary condition }\ V_z = 0\ \text{ on }\ \partial D \tag{7}$$

Using the equations (3), (4), (6) and (7) and Assuming

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = \phi(t). \tag{8}$$

Where $\phi(t)$ is one of the pressure gradients defined above

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial V_z}{\partial} = -\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \gamma_0\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2 V_z \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{d y^2} \tag{10}$$

and $\gamma_0 = \dfrac{\eta_0}{\rho}$ is Kinematic viscosity.

Assuming V_z = Real part of $V_z = V(x, y)\, e^{i\omega t}$ for constant and periodic pressure gradients and $V_z = V(x, y)^{-m^2 t}$ for exponentially decreasing pressure gradient.

Using above defined velocity components and pressure gradients the equation (9) reduces to

$$\nabla^2 V + BV + C = 0 \tag{11}$$

With boundary condition $V = 0$ on ∂D

Where B and C are

For exponentially decreasing pressure gradient

$$B = \frac{m^2}{\gamma_0}\left(\frac{1-\lambda_1 m^2}{1-\lambda_2 m^2}\right), \qquad C = \frac{\alpha}{\gamma_0}\left(\frac{1-\lambda_1 m^2}{1-\lambda_2 m^2}\right).$$

For Periodic pressure gradient

$$B = \frac{\omega}{\gamma_0}\left(\frac{\sin\omega t + \lambda_1\cos\omega t}{\cos\omega t - \lambda_2\sin\omega t}\right), \qquad C = \frac{\omega}{\gamma_0}\left(\frac{\cos\omega t - \lambda_1\omega\sin\omega t}{\cos\omega t - \lambda_2\omega\sin\omega t}\right).$$

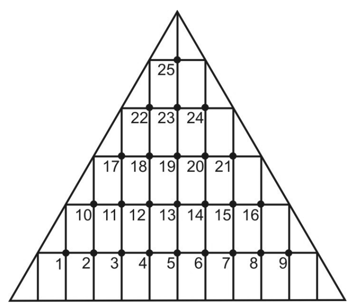
For constant pressure gradient

$$B = \frac{\omega}{\gamma_0}\left(\frac{\sin\omega t + \lambda_1\cos\omega t}{\cos\omega t - \lambda_2\sin\omega t}\right), \qquad C = \frac{\alpha}{\gamma_0}\left(\frac{1}{\cos\omega t + \lambda_2\omega\sin\omega t}\right).$$

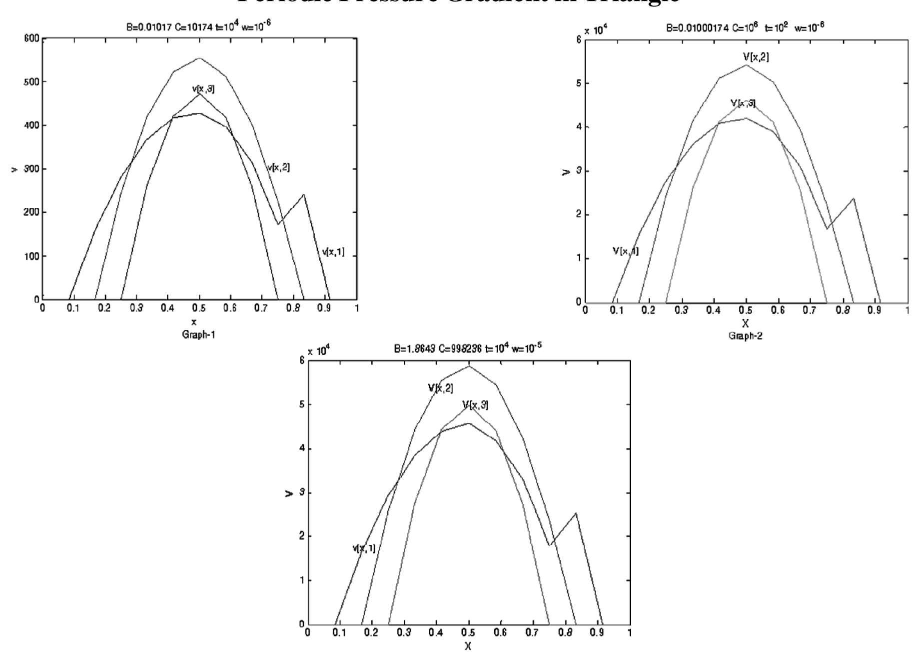
A five point difference analogue for the above equation can be written as

$$\frac{h_2 v(x_i+h_1, y_j) - h_1 v(x_i+h_2, y_j)}{h_1 h_2 (h_1 - h_2)} + v(x_i, y_j)\left[\frac{1}{h_1 h_2} + \frac{1}{k_1 k_2} + B\right] + \frac{k_2 v(x_i, y_j+k_1) - k_1 v(x_i, y_j+k_2)}{k_1 k_2 (k_1 - k_2)} + C = 0. \qquad (12)$$

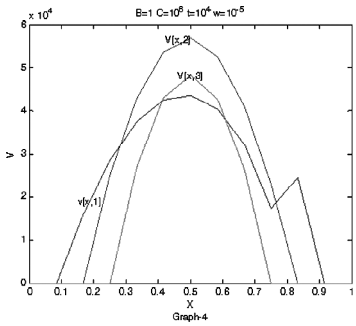
An equilateral triangle of side "1" taken using one side as axis of x and a line perpendicular to its edge as axis of y. The triangular region is discritized with $h = \frac{1}{2}$ along x-axis $K = \frac{\sqrt{3}}{12}$ along y axis which gives rise to 25 internal nodal points. With $V(x, y) = 0$ on boundary, applying five point difference formula (12) at internal nodal points we get 25 is linear equations.

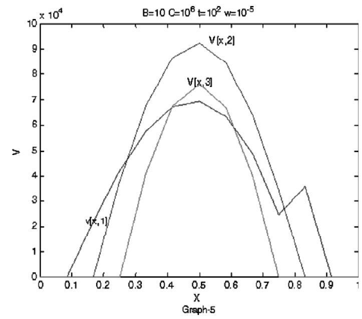


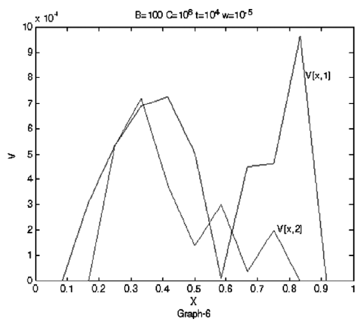
Periodic Pressure Gradient in Triangle



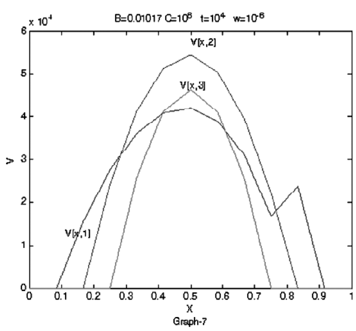
Exponentially Decreasing Pressure Gradient in Tringle

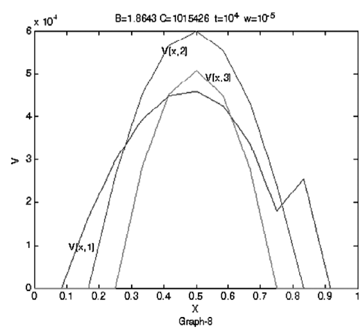






Constant Pressure Gradient in Triangle





4. PARAMETRICAL VALUES

The following values are given to the parameters

$$\lambda_1 = \frac{1}{100}, \gamma_0 = 10^{-6}, m = 10^{-3},$$
$$\lambda_2 = \frac{1}{300}, \alpha = 1.$$

$\frac{\omega}{\gamma_0}$ = 100, 10, 1; $k = 1$ velocities are computed at $t = 10^4$ and 10^2

5. Discussion

Incase of periodic pressure gradient it is observed from Graph (1) represents velocity distribution on $y = \frac{\sqrt{3}}{12}$ when $t = 10^4$ the velocity of fluid increases upto $x = 0.5$ obtaining maximum at $x = 0.5$ and decreases there afterwards the velocity start increasing from $x = 0.8$ upto 0.85 and decreases there onwards. The same phenomena is observed for all values of B, t and ω in this region. Maximum velocity is obtained on $y = \frac{2\sqrt{3}}{12}$ and decreases in all lines parallel to $y = \frac{2\sqrt{3}}{12}$ above and below. The velocity increase on the line $y = 2\frac{\sqrt{3}}{12}$ and taking a parabolic shape with maximum velocity at $x = 0.5$. The velocity found to decrease on line $y = \frac{3\sqrt{3}}{12}$ and taking a parabolic shape. When $t = 100$ the velocity found to increase with same pattern of flow as discussed above Graph (2). Increasing B does not show much variation in the flow pattern Graph (3).

In Case of exponentially decreasing Pressure gradient same type of flow pattern is observed at $y = \frac{\sqrt{3}}{2}$ as in periodic pressure gradient. Velocity profiles take same shape when $B = 1$ Graph (4) and $B = 10$ Graph (5). However when B is increase abnormally a wave like velocity distribution observed in $y = \frac{\sqrt{3}}{12}$ and $y = \frac{2\sqrt{3}}{12}$ Graph (6).

In case of Constant Pressure gradient the velocity profiles found to be of the same pattern as discussed above i.e. $y = \frac{\sqrt{3}}{12}$ same pattern of flow as observed in periodic pressure gradient and also $y = \frac{2\sqrt{3}}{12}$ and $y = \frac{3\sqrt{3}}{12}$. The flow profile is parabolic in nature taking max at $x = 0.5$ Graph (7)&(8) in this case B has no much effect on flow pattern.

6. Conclusion

Finally it is observed that line $y = \frac{\sqrt{3}}{12}$ the velocity profile is not parabolic but subject to variations $x = 0.8$ to $x = 0.85$ and velocities are found to increase as we move from base up to $y = \frac{2\sqrt{3}}{12}$ and taking maximum at $x = 0.5$ decrease there afterwards in all types of pressure gradient velocities are maximum on line throughout the cross section of triangle. In case of exponential pressure gradient an abnormal increase in B i.e. parameter m, kinemetic Viscosity, material constant play very important role and makes the flow un-uniform and velocity profile takes wave like form. In case constant pressure gradient not much variation is observed on basis B-value.

References

[1] Dutta S. K., *J. Angew Math. Mech.*, **41B**, (1961), 219.

[2] Jones J. R., and T. S. Walters, *Mathematics*, **12**, (1965), 246.

[3] Jones J. R., and Walters, *Mathematics*, **13**, (1966), 83.

[4] Langlois W. F., and Rivlin R. S., Report to Office of Ordinance Research U.S. Army, (Brown University), 1959.

[5] Michael C. Williams, and R. Byron, Physics of Fluids (Research Notes), (1962), 1126-1127.

[6] Nand Lal Singh, *Indian Journal of Pure and Applied Mathematics*, **14**(11), (1983), 1362, Now.

[7] Oldroyd J. G., *Proc. Roy. Soc. London Series A*, **245**, (May-July, 1958), 278-297.

[8] Rivlin M., *Quarterly Journal of Mech. Appl. Maths.*, **16**, (1956), 1164.

[9] Rivlin R. S., and Reiner, *Proceedings of Royal Society*, **193a**, (1948), 260.

[10] Rivlin R. S., *Journal of Rational Mech. Anal.*, **5**, (1952), 179.

[11] Introductory Numerical Analysis of Elliptic Boundary value Problems by Green Span, (A Harper International Edidtion).

[12] Elsayed F. *et al*., *IJMMS*, **26**(1), (2001), 21-34.

[13] Paluo J. Olivera, *J. of Non-Newtonian Fluid. Mech.*, **114**, (2003), 33-63.

[14] Danial D. Joseph, *ICTAM*, (2004), 1-2.

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