

Multi Level Thersholding Of Color Images using Heuristic Algorithm

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Abstract : In upcoming research, the segmentation of images in RGB scale itself is becoming more eminent for wide applications. This paper describes an optimal colour image segmentation using various heuristic algorithms. High contrast standard images from openly available data sets have been considered. The algorithm is tested on various methodologies such as Enhanced Bacterial Foraging Algorithm (EBFO), Particle Swarm Optimisation (PSO), Darwinian Particle Swarm Optimisation (DPSO), Fractional Order Darwinian Particle Swarm Optimisation (FODPSO) are considered for implementation. The performance of the segmentation is validated using parameters such as Jmax, Peak Signal to Noise Ratio(PSNR),Root Mean Square Error(RMSE).

Keywords : Color image segmentation, methodology, quality measures, histogram, convergence.

1. INTRODUCTION

Image segmentation [1] generally examines grey scale images but retrieval of complete information from the same. In such cases, color images[2] are used which have a great deal of extra information lending a helping hand to simplify image analysis like object identification and extraction based on color. The input images preferred in this paper are standard RGB database .The database includes high contrast color images Lena, Jet, Mandrill, Couple and House.

The main aim of the optimization technique [3] is to find the set of parameters for the maximum and minimum values for the given image at some specific constraints. A value which is to be chosen should satisfy all the constraints for the set of parameters to obtain a feasible solution. A feasible value obtained from all the techniques is named the optimal solution. Optimal outputs[4] are said to be obtained only if they are within a specified range of pixel density. It is also necessary to ensure that the time consumed to give the best results must be less. In order to attain the above requirements, heuristic algorithm is used. Optimization is widely used in scheduling, sequencing, allocation, decision making, etc. Research in optimization is active in finding new optimization methods. There are several methods [5] used for segmentation out of which, PSO, DPSO, FODPSO and EBFO have been used.

Similar tests have been performed using Otsu based bi-level and multi-level segmentation using improved PSO technique[6] for the same datasets but without converting color images to grey scale. Likewise, controlling of the non-linear system parameters is done optimally by using BFO algorithm.[7]

2. METHODOLOGY

This section presents the methodology adopted in this work. Fig.1 depicts the stages considered

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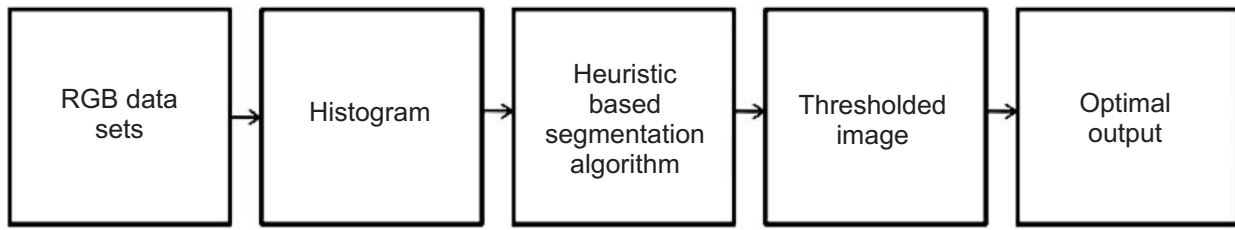


Figure 1: Block diagram of heuristic based segmentation procedure for color image

A. Particle Swarm Optimization

A particle swarm optimization [8] is a stochastic optimization algorithm model. The PSO technique is a implementation of swarm of particles, where each particle represents a possible solution to an optimization problem. Each particle is to be heuristically initialized in this approach. Heuristic algorithm[6] gives the best of all possible methods. In most of the cases only an appropriate value is found. The best position from the previous and neighborhood, is taken into consideration in order to find the best fitness.

$$v_{id}^{t+1} = w \cdot v_{id}^{t+1} + c1 \cdot \phi1 \cdot (p_{id}^t - x_{id}^t) + c2 \cdot \phi2 \cdot (p_{gd}^t - x_{gd}^t) \quad (1)$$

$$X_{id}^{t+1} = X_{id}^{t+1} + v_{id}^{t+1} \quad (2)$$

v_{id}^t – Component in dimension d of the i^{th} particle velocity in iteration t

x_{id}^t – Component in dimension d of the i^{th} particle velocity in iteration t

$c1, c2$ – Constant weight factor

P_i – Best position achieved so long by particle i

P_g – Best position found by the neighbours of particle i

$\phi1, \phi2$ – Random factors in the $[0,1]$ interval

ω – Inertia weight

B. Darwinian Particle Swarm Optimization

A problem with optimization is that the scope of the algorithm is restricted *i.e* particular algorithm may work in one problem but fail in other. The algorithm could be modified to adjust itself to fit into the landscape. In typical implementation of PSO a single swarm of test solution is utilized which is unable to differentiate between local and global optimum. So it cannot be extended to model natural selection. At the end of each swarm update, fitness is used to order the particles. In search of best model selection using PSO, Darwinians Particle Swarm Optimization [8,9] is preferable wherein, the solution may exist at any time. The selection is implemented with a constant changing collection of swarm. The result could be a more robust algorithm with wider applicability.

C. Fractional order Darwinian Particle Swarm Optimization

A new method to control DPSO based on pyrus et al is introduced as FODPSO [8,10]. Fractional calculus attracts the attention of several researcher that can be applied in various scientific fields. The Grunewald Letnikov based concept of fractional differential with fractional coefficient is given by $\alpha \in C$ of general signal $x(t)$ is given by

$$D^\alpha[x(t)] = \text{Lim}_{h \rightarrow 0} \left[\frac{1}{h^\alpha} \sum_{k=0}^{+\infty} \frac{(-1)^k \Gamma[\alpha + 1] x(t - kh)}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} \right] \quad (3)$$

Where Γ is the gamma function

The equation reveals that the integer order derivative implies finite series. Integer derives a local operator while fractional derivative has implicitly, a memory of all past events. However the influence of the equation is represented based on the equation of a discrete time implementation expression defined as

$$D^\alpha[x(t)] = \frac{1}{T^\alpha} \sum_{k=0}^r \frac{(-1)^k \Gamma[\alpha + 1] x(t - kT)}{\Gamma(k + 1) \Gamma(\alpha - k + 1)} \quad (4)$$

The characteristics is revealed by fractional calculus making this very useful for inherent memory property. Considering the inertial property of influence in equation ; assuming $w = 1$, $T = 1$ works based on the expression

$$D^\alpha[v_{i+1}^n] = \rho1r1(g_i^n - x_i^n) + \rho2r2(x_i^n - x_i^n) + \rho3r3(n_i^n - x_i^n) \quad (5)$$

Similar results for $r \geq 4$ were obtained for the algorithm based on preliminary experimental test[8]. The computational requirement increases linearly for r i.e FODPSO present $O(r)$ of memory complexity. Hence, using the first $r = 4$ terms of differential derivative given by the equations,

$$v_{i+1}^n = av_i^n + \frac{1}{2}av_{i-1}^n + \frac{1}{6}\alpha(1-\alpha)v_{i-2}^n + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)v_{i-3}^n \\ + \rho1r1(g_i^n - x_i^n) + \rho2r2(x_i^n - x_i^n) + \rho3r3(n_i^n - x_i^n) \quad (6)$$

Moreover FDPSO may be a collection of FOPSO's in which each swarm individually performs some natural selection tools.

D. Enchanted Bacterial Foraging Optimization Algorithm

This is inspired by the foraging behavior of eschirichia coli bacteria. It is used to find near optimal solutions that simultaneously meet multi objective function. An important generic algorithm proposed by BFO is EBFO [7,11]. It introduces a selection and crossover operation to increase the global optimization of BFO[12]. The way of representation of chemotaxis is used as the position initialization of bacteria in next chemotaxis. By using the different iterative value, the maximum patches of the different images occur in different levels called nodes. The convergence value of the test images at the various levels are to be noted based on the heuristic algorithm obtained. By applying the various threshold values, the parameters are to be analyzed at different position in the given test images. The initial algorithm parameters [7] are assigned as follows

$$N_c = N/2 ; N_s = N_re / 3; N_ed / 4; N_r = N/2$$

$$Ped = \frac{N_ed}{N + N_r}$$

$$d_{\text{attractant}} = W_{\text{attractant}} = \frac{N_s}{N}$$

$$h_{\text{repellent}} = W_{\text{repellent}} = \frac{N_c}{N}$$

N = Number of E.Coli bacteria

N_c = total number of chemotaxis steps

N_s = swim length during the search

N_re = total number of reproduction steps

N_ed = total number of elimination-disposal events

N_r = number of reproduced bacteria





















Ped = the probability than each bacterium will be eliminated/dispersed

n = the run length

3. IMPLEMENTATION OF TECHNIQUES

Characteristics of the image measures image degradation by using several techniques [5]. By applying the different techniques, on changing the threshold value, RGB of the images can be obtained.

Table 1
RGB output images for various threshold values


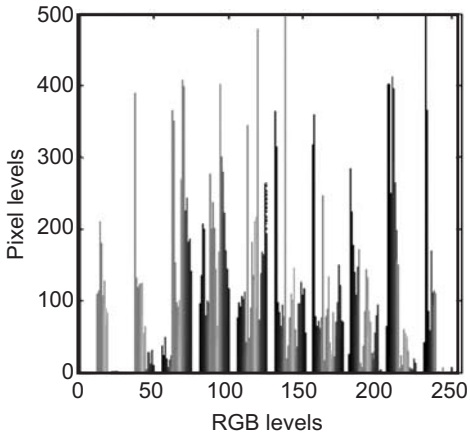
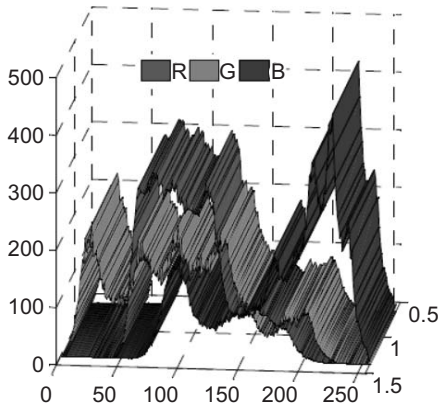

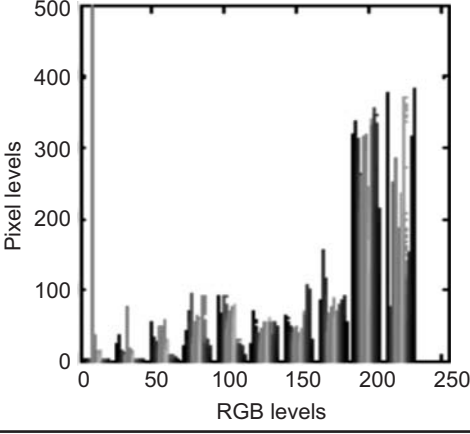
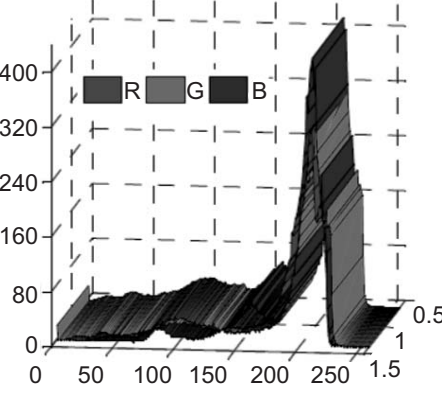

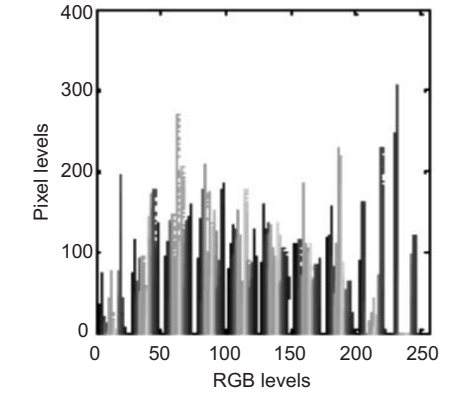
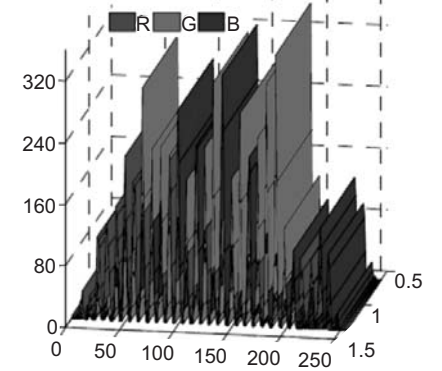

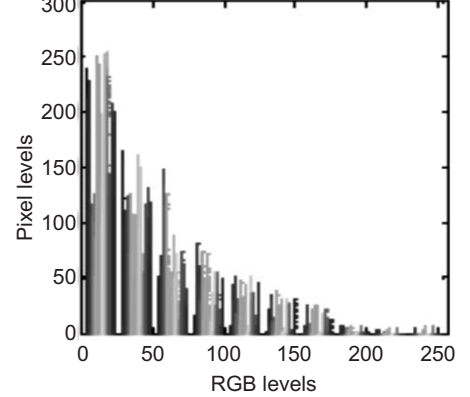
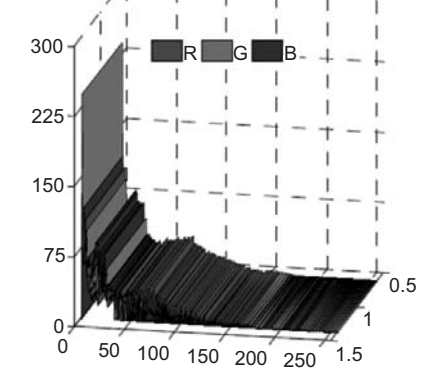
	$m = 2$	$m = 3$	$m = 4$	$m = 5$
Lena				
Jet				
Mandrill				
Couple				
House				

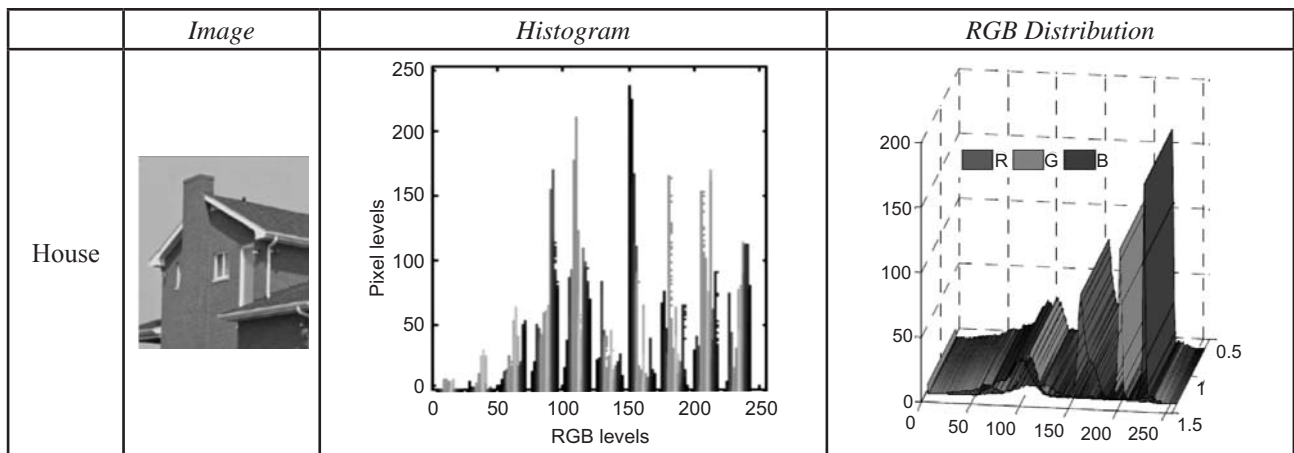
A. Histogram and Thresholding

The probability distribution of a quantitative variable can be estimated by a graphical representation of a numerical data namely histogram [13]. In constructing a histogram the entire range is divided into a series of intervals. The intervals are required to be adjacent to each other and of the same size but need not be of the same width. The total distribution in a image is represented graphically using image histogram[14]. The numbers of pixels are plotted for every tone value. For a dark image, the histogram will have majority of its data points on the left side and centre of the graph.

Conversely, the histogram for a bright image with few dark areas data points on the right side and centre of the graph. For digital images, a colour histogram gives the number of pixels having colours within a fixed limit of color range. The analysis of the RGB color space emphasis on the visual perception of the variation, saturation and intensity values of an image pixel. If an image is at a position say (i, j)

Table 2
RGB histogram and ribbon plot for considered image

	<i>Image</i>	<i>Histogram</i>	<i>RGB Distribution</i>
Lena			
Jet			
Mandrill			
Couple			



$Image(i, j, 1)$ gives the value of RED pixel

$Image(i, j, 2)$ gives the value of BLUE pixel

$Image(i, j, 3)$ gives the value of GREEN pixel

For both image segmentation and histogram generation, the feature extraction method has been applied. The histogram consists of a uniform color transition that provides a window-based smoothing during retrieval. Color lines are robust to color distortion [15] and provide a compact but useful representation of the colors in a scene. Histogram modification has been used for the enhancement of color images, which has been restricted to processing the luminance component and in some cases saturation.

4. RESULTS AND DISCUSSION

The RGB output images for various threshold values are shown in table 1. The color histogram represents the pixel distribution for each tone of the image. Every 2D pixel of the image can be viewed as 3D coordinates in the color space RGB. Better quality output can be obtained by varying the threshold value. The threshold values for levels 2,3,4 and 5 for various images are tabulated in table 3. Based on the threshold values the positions at which the intensity of a specific color is maximum can be observed. With increase in threshold value the number of positions also increases. As the threshold values are increased, the distribution of RGB at various positions can be effectively determined.

Table 3
Objective function and image quality measures for the segmented images

	<i>m</i>		<i>EBFO</i>	<i>PSO</i>	<i>DPSO</i>	<i>FODPSO</i>
Lena	2	R	16, 39	12, 33	14, 110	13, 112
		G	7, 104	8, 107	7, 102	7, 103
		B	3, 150	4, 152	3, 148	3, 149
	3	R	16, 93, 154	11, 42, 138	17, 94, 152	16, 96, 155
		G	9, 68, 146	8, 70, 142	11, 68, 147	10, 72, 144
		B	4, 120, 182	2, 118, 180	4, 121, 182	4, 120, 180
	4	R	18, 80, 123, 142	18, 79, 122, 143	17, 78, 122, 146	16, 80, 124, 142
		G	10, 51, 118, 160	10, 50, 116, 158	11, 50, 114, 157	10, 52, 115, 161
		B	3, 107, 176, 198	4, 108, 171, 194	5, 106, 174, 195	4, 107, 174, 200
	5	R	18, 83, 102, 148, 171	18, 84, 105, 146, 179	18, 82, 104, 146, 168	19, 81, 102, 146, 170
		G	9, 51, 78, 117, 164	10, 53, 76, 119, 162	10, 54, 75, 120, 166	11, 50, 74, 118, 162
		B	4, 112, 153, 179, 202	5, 110, 155, 176, 198	3, 114, 158, 180, 204	4, 110, 156, 178, 200

Jet	2	R	15,177	16,166	14,168	14,165
		G	9,153	10,146	10,145	8,146
		B	4,146	5,138	2,140	4,140
	3	R	14,124,177	15,122,180	14,121,182	15,124,178
		G	8,98,175	10,101,174	9,98,172	8,100,173
		B	2,114,168	4,115,169	4,115,166	3,113,167
	4	R	16,138,176,196	14,134,175,196	14,136,175,197	15,137,174,198
		G	10,68,130,190	11,68,128,192	10,70,128,188	9,71,130,194
		B	4,104,145,185	3,106,144,186	5,102,148,184	4,103,146,188
	5	R	16,116,158,179,201	16,118,160,180,202	15,118,158,182,202	16,120,161,178,201
		G	10,73,110,173,194	9,72,112,175,196	10,74,109,176,194	10,72,75,174,195
		B	2,86,128,167,186	4,85,127,165,188	4,88,128,169,190	5,86,126,168,188
Mandrill	2	R	11,118	10,168	12,130	11,116
		G	7,112	8,114	7,127	6,128
		B	2,123	4,126	4,122	2,122
	3	R	14,81,171	12,82,174	14,83,170	12,80,168
		G	9,94,148	10,92,148	10,95,146	10,94,145
		B	3,108,160	2,110,157	2,110,161	4,107,160
	4	R	16,67,125,181	15,66,122,182	14,68,124,180	14,64,122,182
		G	10,81,120,157	8,80,121,158	9,83,118,158	10,80,121,159
		B	4,78,114,170	2,74,116,168	2,80,115,172	3,76,118,170
	5	R	18,54,95,138,196	15,51,98,134,200	17,50,95,136,202	18,56,96,132,197
		G	12,71,112,130,171	10,70,115,131,174	11,73,114,130,173	12,70,118,127,170
		B	3,66,107,142,184	4,64,104,147,182	4,66,109,144,188	3,62,111,140,184
Couple	2	R	4,53	5,59	5,54	5,58
		G	3,51	3,55	3,46	3,50
		B	1,47	1,50	2,50	2,48
	3	R	8,30,74	8,34,79	7,32,84	10,36,87
		G	4,29,80	5,25,88	4,28,81	4,30,80
		B	2,35,77	2,31,73	2,37,78	2,33,76
	4	R	8,17,47,96	10,20,53,99	8,15,45,98	10,19,51,100
		G	5,15,44,83	5,12,48,85	4,11,48,81	4,15,42,86
		B	2,23,52,90	2,25,58,94	2,20,55,89	2,24,57,93
	5	R	10,23,47,72,122	10,25,48,78,127	11,27,50,79,124	10,25,46,76,125
		G	6,18,44,69,117	8,20,45,71,119	7,18,46,75,120	8,21,40,71,118
		B	3,14,40,65,103	4,16,39,68,109	3,15,42,66,105	4,13,45,62,101
House	2	R	12,152	13,155	13,156	12,159
		G	8,147	10,141	9,144	8,146
		B	4,126	4,132	2,128	4,130
	3	R	16,106,162	13,102,164	15,106,169	14,100,163
		G	11,82,158	10,81,157	10,76,152	10,78,155
		B	6,133,172	4,135,175	6,130,178	5,131,176
	4	R	17,93,130,188	15,90,133,190	17,95,128,186	16,95,136,191
		G	10,85,127,183	10,82,124,180	9,83,125,182	9,82,129,180
		B	5,101,139,176	4,98,141,174	4,103,144,171	5,105,140,173
	5	R	16,91,110,152,192	14,90,115,154,198	16,88,109,150,196	16,88,111,154,197
		G	10,48,85,126,185	10,46,88,124,189	9,44,81,122,181	8,43,85,129,182
		B	6,102,139,160,178	4,98,135,161,177	5,99,136,162,174	5,100,133,164,175

Table 4
Objective function and image quality measures for the segmented images

M	J_{max}				RMSE				PSNR			
	EBFO	PSO	DPSO	FODPSO	EBFO	PSO	DPSO	FODPSO	EBFO	PSO	DPSO	FODPSO
Lena	2	1600.72	1599.85	1600.08	1601.22	74.3562	76.5592	69.1229	10.7045	10.4509	11.2899	11.3384
	3	1602.06	1602.15	1602.73	1602.64	51.0263	61.7629	50.9035	13.9749	12.3162	13.9206	13.9959
	4	1604.11	1603.92	1604.88	1605.50	40.2630	40.5398	40.4025	16.0327	15.9732	15.9429	16.0026
	5	1607.23	1606.68	1608.71	1609.93	35.6976	35.6895	35.1052	17.0780	17.0800	17.0501	17.2234
	2	1791.22	1790.71	1791.50	1791.34	70.2494	69.1471	69.2635	11.1980	11.3353	11.3281	11.3207
Jet	3	1793.07	1791.92	1792.85	1792.77	41.8550	41.7847	41.6246	15.6958	15.7105	15.7417	15.7438
	4	1794.88	1792.21	1794.00	1794.96	30.4917	30.6321	30.6888	18.4472	18.4073	18.4618	18.3912
	5	1795.21	1793.72	1794.91	1795.88	22.7494	22.9560	22.9011	20.9914	20.9129	20.9337	20.9848
Mandrill	2	1248.78	1247.55	1249.18	1248.91	70.6383	76.2128	70.4597	11.1500	10.4902	11.1720	11.1627
	3	1250.52	1250.69	1251.02	1252.71	48.9364	49.0928	48.8668	14.3382	14.3104	14.3312	14.3505
	4	1253.00	1251.71	1254.07	1254.47	37.3176	37.3703	37.2828	16.6925	16.6803	16.7006	16.9592
	5	1256.92	1254.86	1255.98	1257.02	28.9008	28.8959	28.6228	18.9126	18.9141	18.9966	18.9669
	2	653.92	652.54	654.02	654.33	33.5020	29.0568	28.8821	17.6294	18.8658	18.9182	18.9339
Couple	3	654.21	652.99	653.71	654.59	20.8888	21.0348	20.9202	21.7325	21.6720	21.7195	21.7263
	4	655.01	653.57	654.90	655.61	16.3269	16.4533	16.1524	23.8727	23.8058	23.9661	23.8728
	5	655.83	654.11	655.13	656.36	13.7512	13.6875	13.9193	25.3640	25.4043	25.2584	25.4129
	2	1668.11	1667.68	1668.09	1668.47	76.3954	76.4183	76.3266	10.4695	10.4669	10.4773	10.4644
	3	1668.95	1668.42	1669.11	1669.39	53.9952	54.1914	54.2277	13.4837	13.4522	13.4464	13.5042
House	4	1669.44	1669.73	1670.41	1670.54	36.0856	36.2645	36.3601	16.9841	16.9412	16.9183	16.9511
	5	1670.15	1670.32	1671.05	1671.77	30.3813	30.4841	30.2277	18.4787	18.4493	18.5226	18.6041

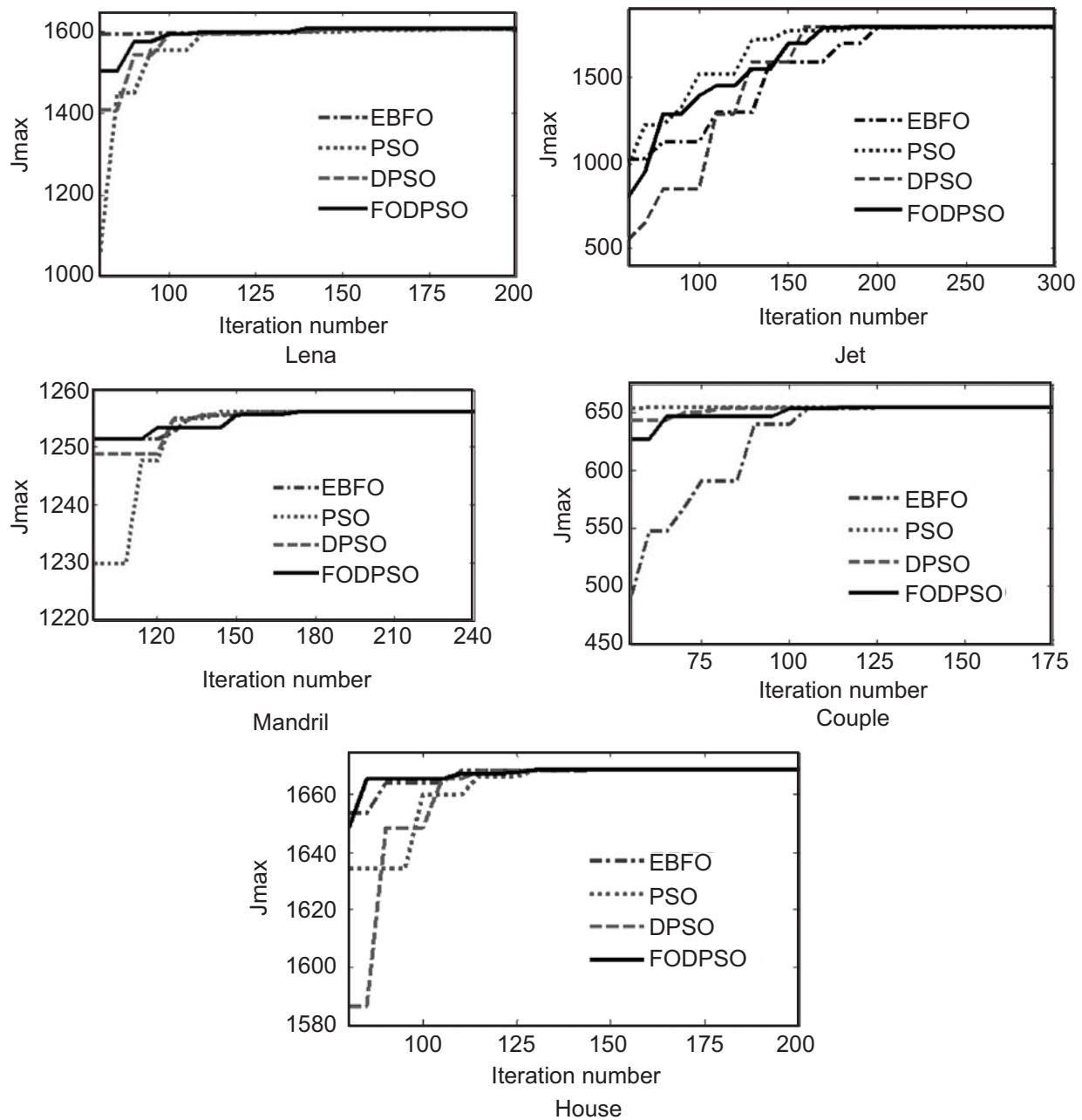


Figure 2: Convergence of heuristic algorithm based search for the RGB test images

Table 2 shows the histogram and ribbon plot for the RGB images. The histogram gives the pixel distribution of the color at various positions. Lena, Mandrill and House gives the distributed RGB levels though the image. In the case of Jet the pixel density is concentrated highly on the right end of the image whereas It is concentrated on the left end for Couple. Table 3 displays the histogram depicting RGB levels. It represents the distribution of RGB colors in the three segments of an image at threshold[6] values 2,3,4 and 5. Lena image alone is considered for this analysis. Initially the histogram is divided into three regions based on threshold values. With increase in threshold values, the distribution of RGB colors within these three regions is obtained.

During analysis of various images, for different threshold values various errors were observed. The values of such errors[16,17] namely J_{max} , Root Mean Square Error(RMSE), Peak signal to Noise Ratio(PSNR) has been tabulated in table4. The value of J_{max} increases with increase in the threshold values irrespective of the method used for analysis. In the error analysis of J_{max} , FODPSO is highly efficient for the Images Lena, Couple, Mandrill, and house. The value of PSNR increases in the same

trend. Hence for both these values, higher value of the parameter (J_{max} ,PSNR) indicates higher quality at the specific position in the image. A decrease in the value of RMSE has been observed with increase in threshold values. In case of RMSE, improved PSO(PSO,DPSO) provides better output for the images House, Lena, Mandrill and couple. For Jet, BFO is optimal.

The fig.2 represents the varying values of RGB

Distribution for each input image for all the methods. The Stability of error analysis depends on the high contrast image considered. When heuristic algorithms of different techniques are used, if the iteration number is high, it is inferred that the Intensity of the image is also high. Based on the intensity

Analysis of the image, Jet shows high resolution in EBFO technique. Likewise, Lena, Mandrill, Couple and House images shows the better resolution for FODPSO technique of which the Couple image gives the stability ratio with less number of iterations. It shows that the Couple image has slightly less intensity compared to other images.

Table 5
Histogram depicting RGB levels

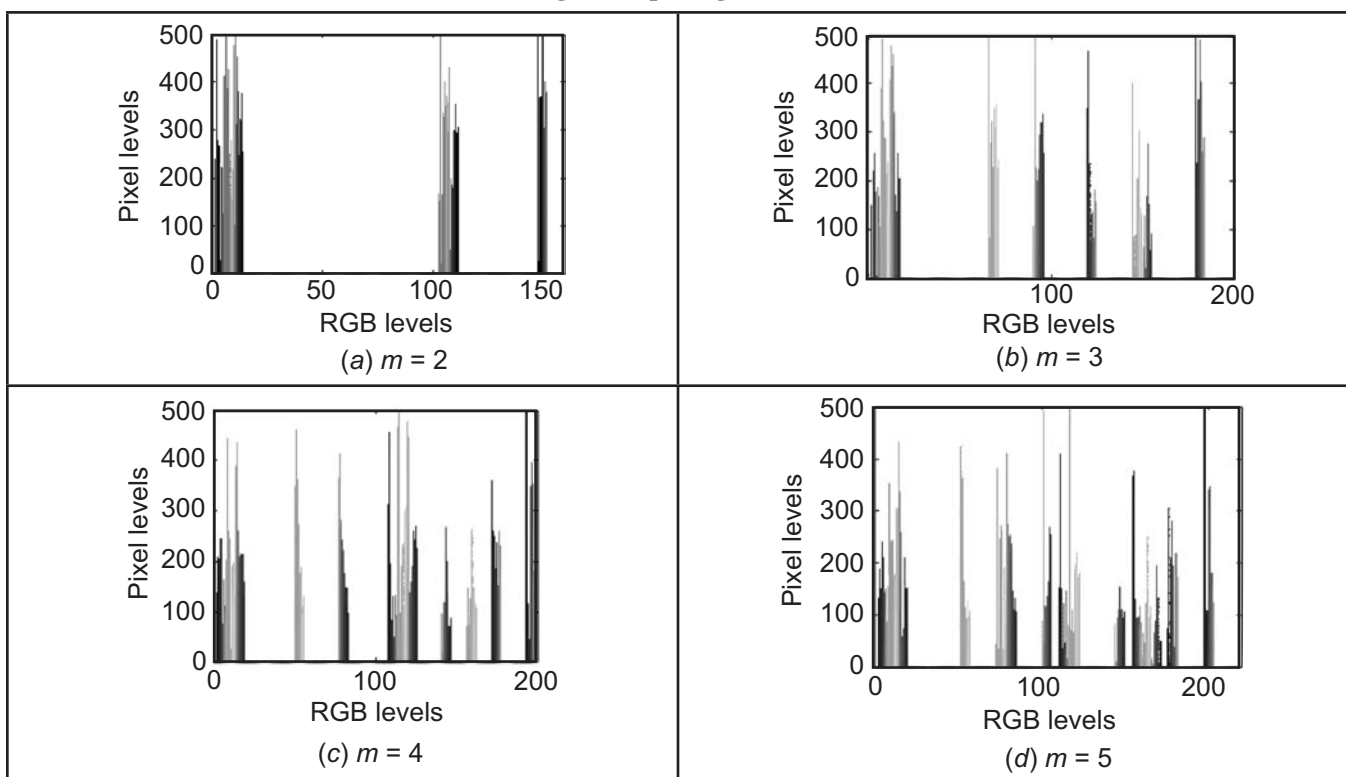


Table 5 displays the histogram depicting RGB levels. It represents the distribution of RGB colors in the three segments of an image at threshold[6] values 2,3,4 and 5.Lena image alone is considered for this analysis. Initially the histogram is divided into three regions based on threshold values. With increase in threshold values, the distribution of RGB colors within these three regions is obtained.

5. CONCLUSION

In the analysis of color image segmentation, the openly available RGB databases have been considered. The non-conversion of images to grey scale ensures in retaining the original intensity of images. For such analysis heuristic algorithms are mostly opted. RGB distribution analysis is done using ribbon graphs where the distribution of each part of the segmented image is performed. Using the ribbon graphs, the image contrasts at different locations can be observed. The image is segmented to many parts and the RGB distribution for each part is noted. The image quality of the segmented image is analyzed using the objectives namely J_{max} , RMSE and PSNR. It is to be noted that the threshold values are considered from 1; as its value increases, it yields better output. This is depicted in histograms form which we can conclude

that optimal output is obtained at high threshold values. As the threshold value increases the precision of the image with the original image increases.

6. REFERENCES

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