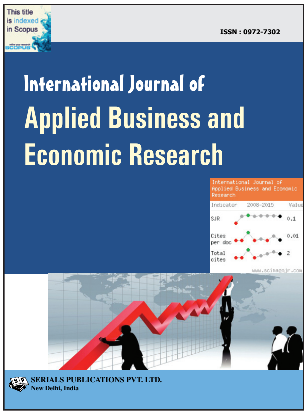

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Optimal Retentions with Ruin Probability Target in the Case of Fire Insurance in Iran

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ABSTRACT

Selecting optimal level of retentions for reinsurance treaties has been a major concern for insurance companies. It is important to choose a risk measure that describes insurer's vulnerability to insolvency. In this paper, by lowering the ruin probability and assuming the initial capital of zero and based on various safety loading factors of insurance and reinsurance premiums, optimal retentions in cases of proportional and non-proportional treaties are found.

We apply the data from fire insurance department of Mellat insurance company in Iran to examine the theory. For the purpose of homogeneity of data, all fire insurance policies are collected from homeowner's fire insurance coverage. For simplicity, commissions and expenses of insurance company are excluded.

Optimal retentions are found in the case of proportional reinsurance. Our findings show that when safety loading factors of ceding company and Reinsurance Company approach to each other, it is better for insurance company to cede more, and as the distance between safety loading factors increase, the ceding company should cede less. In the case of excess of loss reinsurance, the results are not stable but after a critical point in safety loading factors, stable results are derived.

Keywords: Optimal retentions, Ruin probability, Reinsurance, Lundberg's upper bound, Zero initial capital.

1. INTRODUCTION AND LITERATURE REVIEW

Many researchers such as Bühlmann (1970), Grandell (1991), and Asmussen (2000) have comprehensively worked on classical Cramer-Lundberg framework and offered this framework for targeting ruin probability. Centeno (1985) calculated the optimal retention based on minimizing skewness coefficient of insurer's retained risk subject to constraints on variance and expected value of retained risk. However Centeno

(1986) and Dickson (2005) suggest to target an upper bound of ruin probability instead of targeting the exact ruin probability level. They chose the adjustment coefficient as a measure for optimizing reinsurance cover purchased. However, Gajek (1999) in its research focused on comparing various premium calculation principles and its effect on optimal retention. Some other researchers worked on approximations instead of upper bound. De Vylder et. al., (1988) by use of some approximations provided a numerical algorithm for calculating finite time ruin probability in the case of discrete time risk process. Dickson & Waters (1991) presented an algorithm for calculating finite time survival probability and applied it to infinite time survival probability. However, Panjer & Wang (1993) in their work shown that most of recursive algorithms are unstable. Dickson et. al., (1995) in its work presented more stable recursive algorithms comparing to previous ones. Gerber et. al., (1987) worked on both the probability and severity of the ruin. Picard (1994) worked on the maximum severity of ruin. Also, in the concept of distribution of surplus process Dickson (1992) provided the distribution of surplus process prior to ruin and Willmot et. al., (1998) extended the research on this area based on properties of the distribution of surplus process before and after ruin. The Laplace transform of ruin which obtains the probability density function of time of ruin in exponential claims is used in the work of Gerber & Shiu (1998) and Schiff (1999) and the inversion of Laplace transform is suggested in the work of Drekic & Willmot (2003) and Dickson et. al., (2003).

Other researchers focused on revising the limiting assumptions of classical Lundeberg's framework. The classical Lundberg's framework assumed that the claim arrival process is homogenous Poisson process. But this is not the case in which the occurrence of claim has periodic or seasonal behavior. Parisi and Lund (2000) focused on annual arrival cycle and return period properties of land falling Atlantic Basin hurricanes. Based on non-homogenous Poisson process they modeled the seasonality of hurricane arrival time. Garrido et. al., (2004) suggested doubly periodic Poisson model with short-term and long-term trends of seasonality of hurricanes. Charpentier (2010) worked on optimal reinsurance issues and its pitfalls. In his paper he showed by means of deriving an efficient Monte Carlo algorithm there is possible to show that although an insurance company purchases non-proportional reinsurance cover, its ruin probability would increase. This could happen when the claim arrival process follows non-homogenous Poisson process. In this paper we apply Cramer-Lundberg framework on real data. The idea of the paper follows from Dickson (2005). We try to graphically trace the behavior of adjustment coefficient according to different combinations of safety loading factors of insurer and reinsurer in the case of quota-share and excess of loss treaty and by doing so, optimal retentions are found.

2. THE MODEL

In the classical risk model, $\{U(t)\}_{t \geq 0}$ is denoted as surplus of an insurer, in which, $u \geq 0$ is the insurer's surplus at time 0, and c is insurer's rate of premium income per unit time which is assumed to be received continuously and is based on expected value principle with $\theta > 0$ which is insurer's safety loading factor. Also it is assumed $\{N(t)\}_{t \geq 0}$ is a counting process for number of claims occurred in a fixed time period $[0, t]$ which is a Poisson process with parameter λ. The amounts of individual claims $\{Xi\}_{i=1}^{\infty}$ are assumed to be a sequence of independent and identically distributed random variables (i.i.d.) so that X_i is denoted as the amount of ith claim. The surplus process at time t is described as:

$$U(t) = u + ct - S(t) \tag{1}$$

If we denote μ_1 as the mean individual claim amount, in the above model it is assumed $c > \lambda\mu_1$, i.e. the premium income exceeds the expected amount of aggregate claim per unit of time. The premium income is defined as:

$$c = (1 + \theta)\lambda\mu_1 \tag{2}$$

The ultimate ruin probability in infinite time can be defined as:

$$\psi(u) = \Pr(U(t) < 0 \text{ for } t > 0). \tag{3}$$

This equation states that $\psi(u)$ is the probability that an insurer's surplus falls below zero due to exceed of claims outgo relative to initial surplus plus premiums received.

In classical risk process the adjustment coefficient which is denoted by R is defined as a risk measure for surplus process. This measure considers two factors in the surplus process, aggregate amount of claims and premiums received, and is defined to be the unique positive root of equation 4.

$$1 + (1 + \theta)\, \mu_1\, R = M_x(R) \tag{4}$$

in which μ_i is (X_1^i).

Then the adjustment coefficient is needed in Lundberg's inequality for risk process is stated in equation 5.

$$\psi(u) \le \exp\{-Ru\} \tag{5}$$

where R is adjustment coefficient.

The proof of inequality (5) is provided in various forms. Gerber (1979) and Rolski et. al., (1999) proved it by use of martingales. Also other forms of proofs are given in Dickson (2005).

In the case that insurer purchases reinsurance, we assume reinsurance premium is paid continuously at a constant rate and risk process becomes net of reinsurance surplus process $\{U^*(t)\}_{t \ge 0}$, and is given by:

$$U^*(t) = u + c^* t - \sum_{i=1}^{N(t)} X_i^*$$

in which X_i^* denotes the amount the insurer pays on the ith claim, net of reinsurance and c^* is the insurer's premium income per unit time net of reinsurance. If we assume that $c^* > \lambda E[X_1^*]$ and $M_{X_1^*}$ exists, the net of reinsurance adjustment coefficient is given such that

$$\lambda + c^* R^* = \lambda E[\exp\{R^* X_1^*\}].$$

It can be seen that the insurer's ultimate ruin probability is bounded above by $\exp\{-R^* u\}$. If we denote X as the amount of an individual claim with $X \sim F$ and $F(0) = 0$ we can define h as a reinsurance arrangement in which when a claim x occurs the reinsurer contributes the amount of $h(x)$ where $0 \le h(x) \le x$. In the case of proportional reinsurance $h(x) = ax$ where $0 \le a \le 1$. If the insurance company arranges an excess of loss reinsurance with retention level M, this means the insurer pays min(X, M) in occurrence of any claim. In this case $h(x) = \min(X, M)$. Then in order to compare both reinsurance arrangements it should be assumed that

$$E[\min(X, M)] = E[a(X)]. \tag{6}$$

The assumption in equation 6 states the mean individual claim net of reinsurance should be put equal in both reinsurance arrangements. This is a necessary condition for our comparison. The next important assumption is that equation 2 which defines the insurance premium would become as equation 7.

$$c^* = (1+\theta)\lambda E[X] - (1+\varepsilon)\lambda E[X - h(X)], \tag{7}$$

in which

$$c^* > \lambda E[h(X)]. \tag{8}$$

In equation 8, c^* is the difference between premium collected by insurance company and the reinsurance premium. ε is the safety loading factor that reinsurance company charges in order to accept the risks from cedent. Also it is assumed that in equation (7) $\varepsilon \geq \theta > 0$. This assumption ensures that the net adjustment coefficient exists along with the assumption that relevant moment generating function exists. It is necessary to remind that h represents any reinsurance arrangement since based on equation (6) the premium of both reinsurance treaties are kept equal.

2.1. Proportional Reinsurance

In the case of effecting proportional reinsurance, the insurer only pays proportion a of each claim. Then the insurer's net of reinsurance premium income per unit time would be as equation 8.

$$c^* = (1+\theta)\lambda E[X] - (1+\varepsilon)\lambda(1-a)E[X] \tag{9}$$

with the condition that $c^* > \lambda a E[X]$. Based on this condition it can be easily proved that

$$a > 1 - \frac{\theta}{\varepsilon} \tag{10}$$

which means $\theta < \varepsilon$ is a crucial condition for insurance company to purchase reinsurance coverage. If this condition doesn't meet then the insurance company retains all of the risk for itself.

The equation (4) in the case of purchasing proportional reinsurance would look like as equation 10.

$$\lambda + c^* R_p = \lambda E[\exp\{R_p(aX)\}] = \lambda \int_0^\infty \exp\{R_p(aX)\} f(x)dx \tag{11}$$

in which R_p denote the net adjustment coefficient based on proportional reinsurance arrangement.

2.2. Excess of Loss Reinsurance

In the case of purchasing excess of loss reinsurance with retention level M the insurer's net of reinsurance premium income per unit time is as follows:

$$c^* = (1+\theta)\lambda E[X] - (1+\varepsilon)\lambda \int_M^\infty (x - M) f(x)dx$$

The adjustment coefficient based on equation (4) in the case of excess of loss reinsurance would become as equation 11.

$$\lambda + c^* R_e = \lambda E[\exp\{R_e \min(X, M)\}]$$

$$= \lambda\left(\int_0^M \exp\{R_e x\} f(x)dx + \exp\{R_e M\}(1 - F(M))\right). \quad (12)$$

The proof of this equation is given in Dickson 2005.

$$M_{c - S_1}(-R) = 1$$

$$e^{Rc} = M_{S_1}(R) = E(e^{RS_1}) \Rightarrow 1 = e^{-Rc}E(e^{RS_1} - Rc) = E(e^{-R(c - S_1)})$$

For the first claim that occur and for simplicity it is put $\lambda = 1$. This equality is the cornerstone for our work.

If we define function $h(R)$ as a function of adjustment coefficient then :

$$h(R) = E(e^{-R(\lambda c - X_i)}) = 1 \quad (13)$$

It should be noted that the principle of premium calculation used in this work is expected value principle. This premium must satisfy the positive safety loading constraint $E[X_i - c\lambda] < 0$ in the considered time interval, because if the premiums collected do not exceed the amount of losses, ruin has occurred in the time interval.

Also in this thesis it is assumed the initial capital of insurance company is zero. The initial capital has considerable effect on decreasing the ruin probability of insurance company and more capital leads to less demand for reinsurance coverage. By assuming initial capital of zero we can better analyze the portfolio of losses. In this way, the amount of premium is calculated based on expected value of claims. Then based on two factors, premium safety loading factor of insurance company and premium safety loading factor of reinsurance company, the optimal retention for both proportional and non-proportional reinsurance treaties is calculated and compared.

- In the case of proportional reinsurance equation 12 becomes:

$$h(R, a) = E(e^{-R(\lambda c(a) - aX_i)}) = 1 \quad (14)$$

  where, a is the retention rate and $c(a)$ is the premium rate function.

- In the case of excess of loss reinsurance equation 12 becomes:

$$h(R, a) = E(e^{-R(\lambda c(a) - \min(X_i, a))}) = 1 \quad (15)$$

  The proofs of equation 12 and its modifications in equation 13 and 14 are provided in Bowers (1997).

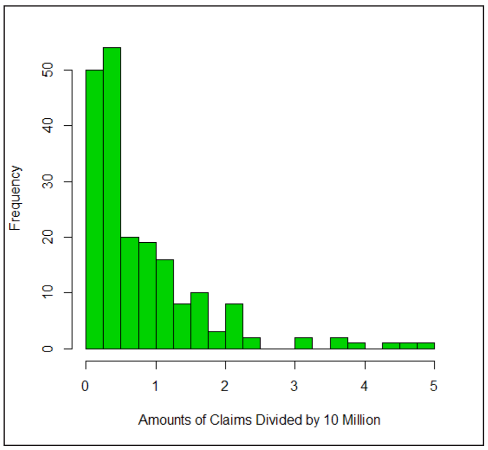
3. EMPIRICAL ANALYSIS

In order to get better understanding of application of ruin theory, real data from home owner's fire insurance policies are chosen. The data includes 2723 insurance policies that are purchased within year 2008 form Mellat insurance company in Iran. In order to have homogenous data, we consciously chose fire insurance policies that cover similar risks, such as insurance policies cover the risk of fire for apartments, domiciles, condominiums, and suites. From above policies, 198 items are caused losses and the insurance company has indemnified for those losses. For illustrative purposes the claims amounts are divided by 10 million Rials and all descriptive statistics and parameter estimations are shown. Table 1 provides descriptive statistics of data.

Table 1
Descriptive statistics of claims

	Mean	*Standard Deviation*	*Minimum*	*Maximum*	*Range*	*Skewness*	*Kurtosis*
Claims	0.79	0.86	0	4.9	4.9	2.19	5.77

Figure 1: Descriptive statistics of claims



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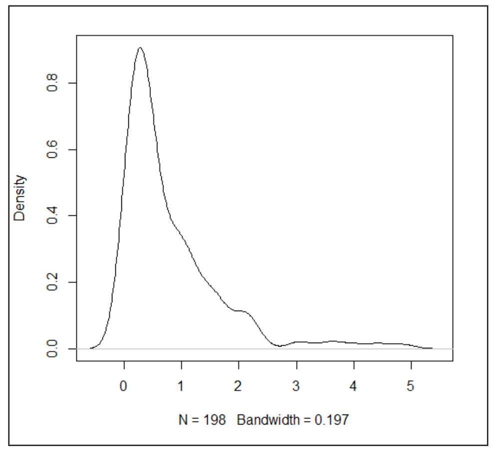

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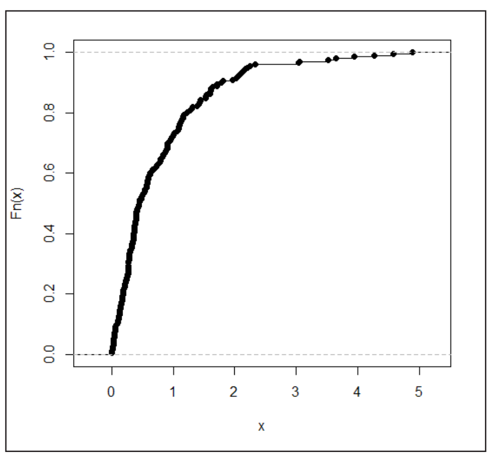


(c)

Figure 1: Descriptive statistics of claims
(a) Histogram of claims data. N = 198. Range = 0.5
(b) Density estimates of claims data. N = 198. Range = 0.5
(c) Empirical cumulative distribution function. N = 198. Range= 0.5

To obtain intuitive knowledge of the distribution of loss amounts, the histogram and density estimate of claims data and empirical cumulative distribution function is given in Figure 1. As can be seen the data is skewed and in order to find the best distribution, the parameters for two famous distributions, exponential and gamma are estimated by maximum likelihood method in Table 2.

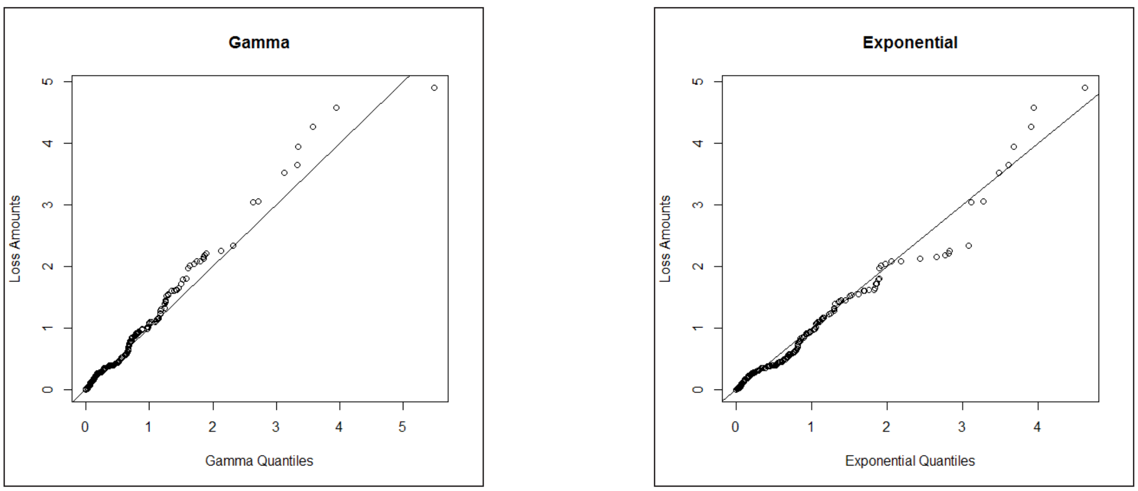
Table 2
Maximum likelihood estimation to fit claims distribution

	Parameters
Gamma	$a = 0.9723027$, $\lambda = 1.22742328$
Exponential	$\lambda = 1.26238761$

The next step in fitting the data is to examine which distribution is the best fit for our data. This is done by probability distribution (Q-Q) plots. The (Q-Q) plots for exponential and gamma distribution is depicted in Figure 2. However, both of these distributions show stable results and make the choice of best fit difficult.

In Table 3 it is shown that according to various Goodness of Fit tests such as Kolmogrov-Smirnov test, Anderson-Darling test and Chi-square test, the best distribution for the data is exponential distribution. So we choose exponential distribution and continue our empirical work based on that.

Figure 2: Q-Q Plots of Loss Amounts Distribution



(a) **(b)**

Figure 2: Q-Q Plots of Loss Amounts Distribution
(a) Gamma with parameters $a = 0.9723$, $\lambda = 1.2274$
(b) Exponential with parameter $\lambda = 1.2623$

Table 3
P-values of goodness-of-fit

	Kolmogrov-Smirnov	*Anderson-Darling*	*Chi-square*
Gamma	0.3305	0.6118	0.123
Exponential	0.4638	0.6632	0.17

As is shown in Table 3 based on goodness of fit tests, the best distribution fitted to our data is exponential distribution with parameter $\lambda = 1.2623$. In what follows, we try to find optimal retention in Quota-share treaties and excess of loss treaties based on various combinations of safety loading factors of insurance company and reinsurance company. In following tables θ indicates safety loading factor of insurance company premium, ε indicates safety loading factor of reinsurance company premium, a is retention of insurance company in the case of quota-share treaty and M is retention of insurance company in the case of excess of loss reinsurance treaty.

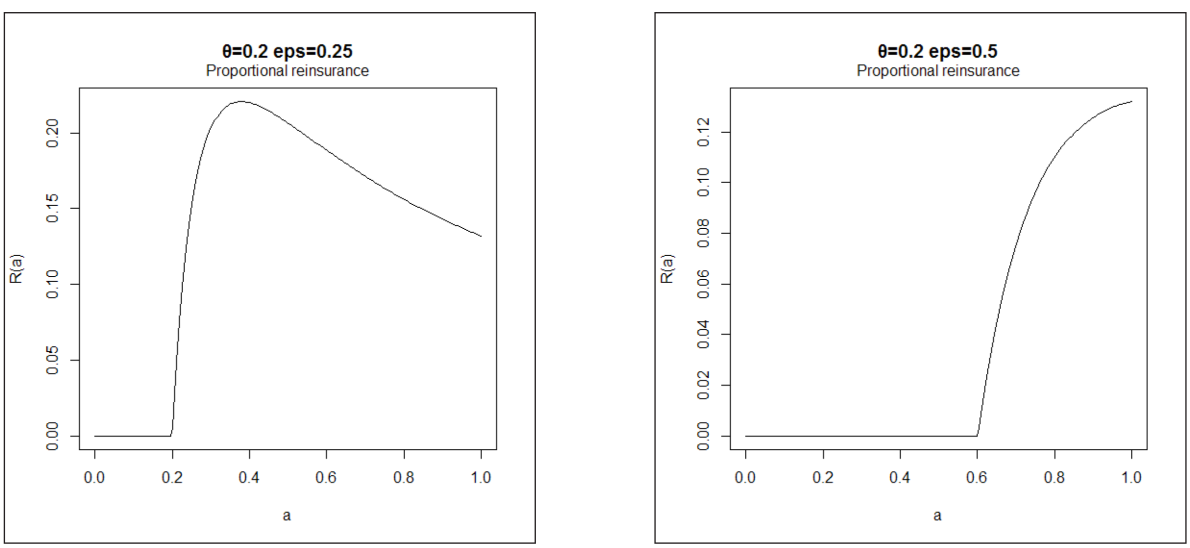
In order to illustrate the effect of θ and ε on optimal retention, in each section three tables are provided. First the amount of θ is kept constant and the effect of increasing ε will be examined. Then the amount of ε is kept constant and the effect of various θ will be examined. Finally, different combinations of θ and ε are examined with 0.05 gap between amount of θ and ε. In first two tables of each kind of treaty it is tried to see the direct effect of changing loading factors. This is the reason that in each table one of loading factors is kept constant. The third table is provided to see keeping the differences between both loading factors constant, by increasing both loading factors whether the amount of retention increases. Also we want to see that whether results are stable in both kinds of treaties.

In Table 4 the amount of θ is fixed and ε increases. As the amount of ε rises the reinsurance coverage becomes expensive for this portfolio of losses. So it is better for insurance company to retain higher percentage of losses for itself. Figure 3 illustrates adjustment coefficient on vertical axis and amount of retention on horizontal axis. The optimal point for retention is where the amount of adjustment coefficient is maximized and consequently the ruin probability is minimized.

Table 4
θ is fixed. Optimal proportional reinsurance

θ	ε	a	R
0.2	0.25	0.39	2.207127e-01
0.2	0.3	0.64	1.556398e-01
0.2	0.4	0.93	1.329532e-01
0.2	0.5	1	--
0.2	0.7	1	--
0.2	0.9	1	--

Figure 3: θ is fixed. Optimal proportional reinsurance



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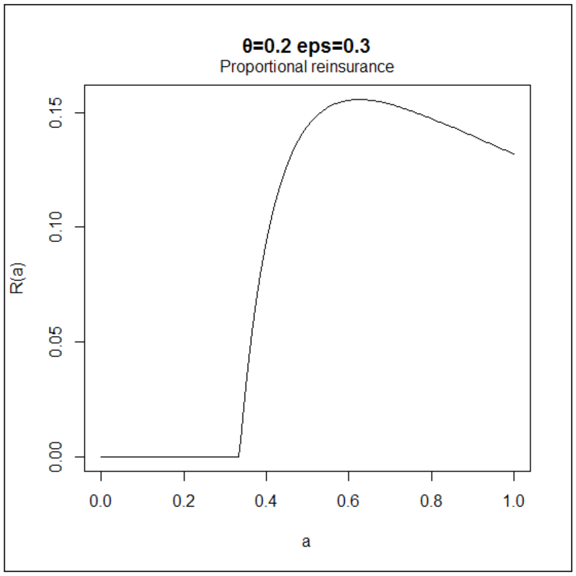

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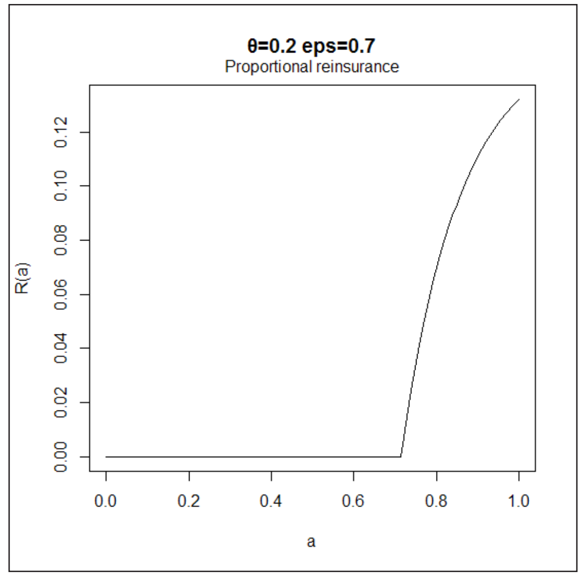

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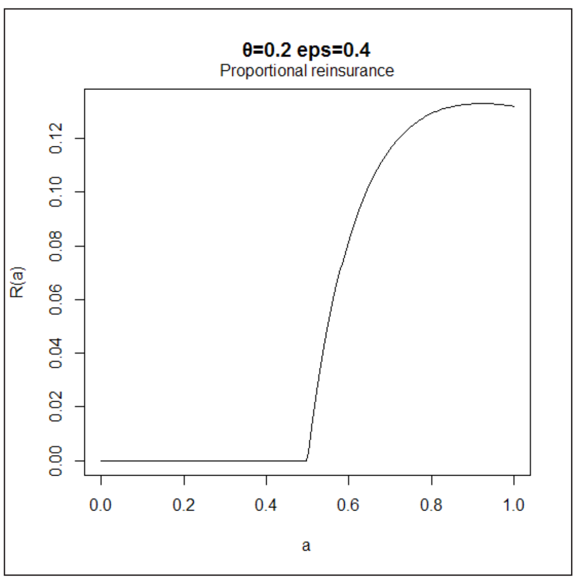

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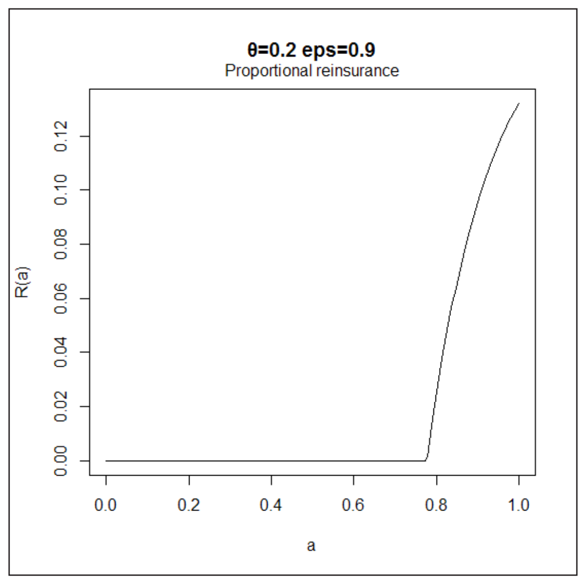


(f)

Figure 3: Optimal proportional reinsurance
(a) θ is fixed at 0.2, ε = 0.25, Optimal a = 0.39
(b) θ is fixed at 0.2, ε = 0.3, Optimal a = 0.64
(c) θ is fixed at 0.2, ε = 0.4, Optimal a = 0.93
(d) θ is fixed at 0.2, ε = 0.5, Optimal a = 1
(e) θ is fixed at 0.2, ε = 0.7, Optimal a = 1
(f) θ is fixed at 0.2, ε = 0.9, Optimal a = 1

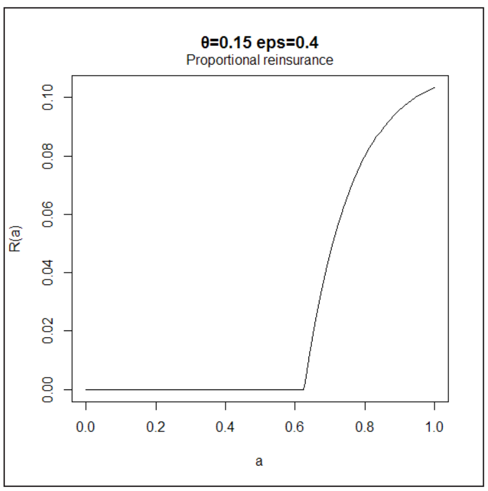
In Table 5 ε is fixed and by increasing safety loading factor of insurance company the retention decreases since reinsurance coverage becomes cheaper and it is better for cedant to retain lower percentage of losses. Also as θ increases the insurance company will have more funds to repay the losses and it can be seen from last column of Table 5 that the ruin probability of whole portfolio decreases. The optimal retentions are illustrated in Figure 4.

Table 5
ε is fixed. Optimal proportional reinsurance

θ	ε	a	R
0.15	0.4	---	---
0.2	0.4	0.93	1.329532e-01
0.25	0.4	0.7	1.772690e-01
0.3	0.4	0.47	2.658976e-01
0.35	0.4	0.24	5.317588e-01
0.37	0.4	0.15	8.860625e-01

Figure 4: Optimal proportional reinsurance

Based on Table 4 and 5, it can be seen that as the distance between θ and ε increases, it is better for insurance company to retain more percentage of the losses for itself. In other words, purchasing expensive quota-share treaty affects the insurance company's vulnerability to ruin and it is optimal reinsurance occurs in the cases in which, the cedent retains more of the losses for itself. However, when the distance decreases, optimal retention occurs when the insurer cedes more percentage of the losses to reinsurer.



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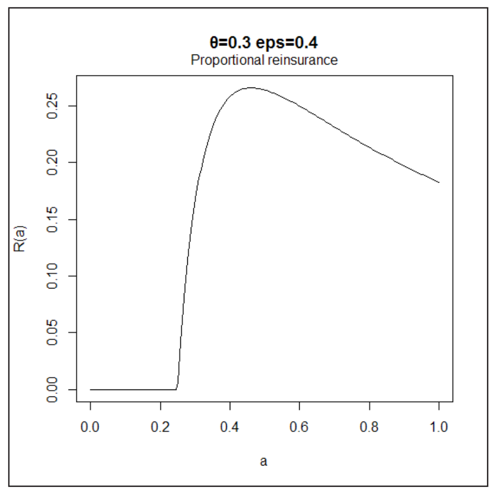

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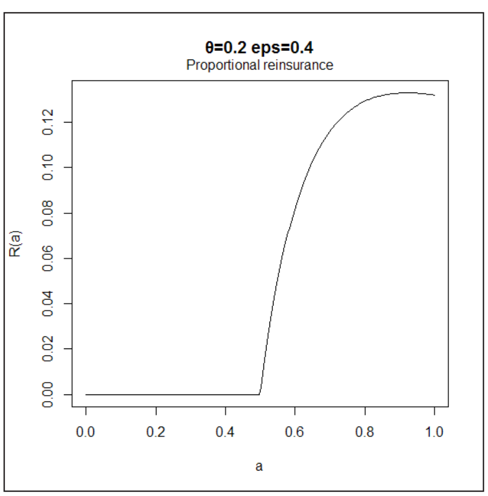

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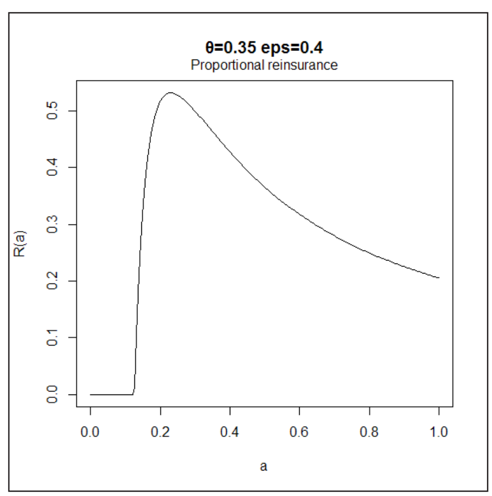

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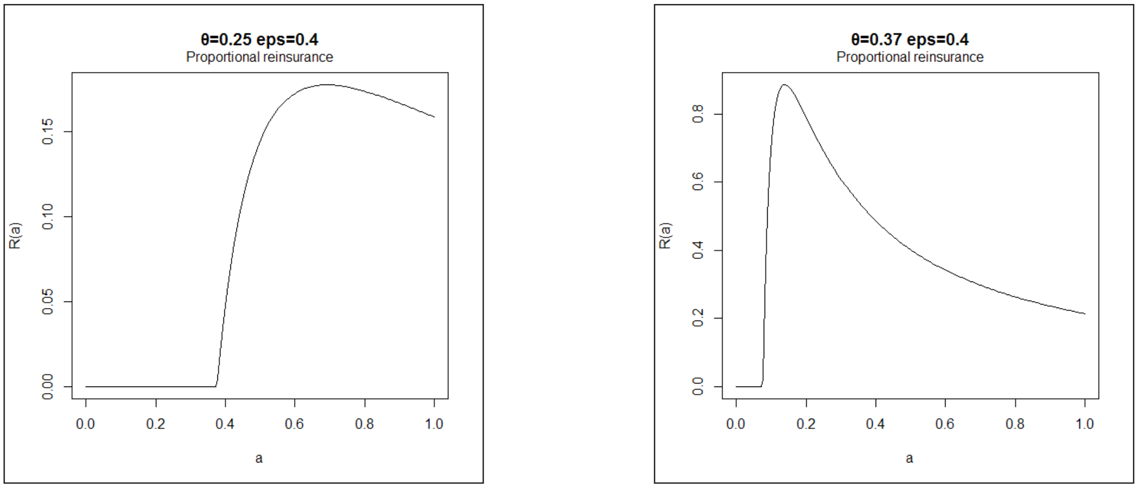


(e) (f)

Figure 4: Optimal Proportional Reinsurance
(a) θ = 0.15, ε is fixed at 0.4, Optimal *a* = 1
(b) θ = 0.2, ε is fixed at 0.4, Optimal *a* = 0.93
(c) θ = 0.25, ε is fixed at 0.4, Optimal *a* = 0.7
(d) θ = 0.3, ε is fixed at 0.4, Optimal *a* = 0.47
(e) θ = 0.35, ε is fixed at 0.4, Optimal *a* = 0.24
(f) θ = 0.37, ε is fixed at 0.4, Optimal *a* = 0.15

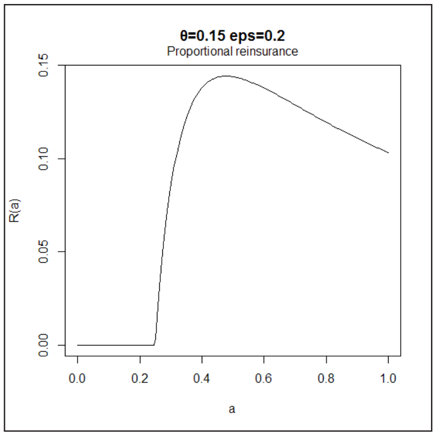
In table 6 the distance between θ & ε is kept constant. The reason to do so is that the effect of increasing both θ & ε on ruin probability and optimal retention being examined. Although the insurance company in all the forms purchases reinsurance coverage with 0.05 difference in loading factor, by growth in loading factor it is better for insurance company to cede more percentage of portfolio to reinsurance company. Accordingly , by increasing loading factors that are collected from policyholders, the adjustment coefficient increases and consequently the ruin probability decreases. In Figure 5 it can be seen that as θ & ε increase the optimal point for adjustment coefficinet shifts to left and this illustrates that retention level of insurance company gradually decreases.

Table 6
The distance bet. θ & ε is fixed at 0.05. Optimal proportional reinsurance

θ	ε	a	R
0.15	0.2	0.49	1.443196e-01
0.2	0.25	0.39	2.207127e-01
0.25	0.3	0.32	3.112504e-01
0.3	0.35	0.28	4.150975e-01
0.35	0.4	0.24	5.317588e-01
0.4	0.45	0.21	6.600351e-01

Figure 5: Optimal proportional reinsurance.

The empirical work is continued in the case of Excess of loss reinsurance. In order to compare the results between proportional reinsurance and non-proportional reinsurance safety loading factors will be provided identical to the case of porportional reinsurance and then the results are illustrated.



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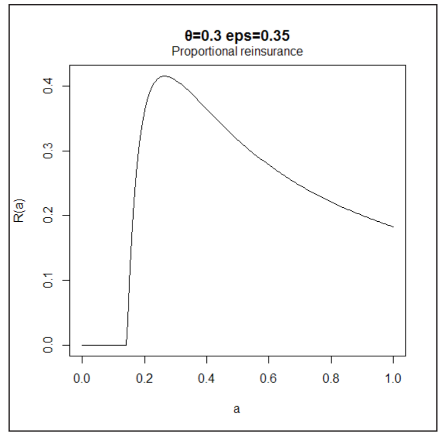

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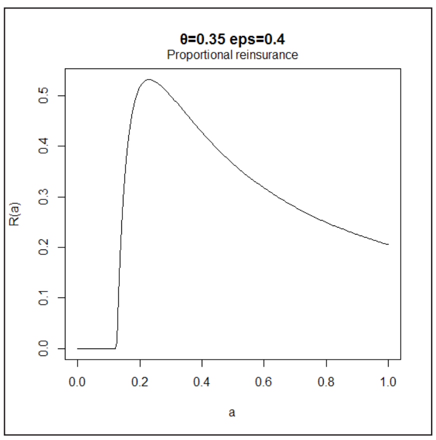

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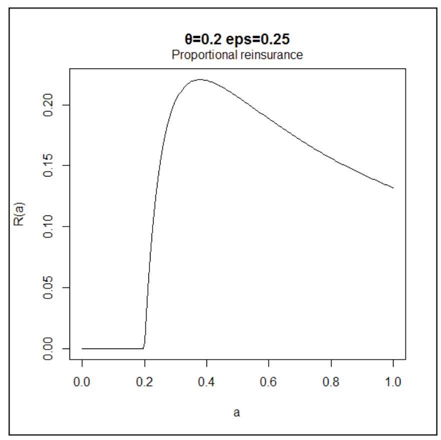

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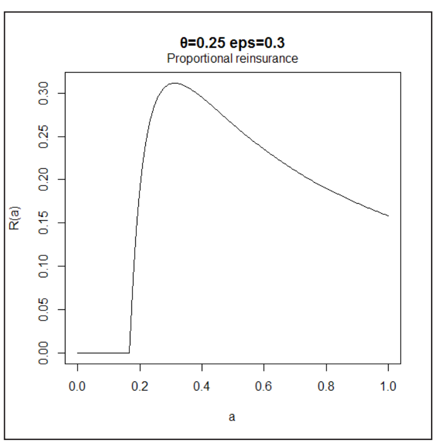

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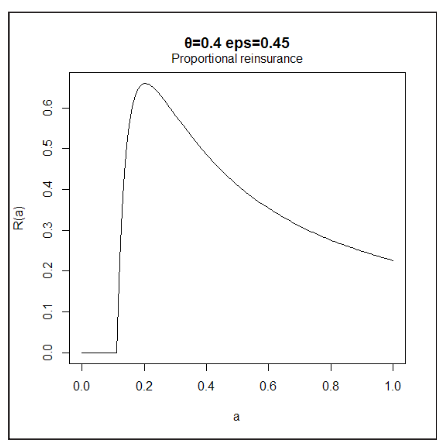


(f)

Figure 5. Optimal proportional reinsurance

(a) $\theta = 0.15$, $\varepsilon = 0.2$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.49$

(b) $\theta = 0.2$, $\varepsilon = 0.25$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.39$

(c) $\theta = 0.25$, $\varepsilon = 0.3$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.32$

(d) $\theta = 0.3$, $\varepsilon = 0.35$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.28$

(e) $\theta = 0.35$, $\varepsilon = 0.4$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.24$

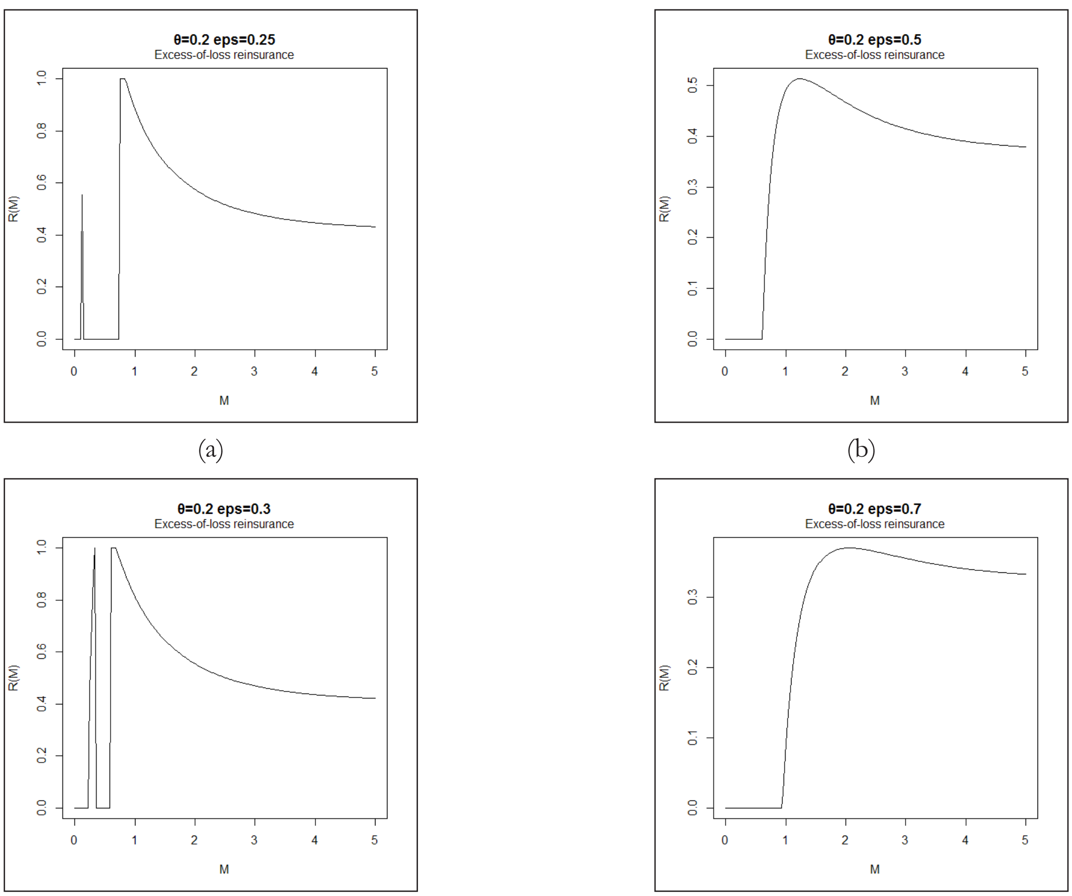
(f) $\theta = 0.4$, $\varepsilon = 0.45$, The distance bet. θ & ε is fixed at 0.5. Optimal $a = 0.21$

Table 7
θ is fixed. Optimal excess of loss reinsurance

θ	ε	M	R
0.2	0.25	0.85	>1
0.2	0.3	0.7	>1
0.2	0.4	0.9	6.735642e-01
0.2	0.5	1.3	5.135373e-01
0.2	0.7	2.1	3.704025e-01
0.2	0.9	3.05	2.905998e-01

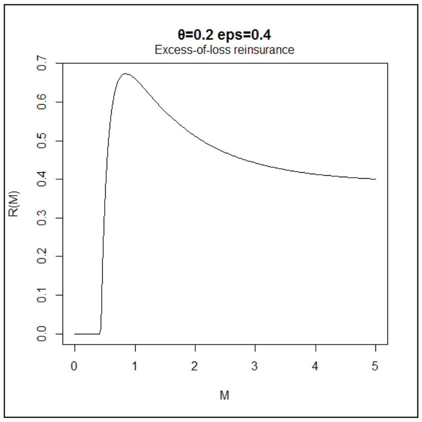
In Table 7, θ is kept constant and by increasing safety loading factor of reinsurance coverage, ε, the price of purchasing excess of loss reinsurance will be increased. As can be seen by increasing ε from 0.25 up to level 0.3 optimal retention decreases and then after level 0.3 to level 0.9 by increasing ε the optimal retention decreases. The Table 7 shows that up to level of ε = 0.3 this method behaves irrational since we expect that by increasing ε, the cedent should cede more of the losses. However, amount of 0.3 for ε is a turning point at which by increasing the price of excess of loss reinsurance, it is beneficial for insurance company to keep more of the losses for itself. From this point, the behavior of adjustment coefficient would be identical to the case of quota-share treaty. The behavior of adjustment coefficient regarding changes in ε is depicted in Figure 6.

Figure 6: Optimal excess of loss reinsurance

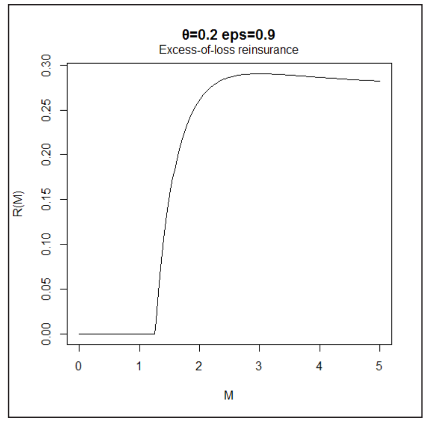


(a) (b)

(c) (d)



(e)



(f)

Figure 6: Optimal Excess of Loss Reinsurance
(a) θ is fixed at 0.2, ε = 0.25, Optimal M = 0.85
(b) θ is fixed at 0.2, ε = 0.3, Optimal M = 0.7
(c) θ is fixed at 0.2, ε = 0.4, Optimal M = 0.9
(d) θ is fixed at 0.2, ε = 0.5, Optimal M = 1.3
(e) θ is fixed at 0.2, ε = 0.7, Optimal M = 2.1
(f) θ is fixed at 0.2, ε = 0.9, Optimal M = 3.05

Now in Table 8 amount of ε is fixed and θ increases gradually. It can be seen that as the gap between θ and ε decreases at first, optimal retention decreases. Up to the point θ reaches to 0.25 optimal retention decreases. The reason could be since θ increases up to the point 0.25, the amount of increase is not enough to pay the losses. But when θ grows more, the cedent becomes capable of retaining more of the losses and because of that the retention increases. Again the decrease of retention M before turning point θ = 0.25 seems to be illogical while after that, by increasing θ there is more tendency to keep more of the losses for insurance company. Optimal retentions for each level are illustrated in Figure 7.

Table 8
ε is fixed. Optimal excess of loss reinsurance

θ	ε	M	R
0.15	0.4	1.25	4.757499e-01
0.2	0.4	0.9	6.735642e-01
0.25	0.4	0.6	9.946662e-01
0.3	0.4	0.95	>1
0.35	0.4	1.15	>1
0.37	0.4	1.25	>1

Figure 7: Optimal excess of loss reinsurance

Now by keeping the distance between θ & ε constant as 0.05 optimal retention is illustrated in Figure 8. It can be seen that as loading factors increases it is better for insurance company to retain more of the losses and cede less. This is a reasonable result since insurance company's safety loading factor increases and because of that, the insurance company has more funds to pay the losses and less need to purchase reinsurance. Optimal retentions are provided in Table 9.

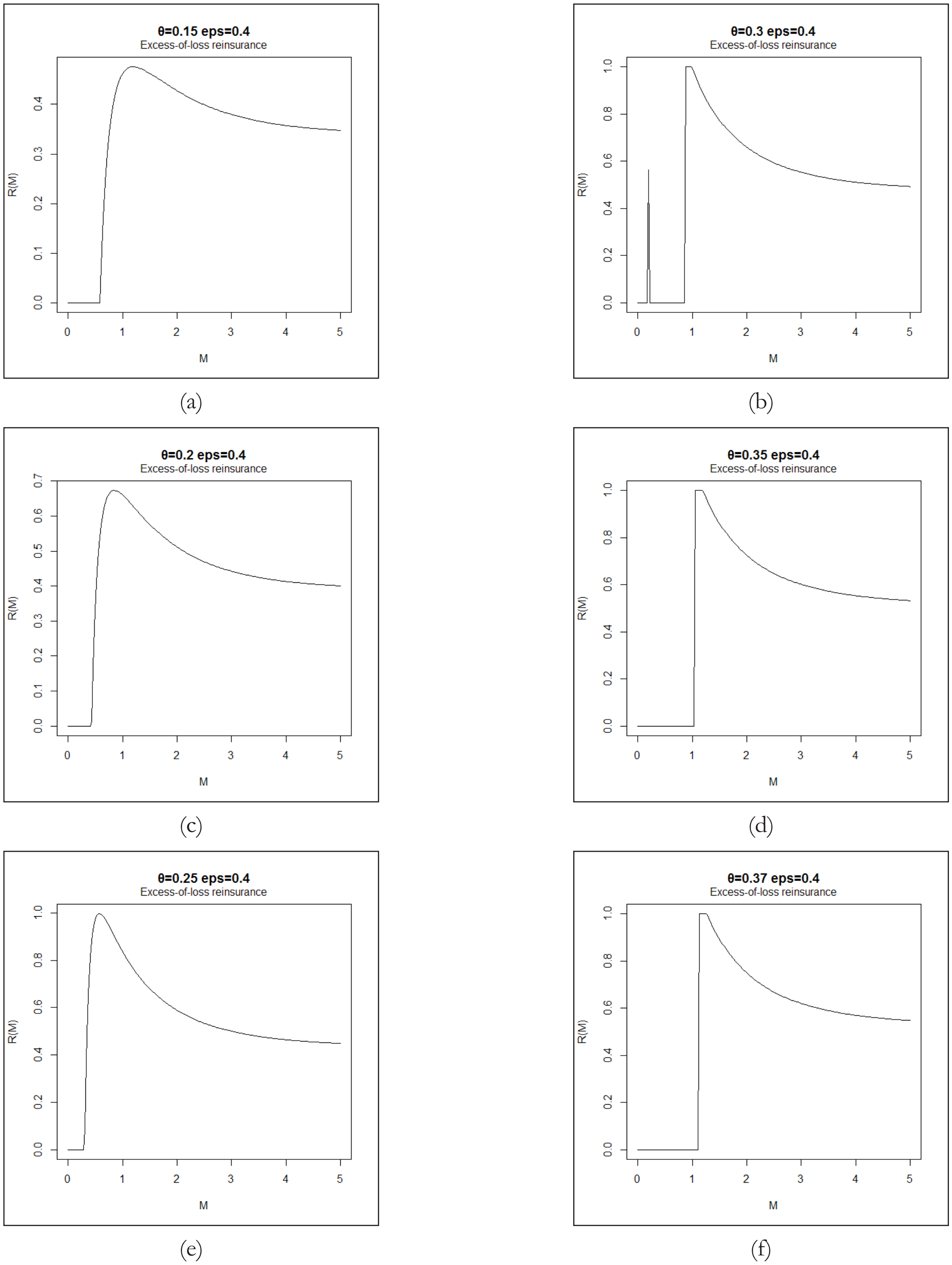


Figure 7: Optimal Excess of Loss Reinsurance
(a) θ = 0.15, ε is fixed at 0.4, Optimal M = 1.25
(b) θ = 0.2, ε is fixed at 0.4, Optimal M = 0.9
(c) θ = 0.25, ε is fixed at 0.4, Optimal M = 0.6
(d) θ = 0.3, ε is fixed at 0.4, Optimal M = 0.95
(e) θ = 0.35, ε is fixed at 0.4, Optimal M = 1.15
(f) θ = 0.37, ε is fixed at 0.4, Optimal M = 1.25

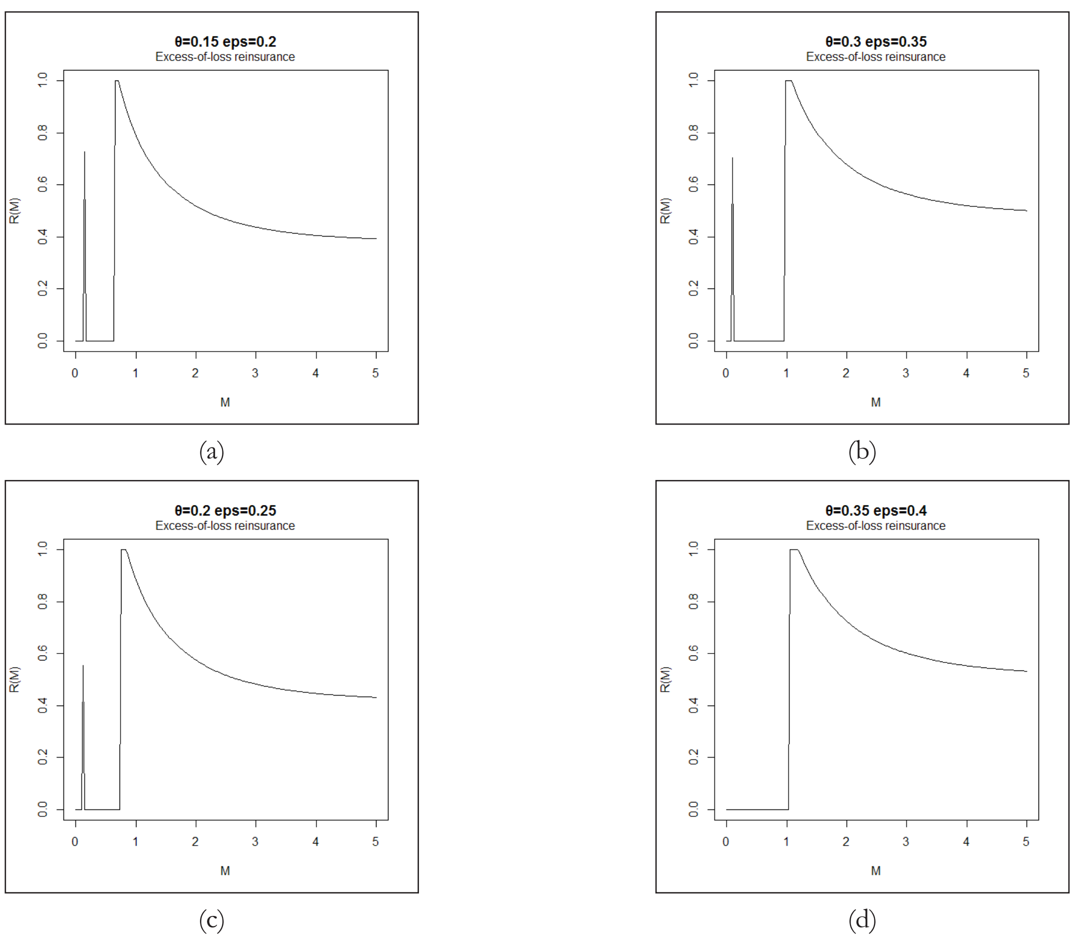
Table 9
The distance bet. θ & ε is fixed at 0.5. Optimal excess of loss reinsurance

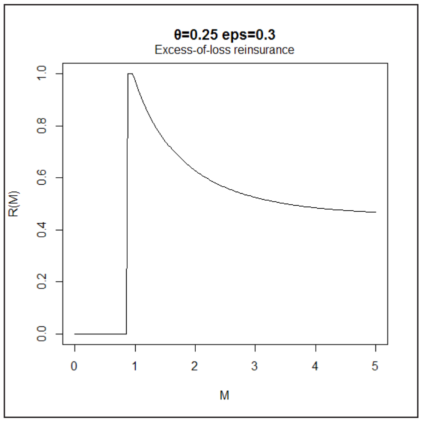
θ	ε	M	R
0.15	0.2	0.75	>1
0.2	0.25	0.85	>1
0.25	0.3	0.95	>1
0.3	0.35	1.1	>1
0.35	0.4	1.2	>1
0.4	0.45	1.3	>1

Figure 8: Optimal excess of loss reinsurance.

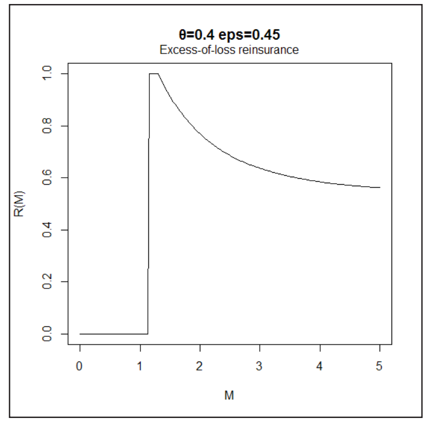
Another important result is shown in table 10. In this table adjustment coefficients of previous combinations for both proportional and excess of loss reinsurance are brought together in one table in order to compare the ruin probability of both of these reinsurance coverages.

As is shown in Table 10 in all the combinations of θ & ε adjustment coefficient is higher in case of excess of loss reinsurance. It should be stated that by considering equal premium for both proportional and excess of loss reinsurance, adjustment coefficient for excess of loss coverage is higher and consequently ruin probability in the case of excess of loss reinsurance is lower compared to the case of proportional coverage. This result is congruent with results in Dickson 2005.





(e)



(f)

Figure 8: Optimal Excess of Loss Reinsurance
(a) θ = 0.15, ε = 0.2, The distance bet. θ & ε is fixed at 0.5. Optimal M = 0.75
(b) θ = 0.2, ε = 0.25, The distance bet. θ & ε is fixed at 0.5.Optimal M = 0.85
(c) θ = 0.25, ε = 0.3, The distance bet. θ & ε is fixed at 0.5. Optimal M = 0.95
(d) θ = 0.3, ε = 0.35, The distance bet. θ & ε is fixed at 0.5. Optimal M = 1.1
(e) θ = 0.35, ε = 0.4, The distance bet. θ & ε is fixed at 0.5. Optimal M = 1.2
(f) θ = 0.4, ε = 0.45, The distance bet. θ & ε is fixed at 0.5, Optimal M = 1.3

Table 10
Adjustment Coefficient Comparison

θ	*ε*	*R(proportional)*	*R(XL)*
0.2	0.25	2.207127e-01	>1
0.2	0.3	1.556398e-01	>1
0.2	0.4	1.329532e-01	6.735642e-01
0.2	0.5	--	5.135373e-01
0.2	0.7	--	3.704025e-01
0.2	0.9	--	2.905998e-01
0.15	0.4	--	4.757499e-01
0.25	0.4	1.772690e-01	9.946662e-01
0.3	0.4	2.658976e-01	>1
0.35	0.4	5.317588e-01	>1
0.37	0.4	8.860625e-01	>1
0.15	0.2	1.443196e-01	>1
0.25	0.3	3.112504e-01	>1
0.3	0.35	4.150975e-01	>1
0.4	0.45	6.600351e-01	>1

Ruin probability has been a powerful tool in analyzing a portfolio of risks. By utilizing ruin theory the probability of insolvency of a risk portfolio can be calculated. The use of Cramer-Lundberg's upper bound simplifies the estimation of maximum probability of ruin and helps to control loading factors that an insurance company should charge on insurance policies in order to make sure losses would not cause insolvency for the company. This theory is also used in risk transfer mechanisms such as reinsurance, and from insurer's point of view, it has helped in supplying the necessary reinsurance coverage for a specific insurance portfolio.

4. CONCLUSIONS

In this paper based on real data and utilizing ruin theory, optimal retentions for both proportional and excess of loss treaties were found. It was also shown that by increasing both safeties loading factor of cedent and Reinsurance Company, optimal retention in both kinds of reinsurance treaties shifts upward and the cedent can retain more of the losses.

In the case of proportional reinsurance, as the price of reinsurance coverage rises there is more tendency to retain the risks and as the prices of both insurance and reinsurance coverage are nearly the same, it is economical for insurance company to cede more of the risk to reinsurance carrier.

In the case of excess of loss reinsurance, the results were not completely stable. When loading factor of reinsurance company increases there is a turning point before which the amount of monetary retention decreases. However after that turning point, reinsurance coverage becomes expensive and retention increases. In the case of increasing loading factor of insurance company, there is also a turning point before which monetary retention decreases and after that point when loading factor of insurance company increases amount of retention increases. From these two results it can be implied when loading factor of cedent and Reinsurance Company is near each other or when they are far from each other, the retention level rises.

Also based on adjustment coefficients found in all the cases, the adjustment coefficient for excess of loss reinsurance is higher comparing to proportional reinsurance. This means excess of loss reinsurance in all the combinations leads to lower ruin probability for insurance company. In other words, keeping the premiums the same for both treaties, excess of loss treaties provide lower probability of ruin.

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