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A MEASURE FOR TIME DEPENDENT QUEUEING SYSTEM WITH FEEDBACK AND SERVICE IN BATCHES OF VARIABLE SIZE

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Abstract

This paper investigates the transient solution of a single channel queueing system with feedback in which the units are occurring singly and are served in batches of variable size. Inter arrival time and service time are exponentially distributed. Time dependent probabilities of exactly i arrivals and j departures are obtained explicitly. The marginal probability of exactly i arrivals and the mean number of arrivals are obtained. Some particular cases of interest are also obtained.

1. INTRODUCTION

The queueing problem considered here is that in which the customers arrive singly and are served in batches of variable size with a further provision that a batch of units are given another chance of service, if required. In the present study, the concept of Pegden and Rosenshine is applied to find out the time dependent probabilities for the exact number of arrivals and departures of a single channel queueing problem in which the batch of units is given the alternative of rejoining the system with a definite probability after being served once .However, it is assumed that batch of units leaves the system definitely after getting the service for the second time.

The practical situation which corresponds the above problem can be that of a dry cleaner, who dry cleans the clothes in a machine. He dry cleans the clothes satisfactorily either in one service or in two services. After dry cleaning the clothes once, some batch of clothes require re cleaning and are sent to the queue for service. The drycleaner can know the total number of clothes to be dry cleaned and the number of clothes dry cleaned *by* a given time.

The queueing system investigated in this paper is described by the following assumptions:

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1. Arrivals are poisson with parameter A and the service time distribution of each batch is exponential with parameter μ.

2. The capacity of the server is a random variable. The size of the batch is determined at the beginning of each service and is either equal to the total number of units or to the capacity of the server detennined afresh before each service whichever is less.The probability that the server can serve m units is a_m so that $\sum_{m=1}^{k} a_m = 1.$ where k is the maximum capacity of the server.

3. The probability of rejoining the batch of units to the system is p and that of leaving the system is q , p + q =1.

4. The batch at the head of the queue either is to be served for the first time or second time. The probability that it joins the server for the first time is assumed to be C_1 and that for the second time is C_2, $C_1+C_2 = 1$.

5. If the queue length is greater than the capacity of the server but the batch at the head of the queue to be served for the second time consists of units less than the capacity of the server, then only that batch consisting of these units to be served for the second time will join the server.

6. The Stochastic processes involved, viz,

   (a) arrival of units

   (b) departure of units

   are statistically independent.

2. DEFINITIONS

$p^{(0)}_{i,j}(t)$ = Probability that there are exactly i arrivals and j departures by time t and the next batch of units is to depart for the first time.

$p^{(l)}_{i,j}(t)$ = Probability that there are exactly i arrivals and j departures by time t and the next batch of units is to depart for the second time.

$P_{i,j}(t)$ = Probability that there are exactly i arrivals and j departures by time t.

$$P_{i,j}(t) = p^{(0)}_{i,j}(t) + p^{(l)}_{i,j}(t) \tag{1}$$

Initially $\quad P^{(0)}_{0,0}(0) = 1$ and $p^{(l)}_{i,i}(t) = 0, \quad i \geq 0$

The difference-differential equations governing the system are

$$\frac{d}{dt}P^{(0)}{}_{0,0}(t) = -\lambda P^{(0)}{}_{0,0}(t) \tag{2}$$

$$\frac{d}{dt}P^{(0)}{}_{i,i-m}(t) = -(\lambda+\mu)P^{(0)}{}_{i,i-m}(t) + \lambda P^{(0)}{}_{i-1,i-m}(t) + \sum_{\gamma=1}^{i-m}\mu\ a_\gamma c_1\left(P^{(1)}{}_{i,i-m-\gamma}(t) + qP^{(0)}{}_{i,i-m-\gamma}(t)\right)$$

$$\sum_{\gamma=1}^{m-1}\mu\, pa_\gamma c_1 P^{(0)}{}_{i,i-m}(t)\quad,\ i \geq m,\ \ 1 \leq m \leq k \tag{3}$$

$$\frac{d}{dt}P^{(0)}{}_{i,j}(t) = -(\lambda+\mu)P^{(0)}{}_{i,j}(t) + \lambda P^{(0)}{}_{i-1,j}(t) + \sum_{\gamma=1}^{j}\mu\, a_\gamma c_1\left(P^{(1)}{}_{i,j-\gamma}(t) + q\,P^{(0)}{}_{i,j-\gamma}(t)\right)$$

$$+\mu pc_1 P^{(0)}{}_{i,j}(t)\ ,\ i \geq j+k,\ 0 < j \leq k \tag{4}$$

$$\frac{d}{dt}P^{(0)}{}_{i,j}(t) = -(\lambda+\mu)P^{(0)}{}_{i,j}(t) + \lambda P^{(0)}{}_{i-1,j}(t) + \sum_{\gamma=1}^{k}\mu a_\gamma c_1\left(P^{(1)}{}_{i,i-1}(t) + q\,P^{(0)}{}_{i,i-1}(t)\right)$$

$$+\mu pc_1 P^{(0)}{}_{i,j}(t)\ ,\ i > j+k\ ,\ j \geq k+1 \tag{5}$$

$$\frac{d}{dt}P^{(0)}{}_{i,i}(t) = -\lambda P^{(0)}{}_{i,i}(t) + \sum_{\gamma=1}^{i}\sum_{m=\gamma}^{k}\mu a_m\left(P^{(1)}{}_{i,i-\gamma}(t) + q\,P^{(0)}{}_{i,i-\gamma}(t)\right),\ 1 \leq i \leq k \tag{6}$$

$$\frac{d}{dt}P^{(0)}{}_{i,i}(t) = -\lambda P^{(0)}{}_{i,i}(t) + \sum_{\gamma=1}^{k}\sum_{m=\gamma}^{k}\mu a_m\left(P^{(1)}{}_{i,i-\gamma}(t) + q\,P^{(0)}{}_{i,i-\gamma}(t)\right),\ i \geq k+1 \tag{7}$$

$$\frac{d}{dt}P^{(1)}{}_{i,i-m}(t) = -(\lambda+\mu)P^{(1)}{}_{i,i-m}(t) + \lambda P^{(1)}{}_{i-1,i-m}(t) + \sum_{\gamma=1}^{i-m}\mu\, a_\gamma c_2\left(P^{(1)}{}_{i,i-m-\gamma}(t) + q\,P^{(0)}{}_{i,i-m-\gamma}(t)\right)$$

$$+\mu pP^{(0)}{}_{i,i-m}(t)\left(\sum_{\gamma=1}^{m-1}a_\gamma c_2 + \sum_{\gamma=m}^{k}a_\gamma\right),\ i \geq m;\ 1 \leq m \leq k \tag{8}$$

$$\frac{d}{dt}P^{(1)}{}_{i,j}(t) = -(\lambda+\mu)P^{(1)}{}_{i,j}(t) + \lambda P^{(1)}{}_{i-1,j}(t) + \sum_{\gamma=1}^{j}\mu\, a_\gamma c_2\left(P^{(1)}{}_{i,j-\gamma}(t) + q\,P^{(0)}{}_{i,j-\gamma}(t)\right)$$

$$+\mu pc_2 P^{(0)}{}_{i,j}(t)\ ,\ i \geq j+k;\ 1 \leq j \leq k \tag{9}$$

$$\frac{d}{dt}P^{(1)}{}_{i,j}(t) = -(\lambda+\mu)P^{(1)}{}_{i,j}(t) + \lambda P^{(1)}{}_{i-1,j}(t) + \sum_{\gamma=1}^{k}\mu a_\gamma c_2\left(P^{(1)}{}_{i,i-\gamma}(t) + q\,P^{(0)}{}_{i,i-\gamma}(t)\right)$$

$$+\mu pc_2 P^{(0)}{}_{i,j}(t)\ ,\ i \geq j+k;\ j \geq k+1 \tag{10}$$

Where

$$\left.\begin{array}{l} P^{(0)}{}_{i,j}(t) = 0 \\ P^{(1)}{}_{i,j}(t) = 0 \end{array}\right. \quad \text{For } j < 0$$

3. SOLUTION OF THE PROBLEM

Using the Laplace transform $\bar{f}(s)$ of f(t) given by $\bar{f}(s) = \int_0^\infty e^{-st}dt,$ Re(s) >0 in the equations (2)-(10) along with the initial conditions and solving recursively, we have

$$\bar{P}^{(0)}{}_{0,0(s)} = \frac{1}{s+\lambda} \tag{11}$$

$$\bar{P}^{(0)}{}_{1,0(s)} = \frac{\lambda^i}{s+\lambda} \prod_{n=0}^{i-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_n)},\ i \geq 1 \tag{12}$$

Where $A_n = \sum_{\gamma=1}^{n} a_\gamma,\ 0 \leq n < k$

$= 1, \quad n \geq k$

$$\bar{P}^{(1)}{}_{i,0(s)} = \frac{\lambda^i \mu p}{(s+\lambda)} \sum_{r=1}^{i} \frac{(1-c_1 A_{r-1})}{(s+\lambda+\mu)^{i-r+1}} \prod_{n=0}^{r-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_n)},\ i \geq 1 \tag{13}$$

$$\bar{P}^{(0)}{}_{i,j}(s) = \sum_{m_j=j}^{i} \frac{\lambda^{i-m_j} \mu c^{1-\delta mj,j}}{(s+\lambda)^{\delta mj,j}} \prod_{\gamma=m_j-j}^{i-j-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} \bar{F}^{(i)}{}_{m_j(s)}, 1 \leq j \leq i \tag{14}$$

$$\bar{P}^{(1)}{}_{i,j}(s) = \sum_{m_j=j+1}^{i} \frac{\lambda^{i-m_j} \mu c_2}{(s+\lambda+\mu)^{i-m_j+1}} \bar{F}^{(i)}{}_{m_j}(s) + \sum_{n_j=j+1}^{i} \sum_{m_j=j}^{n_j} \frac{\lambda^{i-m_j} \mu^2 p(1-c_1 An_j - j-1)}{(s+\lambda)^{(\delta m_j, j)}}$$

$$\frac{C_1^{(1-\delta m_j, j)}}{(s+\lambda+\mu)^{i-n_j+1}} \prod_{\gamma=m_j-j}^{n_j-j-I} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} F^{(i)} m_j(s), 1 \leq j < 1 \tag{15}$$

Where

$$\bar{F}^{(1)}m_r(s) = \left(a_r + \delta m_r, r \sum_{\gamma=r+1}^{k} a_\gamma \right) (\bar{P}^{(1)}m_r, 0(s) + q\bar{P}^{(0)}m_{r,0}(s)), 1 \le r \le k$$

$$\bar{F}^{(j)}m_j(s) = \sum_{\alpha=1}^{j-1} \sum_{m_{j-\alpha}=j-\alpha+1}^{m_j} \frac{\lambda^{m_j - m_{j-\alpha}} c_2 (a_\alpha + \delta m_{j,} j}{(s+\lambda+\mu)^{m_j - m_{j-\alpha}+1}} \sum_{\gamma=1}^{k} a_\gamma) \,.\, \bar{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+\sum_{\alpha=1}^{j-1} \sum_{n_{j-\alpha}=j-\alpha+1}^{m_j} \sum_{m_{j-\alpha}=j-\alpha}^{n_{j-\alpha}} \lambda^{m_j - m_{j-\alpha}} \mu^2 p(a_\alpha + \delta m_j, j \sum_{\gamma=\alpha+1}^{k} a_\alpha)(1 - c_1 A_{n_{j-\alpha}} - j + \alpha - 1)$$

$$\frac{c_1^{(1-\delta m\, j-\alpha, j-\alpha)}}{(s+\lambda)^{\delta m_{j-\alpha}, j-\alpha} (s+\lambda+\mu)^{m_j - n_{j-\alpha}+1}} \prod_{\gamma=m_{j-\alpha}-j-\alpha}^{n_{j-\alpha}-j+\alpha-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} \bar{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+q\sum_{\alpha=1}^{j-1} \sum_{m_{j-\alpha}=j-\alpha}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_1^{(1-\delta m_{j-\alpha}, j-\alpha)} \left(a_\alpha + \delta m_j, j \sum_{\gamma=\alpha+1}^{k} a_\gamma \right)$$

$$\frac{1}{(s+\lambda)^{\delta m_{j-\alpha}, j-\alpha}} \prod_{\gamma=m_{j-\alpha}-j+\alpha}^{m_j-j+\alpha-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} \bar{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+\bar{F}^{(1)}m_j(s), 2 \le j \le k$$

$$\bar{F}^{(j)}m_j(s) = \sum_{\alpha=1}^{k} \sum_{m_{j-\alpha}=j-\alpha+1}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_2 (a_\alpha + \delta m_j, j \sum_{\gamma=\alpha+1}^{k} a_\gamma) \frac{1}{(s+\lambda+\mu)^{m_j - m_{j-\alpha}}} \bar{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+\sum_{\alpha=1}^{k} \sum_{n_{j-\alpha}=j-\alpha+1}^{m_j} \sum_{m_{j-\alpha}=j-\alpha}^{n_{j-\alpha}} \lambda^{m_j - m_{j-\alpha}} \mu^2 p(a_\alpha + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma)(1 - c_1 An_{j-\alpha} - j + \alpha - 1)$$

$$\frac{c_1^{(1-\delta m_{j-\alpha}, j-\alpha)}}{(s+\lambda+\mu)^{m_j - m_{j-\alpha}+1} (s+\lambda)^{\delta m_{j-\alpha}, j-\alpha}} \prod_{\gamma=m_{j-\alpha}-j-\alpha}^{n_{j-\alpha}-j+\alpha-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} \bar{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+q\sum_{\alpha=1}^{k} \sum_{m_{j-\alpha}=j-\alpha}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_1^{(1-\delta m_{j-\alpha}, j-\alpha)} (a_\alpha + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma)$$

$$\frac{1}{(s+\lambda)^{\delta m_{j-\alpha}, j-\alpha}} \prod_{\gamma=m_{j-\alpha}-j+\alpha}^{m_j-j+\alpha-1} \frac{1}{(s+\lambda+\mu-\mu p c_1 A_\gamma)} \bar{F}^{(j-\alpha)}m_{j-\alpha}(s), \; j \ge k+1$$

The laplace transform $\bar{P}_{i,\cdot}(s)$ of the probability that exactly i units arrive by time t can be obtained by using the equations (11) - (15) and is given by

$$\bar{P}_{i,\cdot}(s)=\sum_{j=0}^{i}(\bar{P}^{(1)}{}_{i,j}(s)+\bar{P}^{(0)}{}_{i,j}(s))$$

$$=\frac{\lambda^{i}}{(s+\lambda)^{i+1}},\quad i\geq 0 \tag{16}$$

and the laplace transform of the mean number of arrivals is

$$\sum_{i=0}^{\infty} i\ \bar{P}_{i,\cdot}(s)=\frac{\lambda}{s^2} \tag{17}$$

For finding the inverse transform of the equations (11)-(17) denoting the convolution as usual by * and the inverse transform of

$\prod_{\gamma=r}^{k}\frac{1}{(s+\lambda+\mu-\mu p c_1 A\gamma}$ by $f_{r,r+1,\ldots,k}(t)$ Which can be calculated by making the partial fractions, we have

$$P^{(0)}{}_{0,0}(t)\ =\ e^{-\lambda t} \tag{18}$$

$$P^{(0)}{}_{i,o}(t)\ =\ \lambda^{i}\,e^{-\lambda t}*f_{0,1,2,\ldots i-1}(t),\qquad i\geq 1 \tag{19}$$

$$P^{(1)}{}_{i,0}(t)\ =\ \lambda^{i}\mu p\sum_{r=1}^{i}(1-c_1A_{r-1})\ \left(\frac{1-e^{-\mu t}}{\mu^{i-r+1}}-\sum_{\gamma=1}^{i-r}\ \frac{t^{i-r+1-\gamma}}{\mu^{\gamma}(i-\gamma)!}e^{-\mu t}\right)\ e^{-\lambda t}*f_{\ 0,1,2,\ldots,r-1}(t),\ i\geq 1 \tag{20}$$

$$P^{(0)}{}_{i,j}(t)=\sum_{m_j=j}^{i}\lambda^{i-m}{}_{j}\,\mu c_1^{\,1-\delta m_j,j}\,(e^{-\lambda t})^{m_j,j}*f\ m_j-j,\ldots,i-j-1(t)*F^{(j)}m_j(t),\ 1\leq j\leq i \tag{21}$$

$$P^{(1)}{}_{i,j}(t)\ =\ \sum_{m_j=j+1}^{i}\ \frac{\lambda^{i-m}{}_{j}\,c_2\mu\ t^{i-m}{}_{j}\quad e^{-(\lambda+\mu)t}}{(i-m_j)!}*F^{(j)}m_j(t)$$

$$+\sum_{n_j=j+1}^{i}\ \sum_{m_j=j}^{n_j}\lambda^{i-m}{}_{j}\,\mu^2 p(1-c_1An_j-j-1)c_1^{\,(1-\delta m,\,j)}{}_{j}\ (e^{-\lambda t})\ \frac{\delta^{m_j j}_{j}\ *\ t^{i-n}{}_{j}}{(i-n_j)!}$$

$$e^{-(\lambda+\mu)t}*f\ m_j-j,\ldots,n_j-j-1(t)*F^{(j)}m_j(t),\ 1\leq j<1 \tag{22}$$

Where

$$F^{(1)}m_r(t) = (a_r + \delta m_{r,} r \sum_{\gamma=r+1}^{k} a_\gamma) * (P^{(1)}m_r, 0(t) + q\, P^{(0)}m_r, 0(t)) \quad , 1 \le r \le k$$

$$F^{(j)}m_j(t) = \sum_{\alpha=1}^{j-1} \sum_{m_{j-\alpha}=j-\alpha+1}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_2 (a + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma) \frac{t^{m_j - m_{j-\alpha}}}{(m_j - m_{j-\alpha})!} e^{-(\lambda+\mu)t} * F^{(j-\alpha)} m_{j-\alpha}(t)$$

$$+ \sum_{\alpha=1}^{j-1} \sum_{n_{j-\alpha}=j-\alpha+1}^{m_j} \sum_{m_{j-\alpha}=j-\alpha}^{n_{j-\alpha}} \lambda^{m_j - m_{j-\alpha}} \mu^2 p (a_\alpha + \delta m_j, j \sum_{\gamma=\alpha+1}^{k} a_\gamma)(1 - c_1 A n_{j-\alpha} - j + \alpha - 1)$$

$$c_1^{(1-\delta m_{j-\alpha}, j-\alpha)} (e^{-\lambda t})^{\delta m_{j-\alpha}, j-\alpha} * \frac{t^{m_j - n_{j-\alpha}}}{(m_j - n_{j-\alpha})!} e^{-(\lambda+\mu)t} *$$

$$f\, m_{j-\alpha} - j + \alpha, \ldots, n_{j-\alpha} - j + \alpha - 1(t) * F^{(j-\alpha)} m_{j-\alpha}(t)$$

$$+ q \sum_{\alpha=1}^{j-1} \sum_{m_{j-\alpha}=j-\alpha}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_1^{(1-\delta m_{j-\alpha}, j-\alpha)} (a_\alpha + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma)$$

$$(e^{-\lambda t})^{\delta m_{j-\alpha}, j-\alpha} * F^{(j-\alpha)} m_{j-\alpha}(t) + F^{(1)} m_j(t) \,,\; 2 \le j \le k$$

$$F^{(j)}m_j(t) = \sum_{\alpha=1}^{k} \sum_{m_{j-\alpha}=j-\alpha+1}^{mj} \lambda^{m_j - m_{j-\alpha}} \mu c_2 (a_\alpha + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma) \frac{t^{m_j - m_{j-\alpha}}}{(m_j - m_{j-\alpha})!} e^{-(\lambda+\mu)t} * F^{(j-\alpha)} m_{j-\alpha}(t)$$

$$+ \sum_{\alpha=1}^{k} \sum_{n_{j-\alpha}=j-\alpha+1}^{m_j} \sum_{m_{j-\alpha} j-\alpha}^{n_{j-\alpha}} \lambda^{m_j - m_{j-\alpha}} \mu^2 p (a_\alpha + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma)\,(1 - c_1 A n_{j-\alpha} - j + \alpha - 1)$$

$$e^{-(\lambda+\mu)t} \frac{t^{m_j - m_{j-\alpha}}}{(m_j - m_{j-\alpha})!} c_1^{(1-\delta m\, j-\alpha, j-\alpha)} (e^{-\lambda t})^{\delta m_{j-\alpha}, j-\alpha} * f m_{j-\alpha} - j + \alpha, \ldots, n_{j-\alpha} - j + \alpha - 1(t) * F^{(j-\alpha)} m_{j-\alpha}(t)$$

$$+ q \sum_{\alpha=1}^{k} \sum_{m_{j-\alpha}=j-\alpha}^{m_j} \lambda^{m_j - m_{j-\alpha}} \mu c_1^{(1-\delta m_{j-\alpha}, j-\alpha)} (a_\gamma + \delta m_{j,} j \sum_{\gamma=\alpha+1}^{k} a_\gamma)\; (e^{-\lambda t})^{\delta m_{j-\alpha}, j-\alpha}$$

$$* f\, m_{j-\alpha} - j + \alpha, \ldots, m_j - j + \alpha - 1(t) * F^{(j-\alpha)} m_{j-\alpha}(t), \quad j \ge k+1$$

$$P_{i,.}(t) = \frac{(\lambda t)^i}{i!} e^{-\lambda t}, \qquad i \ge 0 \tag{23}$$

The arrivals are following the Poisson distribution as the probability of the total number of arrivals are not affected by the assumption of the system and the inverse transform of the mean number of arrivals is λt.

From (11)-(15), we have

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i} (\overline{P}^{(1)}{}_{i,j}(s) + \overline{P}^{(0)}{}_{i,j}(s)) = \frac{1}{s}$$

and hence

$$\sum_{i=0}^{\infty} \sum_{j=0}^{i} (P^{(1)}{}_{i,j}(t) + P^{(0)}{}_{i,j}(t)) = 1$$

a verification.

4. PARTICULAR CASES

1. When the units are served singly according to a Poisson distribution with parameter μ the laplace transform of the probabilities $P^{(0)}{}_{i,j}(t)$ and $P^{(1)}{}_{i,j}(t)$ can be obtained by putting $a_k = 1$, $k = l$ and $a_k = 0$ otherwise.

in the equations (11)-(17),we have

$$\overline{P}^{(0)}{}_{0,0}(s) = \frac{1}{s+\lambda} \tag{24}$$

$$\overline{P}^{(0)}{}_{i,0}(s) = \frac{\lambda^i}{(s+\lambda)(s+\lambda+\mu)(s+\lambda+\mu-\mu p c_1)^{i-1}}, \quad i \geq 1 \tag{25}$$

$$\overline{P}^{(1)}{}_{i,0(s)} = \frac{\lambda^i \mu p}{s+\lambda} \sum_{r=0}^{i} \frac{C_2{}^{(1-\delta r,1)}}{(s+\lambda+\mu)^{i-r+2}(s+\lambda+\mu-\mu p c_1)^{r-1}}, \quad i \geq 1 \tag{26}$$

$$\overline{P}^{(0)}{}_{i,j}(s) = \sum_{m_j=j}^{i} \lambda^{i-m}{}_j \mu c_1{}^{1-\delta mj,j} \prod_{\gamma=m_j-j}^{i-j} \frac{1}{(s+\lambda+\mu(1-pc_1) A_\gamma + \mu p c_1 \delta\gamma,1)} \overline{E}^{(j)} m_j(s), 1 \leq j \leq i \tag{27}$$

$$P^{(1)}{}_{i,j}(s) = \sum_{m_j=j+1}^{i} \frac{\lambda^{i-m} j \mu c_2}{(s+\lambda+\mu)^{i-m_j+1}} \overline{E}^{(j)} m_j(s) + \sum_{n_j=j+1}^{i} \sum_{m_j=j}^{n_j} \frac{\lambda^{i-m} j \mu^2 p c_1{}^{1-\delta m_j, j} c_2{}^{1-\delta_{n_j}, j+1}}{(s+\lambda+\mu)^{i-n_j+1}}$$

$$\prod_{\gamma=m_j-j}^{n_j-j-1} \frac{1}{(s+\lambda+\mu(1-pc_1) A_\gamma + \mu p c_1 \delta\gamma,1)} \overline{E}^{(j)} m_j(s), \quad 1 \leq j \leq i \tag{28}$$

for An = 0, n = 0

= 1, otherwise

Where

$$\overline{E}^{(1)}m_r(s) = (\overline{P}^{(1)}m_r, 0(s) + q\overline{P}^{(0)}m_{r,0}(s))$$

$$\overline{E}^{(j)}m_j(s) = \sum_{m_{j-1}=j}^{m_j} \frac{\lambda^{m_j - m_{j-1}} c_2\mu}{(s+\lambda+\mu)^{m_j - m_{j-1}+1}} . \overline{F}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+ \sum_{n_{j-1}=j}^{m_j} \sum_{m_{j-1}=j-1}^{n_{j-1}} \frac{\lambda^{m_j - m_{j-1}} \mu^2 p\, c_1^{1-\delta m,j_j} c_2^{1-\delta m_{j-1},j}}{(s+\lambda+\mu)^{m_j - n_{j-1}+1}}$$

$$\prod_{\gamma=m_{j-1}-j+1}^{n_{j-1}-j+1} \frac{1}{(s+\lambda+\mu(1-pc_1)A_\gamma + \mu pc_1\delta\gamma,1)} \overline{E}^{(j-1)}m_{j-1}(s)$$

$$+q \sum_{m_{j-1}=j-1}^{m_j} \lambda^{m_j - m_{j-1}} \mu c_1^{(1-\delta m_{j-1}, j-1)}$$

$$\prod_{\gamma=m_{j-1}-j+1}^{m_{j-1}-j+1} \frac{1}{(s+\lambda+\mu(1-pc_1)A_\gamma + \mu pc_1\delta\gamma,1)} \overline{E}^{(j-1)}m_{j-1}(s) \quad j \le 2$$

$$\overline{P}_{i,.}(s) = \frac{\lambda^i}{(s+\lambda)^{i+1}} \tag{29}$$

and the laplace transform of the mean number of arrivals is λ/s^2. The results (24)-(29) coincide with the results of Sharda and Garg's(1986)

2. When the units are served singly according to a poisson distribution with parameter m and there is no provision for second service i.e $c_2 = 0$ and $a_k = 1$ for k = 1 and 0 otherwise.

$P^{(1)}_{i,j}(t)$ is zero and hence $\overline{P}^{(1)}_{i,j}(s)$ is also zero and $\overline{P}_{i,j}(s)$ can be obtained by substituting p = 0, q = 1in the equations (11)-(15),we have

$$\overline{P}_{i,0}(s) = \frac{\lambda^i}{(s+\lambda)\,(s+\lambda+\mu)^i}, \; i \ge 0 \tag{30}$$

$$\bar{P}_{i,j}(s) = \sum_{m_j=j}^{i} \frac{\lambda^i \mu}{(s+\lambda+\mu(1-\delta m_{j,}\, j))\ (s+\lambda+\mu)^{i-m_j}} \bar{D}^{(j)} m_j(s) \ ,\ 1 \le j \le i \qquad (31)$$

Where

$$\bar{D}^{(1)} m_r(s) = \frac{1}{(s+\lambda)\ (s+\lambda+\mu)^{m_1}}$$

$$\bar{D}^{(j)} m_j(s) = \sum_{m_{j-1}=j-1}^{m_j} \frac{\mu}{(s+\lambda+\mu)^{m_j - m_{j-1}}\ (s+\lambda+\mu(1-\delta m_{j-1}, j-1))} \bar{D}^{(j-1)} m_{j-1}(s),\ j \ge 2$$

Substituting the value of $\bar{D}m_j(s)$ in the equation (31),we get

$$\bar{P}_{0,0}(s) = \frac{1}{s+\lambda}$$

$$\bar{P}_{i,j}(s) = \left(\frac{\lambda}{s+\lambda+\mu}\right)^i \left(\frac{\mu}{s+\lambda}\right)^j \sum_{k=0}^{j} \frac{(i-k)(i+k-1)!\,(s+\lambda)^{k-1}}{k!\ i!\,(s+\lambda+\mu)^k},\ i \ge 0,\ 0 \le j \le 1 \qquad (33)$$

Which coincides with equation (5) of Pegden and Rosenshine (1982)

3. When the batch of units are served only once and there is no provision for second service i.e C_2= 0 and $P^{(1)}{}_{i,j}(t)$ is zero and hence

$\bar{P}^{(1)}{}_{i,j}(s)$ is also zero,then $\bar{P}_{i,j}(s)$ can be obtained by substituting p = 0, q=1in the equations (12), (14) and (16), we have

$$\bar{P}_{0,0}(s) = \frac{1}{s+\lambda} \qquad (34)$$

$$\bar{P}_{i,0}(s) = \frac{\lambda^i}{(s+\lambda)\ (s+\lambda+\mu)^i},\ i \ge 1 \qquad (35)$$

$$\bar{P}_{i,j}(s) = \sum_{m_j=j}^{i} \frac{\lambda^{i-m_j}\mu}{(s+\lambda)^{\delta mj,j}\,(s+\lambda+\mu)^{i-m_j+1-\delta mj,j}} \bar{H}^{(j)} m_j(s),\ 1 \le j \le i \qquad (36)$$

Where

$$\bar{H}^{(1)} m_r(s) = \frac{\lambda^{m_r}}{(s+\lambda)(s+\lambda+\mu)^{m_r}} \left(a_r + \delta m_{r,}\, r \sum_{\gamma=r+1}^{k} a_\gamma\right) \qquad 1 \le r \le k$$

$$\bar{H}^{(j)}m_j(s)=\sum_{\alpha=1}^{j-1}\sum_{m_{j-\alpha}=j-\alpha}^{m_j}\lambda^{m_j-m_{j-\alpha}}\mu(a_\alpha+\delta m_{j,}j\sum_{\gamma=\alpha+1}^{k}a\gamma)\frac{1}{(s+\lambda)^{\delta m_{j-\alpha},j-\alpha}(s+\lambda+\mu)^{m_j-m_{j-\alpha}+1-\delta m_{j-\alpha},j-\alpha}}\bar{H}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$+\bar{H}^{(1)}m_j(s)\ ,\ 2\le j\le k$$

and

$$\bar{H}^{(j)}m_j(s)=\sum_{\alpha=1}^{k}\sum_{m_{j-\alpha}=j-\alpha}^{mj}\lambda^{m_j-m_{j-\alpha}}\mu(a_\alpha+\delta m_{j,}j\sum_{\gamma=\alpha+1}^{k}a_\gamma)\frac{1}{(s+\lambda)^{\delta m_{j-\alpha},j-\alpha}(s+\lambda+\mu)^{m_j-m_{j-\alpha}+1-\delta m_{j-\alpha},j-\alpha}}\bar{H}^{(j-\alpha)}m_{j-\alpha}(s)$$

$$,\ j\ge k+1$$

$$\bar{P}_{i,}(s)=\frac{\lambda^i}{(s+\lambda)^{i+1}}\ ,\ i\ge 0 \qquad (37)$$

The results (33)-(37) coincide with the results of Garg (2003).

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