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A New Image Encryption Using Chaotic Attractor with Bounded Function

Nahed Aouf Souayed*, Kais Bouallegue, Nabil Ben Slimane*, Mohsen Machhout***

ABSTRACT

In this paper, a new chaotic attractor using Chua attractorcombined with Julia set and bounded function tested for encryption image. The proposed implementation gives us an efficient results in encryption and high level of security. The new system use simplicity of implementation, high quality and high security. Results confirm that the proposed system in unbreakable with the differential power attacks (DPA).

Keywords: Chaotic attractor, Chua's attractor, Julia processes, Image Encryption, bounded function,Cryptanalysis, DPA.

1. INTRODUCTION

During the last four decades,the behavior of chaotic attractor has been competently studied([2-3], [6], [10-11], [39-44]). Itreaches many natural and artificial dynamic systemssuch as human heart, mechanical system, electroniccircuits, etc [22]. There are so many classical attractorswhich are known until now such us Lorenz, Rössler, Chua , Chen, and others ([11], [20], [25], [26]). Some recent control methods are discussed in [39-45].

Our approach in thispaper is to generate a new behavior of chaoticattractor using Chua attractor combined with JuliaProcess and bounded function. This new chaotic attractor became a desired goal for manyengineeringapplication [15] especially for encryption image. That is why we opt to apply an example of these algorithms in an image encryption and we test their efficiency besides the differential power attacks

This paper has been compactly arranged in five sections. Section 1 gives a brief overview of the chaotic attractors and bounded function. Section 2deals with the mathematical formulation for the Julia process.

Section 3 is concerned with the results of implementation when we combine Chua's attractor,Lorenz attractor with Julia processes and bounded function. Section 4 presents the application of the research in a proposed cryptosystem and evaluated using various security and statistical analysis. The paper is concluded with a summary of main results in Section 5.

2. CHAOTIC ATTRACTOR

2.1. Chua Attractor

In 1983, Chua invented the Chua circuit in order to response to two unsuccessful quest between many researchers on chaos concerning two wanting aspects of the Lorenz equations (Lorenz, 1963). The existence of the chaotic attractor from the Chua circuit was confirmed numerically by Matsumoto (1984), and observed experimentally by byZhong and Ayrom (1985), and proved strictly by Chua and al(1986). The basic approach

* Electronic and micro-electronics laboratory, Faculty of science of Monastir, University of Monastir Monastir 5000, Tunisia, *Emails: nahedaouf@gmail.com, machhout@yahoo.fr*

** Departement of Electrical Engineering, Higher Institute of Applied Sciences and Technology of Sousse, University of Sousse, 4003 Sousse, Tunisia, *Email: kais_bouallegue@yahoo.fr*

of the proof is illustrated in a guided exercise on Chua's circuit in the well-known textbook by Hirsch, Smale and Devaney (2003). [4] This system has become one of the models in the research of chaos and is described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a\left(x_1 - x_2 - x_3 + f\left(x_1\right)\right) \end{cases} \tag{1}$$

Where $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$ are the first time derivatives and 'a' is a real parameter. Where$f(x1)$ is a statured function as follows:

$$f\left(x\right) = \begin{cases} k,\ if\ \ x > 1 \\ kx,\ if\ \ \left|x\right| < 1 \\ -k,\ if\ \ x < -1 \end{cases} \tag{2}$$

The figure bellow shows as a classical Chua attractor with two scrolls:

Figure 1: classical Chua attractor with 2 scrolls

2.2. Bounded function

In this subsection, we present the four bounded functions used in this article. The first one is described by:

$$f_1(x) = tangh\ (x);\ -1 < f_1 < 1$$

Then $f_2\left(x\right) = \dfrac{x^2 - 1}{x^2 + 1}; -1 < f_2 < 1$

Then $f_3\left(x\right) = e - \dfrac{1}{x^2 + 1}; e^{-1} < f_3 < 1$

Finally $f_4\left(x\right) = \dfrac{\text{asin}\left(x^2\right) - 1}{\text{asin}\ \left(x^2\right) + 1}; -1 < f_4 < 1$

All this functions are bounded and in the fourth section we will check there influence in the behavior of the new chaotic attractor. [6] [7] [8]

3. MATHEMATIC FORMULATION OF FRACTAL MODELS

3.1. Julia set

The interest in Julia set began in the 1920's with Gaston Julia. Recently, this process has been studied by many researchers and developed in different applications and controls. In this section, we will present an algorithm inspired from Julia processes [3]. So, in order to generate a Julia processes, some properties are well considered ([1], [12], [31]):

1) The Julia set is a repellor.
2) The Julia set is invariant.
3) An orbit on Julia set is either periodic or chaotic.
4) All unstable periodic points are on Julia set.
5) The Julia set is either wholly connected or wholly disconnected.

All sets generated only with Julia sets combination have fractal structure([3], [4], [21]). Real and imaginary parts of the complex numbers are separately calculated.

$$\begin{cases} x_{i+1} = x_i^2 - y_i^2 + \alpha \\ y_{i+1} = x_i y_i + \beta \end{cases} \tag{3}$$

Algorithm 1: $(x_{i+1}; y_{i+1}) = Pj(x_i; y_i)$

1: **If** $x_n > 0$ **Then**

2: $\begin{cases} x_{n+1} = \dfrac{\sqrt{\sqrt{x_n^2 + y_n^2} + a}}{2} \\ y_{n+1} = \dfrac{b}{2x_n} \end{cases}$

3: **end if**

4: **If** $x_n < 0$ **Then**

5: $\begin{cases} y_{n+1} = \dfrac{\sqrt{\sqrt{x_n^2 + y_n^2} - x_n}}{2} \\ x_{n+1} = \dfrac{b}{2y_n} \end{cases}$

6: **end if**

7: **If** $x_n = 0$ **Then**

8: $\begin{cases} x_{n+1} = \sqrt{\dfrac{|y_n|}{2}} \\ y_{n+1} = 0 \end{cases}$

9: **end if**

10: **If** $k < 1300$ **Then** $\begin{cases} x_{n+1} = x_n \\ y_{n+1} = y_n \end{cases}$

Else $\begin{cases} x_{n+1} = -x_n \\ y_{n+1} = -y_n \end{cases}$

11: **end if**

12: **end if**

3.2. Mathematic formulation

In this section, we study two chaotic attractors using transformation by Julia's process as cited in [21]

$$(x_{i+1}, y_{i+1}) = Pj(x_i, y_i)$$

Our method is to apply the methodology cited in paper [21] we conceder this two systems which aredefined for two different chaotic attractors respectively: the first chaotic attractor is given by

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, x_3) + \alpha_1 \\ \dot{x}_2 = f_2(x_1, x_2, x_3) + \beta_1 \\ \dot{x}_3 = f_3(x_1, x_2, x_3) \end{cases} \tag{4}$$

We use the same chaotic attractor and we modify the value of α_1 and β_1. So we consider the second one as follows:

$$\begin{cases} \dot{y}_1 = f_1(y_1, y_2, y_3) + \alpha_2 \\ \dot{y}_2 = f_2(y_1, y_2, y_3) + \beta_2 \\ \dot{y}_3 = f_3(y_1, y_2, y_3) \end{cases} \tag{5}$$

We treat the first system with a Julia's fractal Process. The results are obtained as follows:

$$(u, v) = P_j(\dot{x}_1, \dot{x}_2)$$

$$(u, v) = P_j\left(f(x_1, x_2, x_3) + \alpha_1, f_2(x_1, x_2, x_3) + \beta_1\right) \tag{6}$$

The second system is considered as follows:

$$(p, q) = P_j(\dot{y}_1, \dot{y}_2)$$

$$(p, q) = P_j\left(f(y_1, y_2, y_3) + \alpha_2, f_2(y_1, y_2, y_3) + \beta_2\right) \tag{7}$$

After that, we generate two outputs (X_G, Y_G)

$$(X_G, Y_G) = P_j((u - p), (v - q)) \tag{8}$$

P_j switches between two cases:

The first one when $\dot{x}_1 > 0$

The second case is when $\dot{x}_1 < 0$

If $\dot{x}_1 > 0$, the outputs are computed as follows

$$\begin{cases} u = \sqrt{\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2}} \\ v = \dfrac{\dot{x}_2}{2\dot{x}_1} \end{cases} \tag{9}$$

If $\dot{x}_1 < 0$, the outputs are computed as follows

$$\begin{cases} u = \dfrac{\dot{x}_2}{2\dot{x}_2} \\ v = \sqrt{\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2}} \end{cases} \tag{10}$$

The second system is as follows:

$$(p,q) = P_j(\dot{y}_1, \dot{y}_2)$$

If $\dot{y}_1 > 0$, the outputs are computed as follows

$$\begin{cases} p = \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}} \\ q = \dfrac{\dot{y}_2}{2\dot{y}_2} \end{cases} \tag{11}$$

If $\dot{y}_1 < 0$, the outputs are computed as follows

$$\begin{cases} p = \dfrac{\dot{y}_1}{2\dot{y}_2} \\ q = \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}} \end{cases} \tag{12}$$

We conceder (X_G, Y_G), the output vector generated as follows:

$$(X_G, Y_G) = Pj((u-p), (v-q)) \tag{13}$$

In this stage, we have four cases of switching:

Case 1: when $(x_1 > 0)$ and $(y_1 > 0)$

Case 2: when $(x_1 > 0)$ and $(y_1 < 0)$

Case 3: when $(x_1 < 0)$ and $(y_1 > 0)$

Case 4: when $(x_1 < 0)$ and $(y_1 < 0)$

weanalyse only the first one:

when $(x_1 > 0)$ and $(y_1 > 0)$so $(u; v)$ takes this result:

$$\begin{cases} u = \sqrt{\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2}} \\ v = \dfrac{\dot{x}_2}{2\dot{x}_1} \end{cases} \tag{14}$$

$$\begin{cases} p = \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}} \\ q = \dfrac{\dot{y}_2}{2\dot{y}_1} \end{cases} \tag{15}$$

$$\Rightarrow \left(X_G, Y_G\right) = \begin{cases} X_G = \dfrac{\sqrt{\sqrt{\left(u-p\right)^2 + \left(v-q\right)^2} + \left(u-p\right)}}{2} \\ Y_G = \dfrac{\left(v-q\right)}{2\left(u-p\right)} \end{cases} \tag{16}$$

$$\Rightarrow \begin{cases} \sqrt{\sqrt{\left(\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2} - \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}}\right)^2 + \left(\dfrac{f_2}{2f_1} - \dfrac{g_2}{2g_1}\right)^2} + \dfrac{\sqrt{\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2}} - \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}}}{2}} \\ Y_G = \dfrac{\dfrac{\dot{x}_2}{2\dot{x}_1} - \dfrac{\dot{y}_2}{2\dot{y}_1}}{2\left(\sqrt{\dfrac{\sqrt{\dot{x}_1^2 + \dot{x}_2^2} + \dot{x}_1}{2}} - \sqrt{\dfrac{\sqrt{\dot{y}_1^2 + \dot{y}_2^2} + \dot{y}_1}{2}}\right)} \end{cases} \tag{17}$$

4. IMPLEMENTATION RESULTS

4.1. Behavior of chaotic attractor with Chua's attractor and Julia processes

In our approach we apply a combination of Julia's process [21] with Chua attractor as follow:

$$P_j\left(X_k, Y_k\right) = \left(X_{k+1}, Y_{k+1}\right) \tag{18}$$

Let ε be the complete metric unit, ϕ a fractal-processed system of ε in ε such as

$$\varepsilon \to \varepsilon$$
$$\mathtt{f}: (f_1, f_2) \to (X_G, Y_G)$$

The fractal-processed system ϕ is represented by figure 2, it shows us three different behaviors of chaotic attractors.

The figure 2.a represent the implementation of the following system

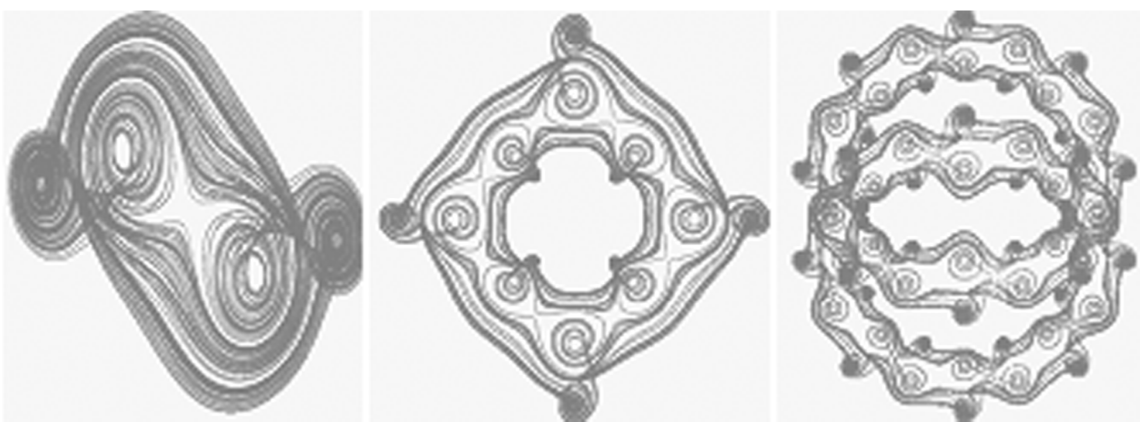
$$\phi_1 : (X_G, Y_G) = P_j((y_1 + \beta_1), (y_2 + \beta_2)) \tag{19}$$

The figure 2.b represent the implementation of the following system

$$\phi_2 : \begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2)) \\ (U_2, V_2) = P_j((y_1 + \alpha_1), (y_2 + \alpha_2)) \\ (X_G, V_G) = P_j((U_2 - U_1), (V_2 - V_1)) \end{cases} \tag{20}$$

The figure 2.c represent the implementation of the following system

$$\phi_3 : \begin{cases} (U_1, V_1) = P_j((y_1 + \beta_1), (y_2 + \beta_2)) \\ (U_2, V_2) = P_j((y_1 + \alpha_1), (y_2 + \alpha_2)) \\ (U_3, V_3) = P_j((U_2 - U_1), (V_2 - V_1)) \\ (U_4, V_4) = P_j(U_3, V_3) \\ (X_G, V_G) = P_j((U_4 - U_3), (V_4 - V_3)) \end{cases} \tag{21}$$



(a) (b) (c)

Figure 2: Multi-scroll Chua attractor

The simulation results allows us to obtain a chaotic attractor in a convex space. The number of scrolls are increased. The simulation is done by Microsoft Visual Studio 2010.[7]

4.2. Behavior of chaotic attractor with Chua's attractor and bounded function

After the combination between Chua's attractor and Julia process we add the four bounded function presented before, in order to choose one of them to encrypt image.[6]

i. behavior of chaotic attractor with chua's attractor and tanh(x)

When we apply hyperbolic tangent in the presented system we obtain three results as follow.

The figure 3. a represent the implementation of this system

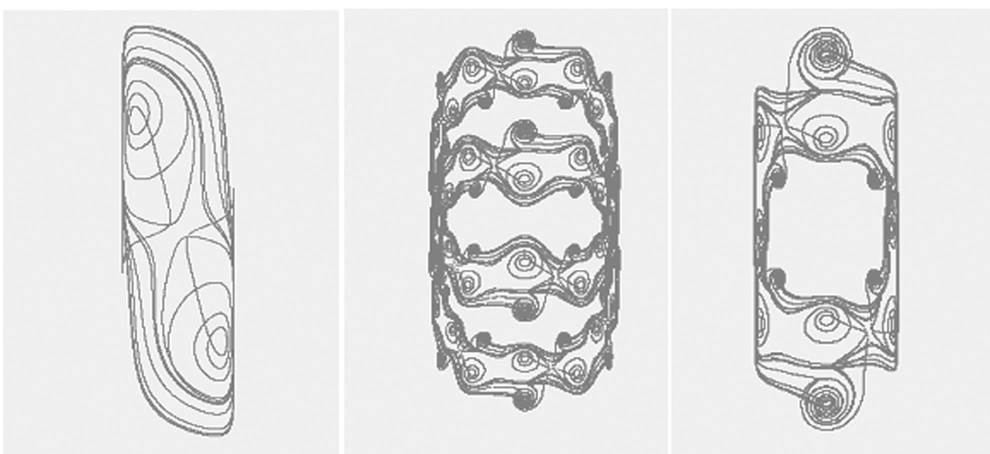
$$(X_G, Y_G) = P_j(tanh(y_1 + \beta_1), (y_2 + \beta_2)) \tag{22}$$

The figure 3.b represent the implementation of this system

$$\begin{cases} (U_1,\ V_1) = P_j\left((y_1+\beta_1),(y_2+\beta_2)\right) \\ (U_2,\ V_2) = P_j\left((y_1+\alpha_1),(y_2+\alpha_2)\right) \\ (X_G,\ V_G) = P_j\left(\tanh(U_2-U_1),(V_2-V_1)\right) \end{cases} \tag{23}$$

The figure 3.c represent the implementation of this system

$$\begin{cases} (U_1,\ V_1) = P_j\left((y_1+\beta_1),(y_2+\beta_2)\right) \\ (U_2,\ V_2) = P_j\left((y_1+\alpha_1),(y_2+\alpha_2)\right) \\ (U_3,\ V_3) = P_j\left((U_2-U_1),(V_2-V_1)\right) \\ (U_4,\ V_4) = P_j\left(U_3,\ V_3\right) \\ (X_G,\ V_G) = P_j\left(\tanh(U_4-U_3),(V_4-V_3)\right) \end{cases} \tag{24}$$



(a) (b) (c)

Figure 3: Multi-scroll Chua attractor with bounded function (tanh(x))

ii. Behavior of chaotic attractor with Chua's attractor and $\left(\frac{x^2-1}{x^2+1}\right)$

When we replace the hyperbolic tangent $\left(\frac{x^2-1}{x^2+1}\right)$ in the presented system we obtain three results as follow.

The figure 4.a represent the implementation of this system

$$\begin{cases} (U_1,\ V_1) = P_j\left((y_1+\beta_1),(y_2+\beta_2)\right) \\ U_2 = \dfrac{U_1^2-1}{U_1^2+1} \\ (X_G,\ V_G) = (U_1,\ V_1) \end{cases} \tag{25}$$

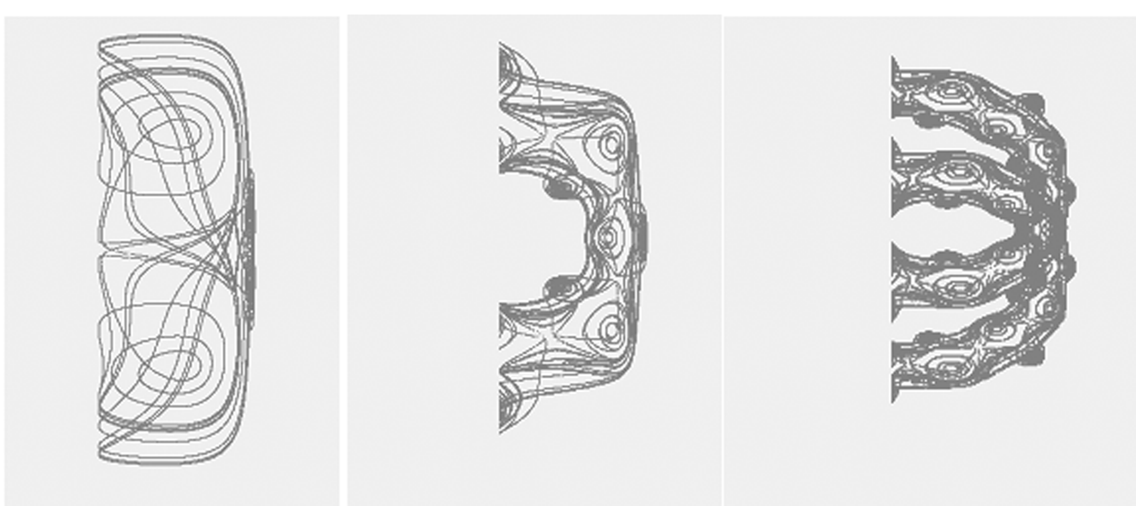
The figure 4.b represent the implementation of this system

$$\begin{cases} (U_1,\ V_1) = P_j\left((y_1+\beta_1),(y_2+\beta_2)\right) \\ (U_2,\ V_2) = P_j\left((y_1+\alpha_1),(y_2+\alpha_2)\right) \\ (U_3,\ V_3) = P_j\left((U_2-U_1),(V_2-V_1)\right) \\ U_4 = \dfrac{U_3^2-1}{U_3^2+1} \\ (X_G,\ Y_G) = (U_4,\ V_3) \end{cases} \tag{26}$$

The figure 4.c represent the implementation of this system

$$\begin{cases} (U_1,\ V_1) = P_j\left((y_1+\beta_1),(y_2+\beta_2)\right) \\ (U_2,\ V_2) = P_j\left((y_1+\alpha_1),(y_2+\alpha_2)\right) \\ (U_3,\ V_3) = P_j\left((U_2-U_1),(V_2-V_1)\right) \\ (U_4,V_4) = P_j(U_3,V_3) \\ (U_5,\ V_5) = P_j\left((U_4-U_3),(V_4-V_3)\right) \\ U_6 = \dfrac{U_5^2-1}{U_5^2+1} \\ (X_G,\ Y_G) = P_j(U_6,\ V_5) \end{cases} \tag{27}$$

iii. behavior of chaotic attractor with Chua's attractor and $\left(\dfrac{asin(x^2)-1}{asin(x^2)+1}\right)$ or $e-\dfrac{1}{x^2+1}$



(a) **(b)** **(c)**

Figure 4: Multi-scroll Chua attractor with bounded function $\left(\dfrac{x^2-1}{x^2+1}\right)$

when we replace $\left(\frac{x^2-1}{x^2+1}\right)$ by $\left(\frac{asin\left(x^2\right)-1}{asin\left(x^2\right)+1}\right)$ or $e-\frac{1}{x^2+1}$ we obtain the same figures approximately with different sizes.

So we observe that only hyperbolic tangent can conserve the symmetry of the chaotic attractor with a bounded zone. However, the three other functions divide the figure in a half. For the Julia process, let the bounded function to conserve the multiplication of the number of scrolls as we can see in figure 3 and 4. For these reasons we choose to apply the encryption using hyperbolic tangent and we will check the results in the following section.

5. THE PROPOSED ENCRYPTION SCHEME

The proposed encryption algorithm, based on combination between Chua attractor, Julia set and Bounded function (hyperbolic tangent) .the image is XORED by chaotic matrix to modify the values of each pixel in order to obtain ciphered image.

The hyperbolic tangent is one time applied on the first variable (x) we call it algorithm 1 and when we apply it in the second variable (y) we call it algorithm 2.

For having a good encryption, the cryptosystem should confuse the cipher image enough in order that the attacks can't explore any useful information. We illustrate bellow the statistical analyses from four indicators : the histogram, correlation of two adjacent pixels,the information entropy and the differential attacks.

5.1. Histogram

The strong resistance to statistical attacks of our proposed algorithms is shown on the histogram of enciphered image. Many gray-scale images of size 256 × 256 are selected for this purpose and their histograms are compared with their corresponding encrypted image. For our case we have used two typical examples "Lena" and "Elena" as shown in figure 7. The histogram of the original images contains large spikes but the

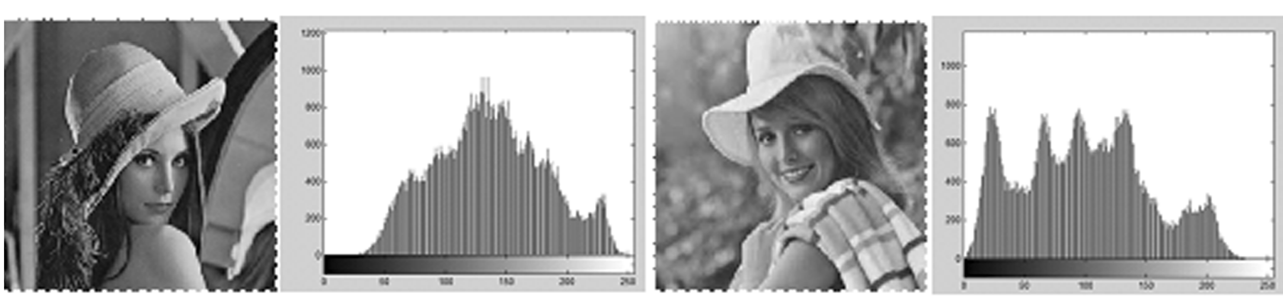


Figure 5: Histogram of Original Image

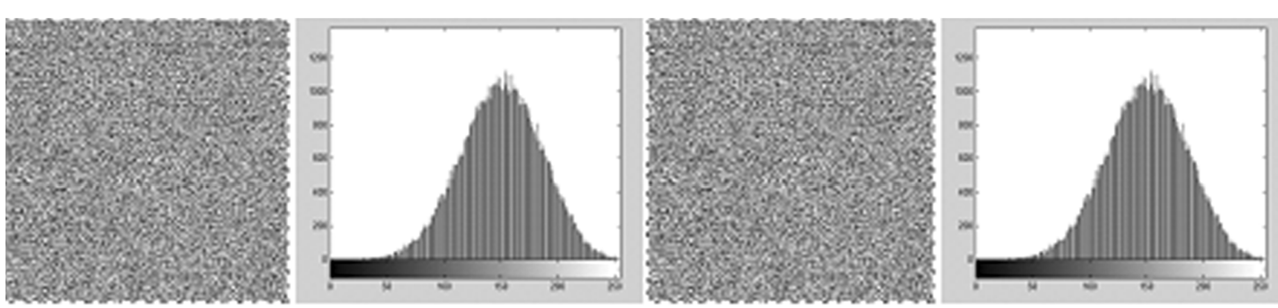


Figure 6: Histogram of Encrypted Image

histogram of the cipher images as shown in figure 8 is more uniform .It is clear that the histogram of the encrypted images is, significantly different from the respective histogram of the tow original images and bears no statistical resemblance to the plain image. Hence statistical attack on the proposed image encryption procedure is difficult.

5.2. Correlation of two adjacent pixels

We calculate the correlation coefficients of adjacent pixels for both plain image and cipher image for two kind of image (Lena and Elena) and with the two different algorithm. It is well known that adjacent image pixels are highly correlated either in horizontal, vertical ordiagonal directions. Such high correlation property can be quantified by means of correlation coefficients which are given by:

Table 1
Correlation coefficients comparison of the proposed algorithm for image encryption with others.

image name	*Direction of adjacent pixels*	*correlation plain image and*	*correlation of cipher image ALGORITHM 1*	*correlation of cipher image ALGORITHM 2*	*[36]*	*[9]*
Lena	Horizontal	0.9961	0.00324	0.00292	0.0802	0.032
	Vertical	0.9928	0.00297	0.00211	0.0706	0.027
	Diagonal	0.9841	0.00107	0.00097	0.0738	0.038
Elaine	Horizontal		0.00610	0.00287	NA	NA
	Vertical		0.00102	0.00305	NA	NA
	Diagonal		0.00298	0.00101	NA	NA

According to the values of correlation listed in table 1 both of our proposed algorithm have a minimum absolute value of correlation coefficient in all used images for encryption test. It means our system is secure and efficient.

5.3. Information entropy

Entropy information is amathematical theory for data communication and storage.Now, information theory is interested with correction oferrors, compression of data and cryptography.the entropy H(m) is computed by the following equation

$$H(m) = \sum_{i=0}^{2^N} P(m) \log_2 \frac{1}{P(m_i)} bits \tag{28}$$

Where $P(m_i)$ is the probability of symbol *mi* and theentropy is measured in bits. The entropyanalysis testare executed on two images test, the Table 2 show theresult which means that the proposed cryptosystem for imageencryption is nearly to proved theoretical entropy valuewhich equals 8, to conclude that our cryptosystem respect the entropy attack. Table 2 show differents values of entropies.

Table 2
Entropies analysis of the proposed image cryptosystem

Image test		*Lena*	*Elaine*
Entropy	Algorithm 1	7.9845	7.9809
	Algorithm 2	7.9859	7.9837

5.4. Differential attack

As a general requirement for all the image encryption schemes, the encrypted image should begreatly different from its original form. Such difference can be measured by means of two criterianamely, the NPCR (Number of Pixel Change Rate) and the UACI (Unied Average ChangingIntensity) [30]. The NPCR is used to measure the number of pixels in difference between twoimages. The second criterion, UACI, is used to measure the average intensity difference .

The NPCR and UACI measured between the plain images of Lena and Elaine with the proposed cryptosystem are given in the table 3. So our cryptosystem is securely resistant against differential attacks.

Table 3
Value of plaintext sensitivity

Image test		*Lena*	*Elaine*
NPCR %	Algorithm 1	99.5876%	99.5797%
	Algorithm 2	99.5937%	99.5912%
UACI %	Algorithm 1	30.0820%	30.0886%
	Algorithm 2	30.1483%	30.1527%

6. CONCLUSION

In this work, an efficient, secure and robust cryptosystemfor image encryption is reported which is realized usingChua attractor combined with Julia process and hyperbolic tangent. Our experimental resultsshow a good cryptographic features of level security andspeed in image encryption.

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