

Some New Results on Extended Medium Domination Number of Few class of Graphs

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Abstract : In Computer Communication Net work, each node(computer) is capable of safeguarding every node (computer) in its neighbourhood and in domination every vertex is required to be protect. The resistance of network is the response to any disruption in some of the work stations. The concept of Extended Medium domination number plays an vital role. Let $G = (V,E)$ be the graph. The manipulations of the Extended Medium domination number has variety of applications in computer communication Networks. For some more real life situation and efficient communication, G. Mahadevan, V. Vijayalakshmi and C. Sivagnanam. introduced the concept of extended medium domination number of a graph. It is defined as $EMD(G)$

$$= \frac{ETDV(G)}{\binom{p}{2}} \text{ where } ETDV(G) = \sum_{u,v \in V(G)} edom(u, v)$$

Keywords : Extended medium domination number (EMD), mirror graph, fire cracker, banana tree.

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1. INTRODUCTION

Graph theory especially the theory of domination plays an vital role as far as application side is concern. It is one of the most prominent areas of research because of its applications in Computer science, Communication Net works, Electrical Engineering and Operation research. Any real life problem can be converted in to graph theoretical model. With the help of the existing and/or new one algorithm in domination, we can apply and get the result there by interpret with the real life physical situation. Many different types of domination parameters are available in the literature. The concept of Medium domination number was introduced by Duygu Vargor and Pinar Dundar in [1] with application in communication network. Motivated by the above, in [9], the authors introduced another concept called Extended Medium domination number of a graph. Here we impose one additional condition that $edom(u, v)$ is the sum of number of u-v paths of length one, two and three. We define $ETDV(G) = \sum_{u,v \in V(G)} edom(u, v)$ for all u, v

$\in V(G)$ and the extended medium domination number is defined as $EMD(G) = \frac{ETDV(G)}{\binom{p}{2}}$. Because of this

additional condition, many conflicts in communication networks can be avoided. For certain practical purpose, we need this extended medium domination for some special types of graphs. Hence in this paper we investigate the extended medium domination number of some interesting special types of graphs.

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Definition 1.1 : [1] In any simple graph G of p number of vertices, the medium domination number of G is defined as $EMD(G) = \frac{ETDV(G)}{\binom{p}{2}}$

Example 1.2 :

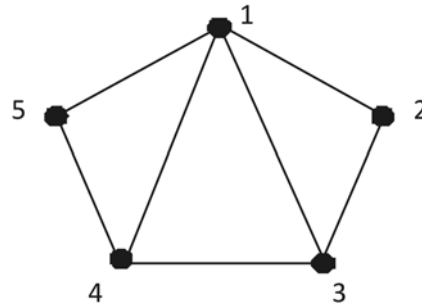


Fig. 1.

From the above figure 1.1, $edom(1, 2) = 3$; $edom(1, 3) = 4$; $edom(1, 4) = 4$; $edom(1, 5) = 3$; $edom(2, 3) = 3$; $edom(2, 4) = 4$; $edom(2, 5) = 4$; $edom(3, 4) = 4$; $edom(3, 5) = 5$; $edom(4, 5) = 3$. $ETDV(G) = 37$;

$$EMD(G) = \frac{ETDV(G)}{\binom{n}{2}} = \frac{37}{10}.$$

Definition 1.3 : Let G be a graph. Let G' be a copy of G. The mirror graph M(G) of G is defined as the disjoint union of G and G' with additional edges joining each vertex of G to its corresponding vertex in G'.

Notation 1.4 : $F_{m,n}$ is a fire cracker graph obtained by attaching any one of pendent vertex of m copy of the star $K_{1, n-1}$.

Notation 1.5 : $B_{m,n}$ is a banana tree graph obtained by attaching any one of pendent vertex of the star $K_{1,n}$ to each pendent vertex of the star $K_{1,m}$.

Notation 1.6 : K-nested triangle graph is joining K copy of triangle $a_i b_i c_i$ for $i = 1$ to K. Connecting the vertices a_i to a_{i+1} , b_i to b_{i+1} , c_i to c_{i+1} for $i = 1$ to K-1.

Observation 1.7 : [9] $ETDV(C_m) = 3m$.

Observation 1.8 : [9] $ETDV(P_m) = 3m-6$.

Observation 1.9 : [9] $ETDV(K_n) = \binom{n(n-1)}{2} (n^2 - 4n + 5)$.

2. MAIN RESULT

The concept of extended medium domination number plays an vital role in communication network subject to some specified conditions. In [9,10], the authors obtained extended medium domination number of various types of some basic graphs. In order to interpret with real life situation, we need this number for various special types of graphs with real life situation. Hence it is necessary to obtain this number for some specialized types of graphs, like $M(P_n)$, $M(C_n)$, $M(K_n)$ $P_m \odot K_n^c$, m-nested triangle, firecracker and Banana tree. In this section, we obtain the extended medium domination number of all the above said special types of graphs.

Theorem 2.1 : If $G = M(P_t)$ then $EMD(G) = \frac{21t - 32}{\binom{2t}{2}}$ where $t \geq 3$.

Proof : Consider the mirror graph of a path P_t . Let the vertices of P_t are A_1, A_2, \dots, A_t and the vertices of P_t' are B_1, B_2, \dots, B_t . Now join A_i to B_i for $i = 1$ to t. $ETDV(G) = \sum edom(u, v)$ for $u, v \in V(G)$.

For any path P_t , $ETDV(P_t) = 3t - 6$ for any t . we have two paths P_t and P_t' .

$$\text{edom}(A_i, B_i) = 3 \text{ for } i = 2 \text{ to } t - 1; \text{ Therefore } \sum_{i=2}^{t-1} \text{edom}(A_i, B_i) = 3(t - 2);$$

$$\text{edom}(A_i, B_i) = 2 \text{ for } i = 1 \text{ and } t; \text{ Therefore } \sum_{i=1, t} \text{edom}(A_i, B_i) = 4;$$

$$\text{edom}(A_i, A_{i+1}) = 1 \text{ for } i = 1 \text{ to } t - 1; \text{ Therefore } \sum_{i=1}^{t-1} \text{edom}(A_i, B_{i+1}) = t - 1;$$

$$\text{edom}(B_i, B_{i+1}) = 1 \text{ for } i = 1 \text{ to } t - 1; \text{ Therefore } \sum_{i=1}^{t-1} \text{edom}(A_i, B_{i+1}) = t - 1;$$

$$\text{edom}(A_i, B_{i\pm 1}) = 2 \text{ for } i = 2 \text{ to } t - 1; \text{ Therefore } \sum_{i=2}^{t-1} \text{edom}(A_i, B_{i\pm 1}) = 4(t - 2);$$

$$\text{edom}(A_i, B_{i\pm 2}) = 3 \text{ for } i = 3 \text{ to } t - 2; \text{ Therefore } \sum_{i=3}^{t-2} \text{edom}(A_i, B_{i\pm 2}) = 6(t - 4);$$

$$\text{edom}(A_1, B_2) = 2; \text{edom}(A_n, B_{n-1}) = 2; \text{edom}(A_1, B_3) = 3; \text{edom}(A_2, B_4) = 3; \text{edom}(A_n, B_{n-2}) = 3; \text{edom}(A_{n-1}, B_{n-3}) = 3.$$

$$\begin{aligned} \text{ETDV}(G) &= 2(3t - 6) + 3(t - 2) + 2(t - 1) + 4(t - 2) + 6(t - 4) + 20 \\ &= 6t - 12 + 3t - 6 + 2t - 2 + 4t - 8 + 6t - 24 + 20 = 21t - 32. \end{aligned}$$

$$\text{EMD}(G) = \frac{\text{ETDV}(G)}{\binom{p}{2}} = \frac{21t - 32}{\binom{2t}{2}}.$$

Theorem 2.2 : If $G = M(C_m)$ then $\text{EMD}(G) = \frac{21m}{\binom{2m}{2}}$ where $m \geq 3$.

Proof : Consider the mirror graph of a cycle C_m . Let the vertices of C_m are a_1, a_2, \dots, a_m and the vertices of C_m' are b_1, b_2, \dots, b_m . Now join a_i to b_i by using the edge $a_i b_i$ for $i = 1$ to m . In this graph consider the vertex $a_{m+1} = a_1; b_{m+1} = b_1; a_m = a_0; b_m = b_0$.

$$\text{ETDV}(G) = \sum \text{edom}(u, v) \text{ for } u, v \in V(G).$$

For any cycle C_m , $ETDV(C_m) = 3m$ for any m . we have two cycles C_m and C_m' .

$$\text{edom}(a_i, b_i) = 3 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(a_i, b_i) = 3m;$$

$$\text{edom}(a_i, b_{i\pm 1}) = 2 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(a_i, b_{i\pm 1}) = 4m;$$

$$\text{edom}(a_i, b_{i\pm 2}) = 3 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(a_i, b_{i\pm 2}) = 6m;$$

$$\text{edom}(a_i, a_{i+1}) = 1 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(a_i, a_{i+1}) = m;$$

$$\text{edom}(b_i, b_{i+1}) = 1 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(b_i, b_{i+1}) = m; \text{ETDV}(G) = 21m.$$

$$\text{EMD}(G) = \frac{\text{ETDV}(G)}{\binom{p}{2}} = \frac{21m}{\binom{2m}{2}}$$

Theorem 2.3 : If $G = M(K_n)$ then $EMD(G) = \frac{n^4 - 3n^3 + 7n^2 - 4n}{\binom{2n}{2}}$ where $n \geq 4$.

Proof : Consider the mirror graph of a complete graph K_n . Let the vertices of K_n are a_1, a_2, \dots, a_n and the vertices of K_n' are b_1, b_2, \dots, b_n . Now join a_i to b_i by using the edge $a_i b_i$ for $i = 1$ to n .

$$ETDV(G) = \sum \text{edom}(u, v) \text{ for } u, v \in V(G).$$

For any complete graph K_n , $ETDV(K_n) = \left(\frac{n(n-1)}{2}\right)(n^2 - 4n + 5)$ for any n .

We have two complete graph K_n and K_n' .

$$\text{edom}(a_1, x) = 1 \text{ for } x = a_2 \text{ to } a_n; \text{ Therefore } \sum_{x=a_2}^{a_n} \text{edom}(a_1, x) = (n-1);$$

$$\begin{aligned} \text{edom}(a_2, x) &= 1 \text{ for } x = a_3 \text{ to } a_n; \text{ Therefore } \sum_{x=a_3}^{a_n} \text{edom}(a_2, x) \\ &= (n-2); \dots \dots \text{edom}(a_{n-1}, a_n) = 1; \end{aligned}$$

$$\text{edom}(b_1, x) = 1 \text{ for } x = b_2 \text{ to } b_n; \text{ Therefore } \sum_{x=b_2}^{b_n} \text{edom}(b_1, x) = (n-1);$$

$$\begin{aligned} \text{edom}(b_2, x) &= 1 \text{ for } x = b_3 \text{ to } b_n; \text{ Therefore } \sum_{x=b_3}^{b_n} \text{edom}(b_2, x) \\ &= (n-2); \dots \dots \text{edom}(b_{n-1}, b_n) = 1; \end{aligned}$$

$$\text{edom}(a_i, b_i) = n \text{ for } i = 1 \text{ to } n; \text{ Therefore } \sum_{i=1}^{b_n} \text{edom}(a_i, b_i) = n^2;$$

$$\text{edom}(a_1, b_j) = 2(n-1) \text{ for } j = 2 \text{ to } n; \text{ Therefore } \sum_{j=2}^n \text{edom}(a_1, b_j) = 2(n-1)^2; j \neq 1;$$

$$\text{edom}(a_2, b_j) = 2(n-1) \text{ for } j = 1 \text{ to } n; \text{ Therefore } \sum_{j=1}^n \text{edom}(a_2, b_j) = 2(n-1)^2; j \neq 2; \dots \dots$$

$$\text{edom}(a_n, b_j) = 2(n-1) \text{ for } j = 1 \text{ to } n; \text{ Therefore } \sum_{j=1}^n \text{edom}(a_n, b_j) = 2(n-1)^2; j \neq n;$$

$$\begin{aligned} ETDV(G) &= 2\left(\frac{n(n-1)}{2}\right)(n^2 - 4n + 5) + n(n-1) + n^2 + 2n(n-1)^2 \\ &= n^4 - 4n^3 + 5n^2 - n^3 + 4n^2 - 5n + 2n^2 - n + 2n^3 - 4n^2 + 2n \\ &= n^4 - 3n^3 + 7n^2 - 4n \end{aligned}$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{n^4 - 3n^3 + 7n^2 - 4n}{\binom{2n}{2}}$$

Theorem 2 : 4 If G be a graph $F_{m,n}$, then $EMD(G) = \frac{m(n^2 + 3n + 8) - 2(2n + 9)}{2 \binom{mn}{2}}$ where $m, n \geq 4$.

Proof : Let $F_{m,n}$ be a fire cracker graph obtained by attaching any one of the pendent vertex of the star $K_{1,n}$ to all the vertices of the path P_m . Let $K_{1,n}$ be a star with n pendent vertices and P_m be a path with m vertices. Let (a_1, a_2, \dots, a_m) be the vertices of the path P_m . Let $(f_1, f_2, \dots, f_{n-1})$ be the $n-1$ pendent vertices and f_n be the root vertex of the star S_1 , $(f_{n+1}, f_{n+2}, \dots, f_{2n-1})$ be the $n-1$ pendent vertices and f_{2n} be the root vertex of the star $S_2, \dots, (f_{(m-1)n+1}, \dots, f_{mn-1})$ be the $n-1$ pendent vertices and f_{mn} be the root vertex of the star S_m . Now attach the pendent vertex f_1 to a_1, f_{n+1} to $a_2, \dots, f_{(m-1)n+1}$ to a_m respectively.

$$ETDV(G) = \sum \text{edom}(u, v) \text{ for } u, v \in V(G).$$

For any path P_m , $ETDV(P_m) = 3m - 6$;

For any star $K_{1,n-1}$, $ETDV(K_{1,n-1}) = \frac{n(n-1)}{2}$, We have m copy of $K_{1,n-1}$;

$$\text{edom}(f_i, f_{i(n+1)}) = 1 \text{ for } i = 1 \text{ to } m-1; \text{ Therefore, } \sum_{i=1}^{m-1} \text{edom}(f_i, f_{i(n+1)}) = m-1;$$

$$\begin{aligned} \text{edom}(f_{i+1}, f_{(i \pm 1)n+j}) &= 1 \text{ for } i = 2 \text{ to } m-3; j = 2 \text{ to } n \ \& \ 2n; \text{ Therefore, } \sum \text{edom}(f_{i+1}, f_{(i \pm 1)n+j}) \\ &= 2n(m-4) \text{ for } i = 2 \text{ to } m-3; j = 2 \text{ to } n \ \& \ 2n; \end{aligned}$$

$$\text{edom}(f_1, f_{n+i}) = 1 \text{ for } i = 2 \text{ to } n \ \& \ 2n; \text{ Therefore } \sum \text{edom}(f_1, f_{n+i}) = n \text{ for } i = 2 \text{ to } n \ \& \ 2n;$$

$$\begin{aligned} \text{edom}(f_{n+1}, f_{(n \pm n)+i}) &= 1 \text{ for } i = 2 \text{ to } n; \text{ Therefore, } \sum_{i=2}^n \text{edom}(f_{n+1}, f_{(n \pm n)+i}) \\ &= 2(n-1); \text{edom}(f_{n+1}, f_{4n}) = 1; \text{edom}(f_{(m-1)n+1}, f_{(m-2)n+i}) \\ &= 1 \text{ for } i = 2 \text{ to } n \ \& \ -n; \text{ Therefore, } \sum \text{edom}(f_{(m-1)n+1}, f_{(m-2)n+i}) \\ &= n \text{ for } i = 2 \text{ to } n \ \& \ -n; \text{edom}(f_{(m-2)n+1}, f_{(m-2 \pm 1)n+i}) \\ &= 1 \text{ for } i = 2 \text{ to } n; \text{ Therefore, } \sum_{i=2}^n \text{edom}(f_{(m-2)n+1}, f_{(m-2 \pm 1)n+i}) = 2(n-1); \end{aligned}$$

$$\text{edom}(f_{(m-2)n+1}, f_{(m-3)n}) = 1.$$

$$ETDV(G) = 3m - 6 + m \left(\frac{n(n-1)}{2} \right) + (m-1) + 2n(m-4) + 2n + 4(n-1) + 2$$

$$= \left(\frac{mn^2 - mn}{2} \right) + 4m + 2mn - 2n - 9$$

$$= [mn^2 - mn + 8m + 4mn - 4n - 18] / 2$$

$$= \frac{m(n^2 + 3n + 8) - 2(2n + 9)}{2}$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{m(n^2 + 3n + 8) - 2(2n + 9)}{2 \binom{mn}{2}}$$

Theorem 2.5 : If G be a graph $B_{m,n}$ then $EMD(G) = \frac{m(n^2 + 3m + 3n - 1)}{2 \binom{m(n+1)+1}{2}}$ where $m, n \geq 3$.

Proof : Let $B_{m,n}$ be a graph obtained by attaching any one of pendent vertex of the star $K_{1,n}$ to each pendent vertex of the star $K_{1,m}$. Let $K_{1,m}$ be a star with m pendent vertices. Let (l_1, l_2, \dots, l_m) be the pendent vertices and a_1 be the root vertex of the star S. Let (b_1, b_2, \dots, b_n) be the pendent vertices and C_1 be the root vertex of the star S_1 , $(b_{n+1}, b_{n+2}, \dots, b_{2n})$ be the pendent vertices and C_2 be the root vertex of the star $S_2, \dots, (b_{(m-1)n+1}, \dots, b_{mn})$ be the pendent vertices and C_m be the root vertex of the star S_m . Now attach the pendent vertices l_1 to b_n and name that vertex as b_n, l_2 to b_{2n} and name that vertex as b_{2n}, \dots, l_m to b_{mn} and name that vertex as b_{mn} .

$$ETDV(G) = \sum \text{edom}(u, v) \text{ for } u, v \in V(G).$$

For any star $K_{1,n}$, $ETDV(K_{1,n}) = \frac{n(n+1)}{2}$, We have m copy of $K_{1,n}$;

For any star $K_{1,m}$, $ETDV(K_{1,m}) = \frac{m(m+1)}{2}$;

$$\begin{aligned} \text{edom}(a_1, b_{ni+j}) &= 1 \text{ for } i = 0 \text{ to } m-1; j = 1 \text{ to } n-1; \text{ Therefore, } \sum \text{edom}(a_1, b_{ni+j}) \\ &= m(n-1) \text{ for } i = 0 \text{ to } m-1; j = 1 \text{ to } n-1; \end{aligned}$$

$$\text{edom}(a_1, c_i) = 1 \text{ for } i = 1 \text{ to } m; \text{ Therefore } \sum_{i=1}^m \text{edom}(a_1, c_i) = m;$$

$$\begin{aligned} \text{edom}(b_{in}, c_j) &= 1 \text{ for } i = 1 \text{ to } m; j = 1 \text{ to } m; i \neq j; \text{ Therefore, } \sum \text{edom}(b_{in}, c_j) \\ &= m(m-1) \text{ for } i = 1 \text{ to } m; j = 1 \text{ to } m; i \neq j; \end{aligned}$$

$$\begin{aligned} ETDV(G) &= \frac{n(n+1)}{2}(m) + \frac{m(m+1)}{2} + mn - m + m + m^2 - m \\ &= [mn^2 + mn + m^2 + m + 2m^2 + 2mn - 2m] / 2 \\ &= [3m^2 + mn^2 + 3mn - m] / 2 \\ &= m(n + 3m + 3n - 1) / 2 \end{aligned}$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{m(n^2 + 3m + 3n - 1)}{2 \binom{m(n+1)+1}{2}}$$

Theorem 2.6 : If G be a m-nested triangle graph then $EMD(G)$ is $\frac{3(21m-28)}{\binom{3m}{2}}$ where $m \geq 3$.

Proof : Consider the graph obtained by attaching m copy of triangles. Let (a_{i1}, a_{i2}, a_{i3}) be the vertices of i^{th} triangle for $i = 1$ to m . Now join a_{i1} to $a_{(i+1)1}$, a_{i2} to $a_{(i+1)2}$, a_{i3} to $a_{(i+1)3}$ for $i = 1$ to $m-1$.

$$ETDV(G) = \sum \text{edom}(u, v) \text{ for } u, v \in V(G).$$

$$\text{edom}(a_{i1}, a_{(i+1)1}) = 3 \text{ for } i = 1 \text{ to } m-1; \text{ Therefore, } \sum_{i=1}^{m-1} \text{edom}(a_{i1}, a_{(i+1)1}) = 3(m-1);$$

$$\text{edom}(a_{i2}, a_{(i+1)2}) = 3 \text{ for } i = 1 \text{ to } m-1; \text{ Therefore, } \sum_{i=1}^{m-1} \text{edom}(a_{i2}, a_{(i+1)2}) = 3(m-1);$$

$$\text{edom}(a_{i3}, a_{(i+1)3}) = 3 \text{ for } i = 1 \text{ to } m - 1; \text{ Therefore, } \sum_{i=1}^{m-1} \text{edom}(a_{i3}, a_{(i+1)3}) = 3(m - 1);$$

$$\text{edom}(a_{i1}, a_{i2}) = \text{edom}(a_{i1}, a_{i3}) = \text{edom}(a_{i3}, a_{i2}) = 4 \text{ for } i = 2 \text{ to } m - 1;$$

$$\text{Therefore } \sum_{i=2}^{m-1} [\text{edom}(a_{i1}, a_{i2}) + \text{edom}(a_{i1}, a_{i3}) + \text{edom}(a_{i3}, a_{i2})] = 12(m - 2);$$

$$\begin{aligned} \text{edom}(a_{11}, a_{12}) &= \text{edom}(a_{11}, a_{13}) = \text{edom}(a_{13}, a_{12}) = \text{edom}(a_{m1}, a_{m2}) = \text{edom}(a_{m1}, a_{m3}) \\ &= \text{edom}(a_{m3}, a_{m2}) = 3; \end{aligned}$$

$$\text{edom}(a_{i1}, a_{(i+2)1}) = \text{edom}(a_{i2}, a_{(i+2)2}) = \text{edom}(a_{i3}, a_{(i+2)3}) = 1 \text{ for } i = 1 \text{ to } m - 2;$$

$$\text{Therefore } \sum_{i=1}^{m-2} [\text{edom}(a_{i1}, a_{(i+2)1}) + \text{edom}(a_{i2}, a_{(i+2)2}) + \text{edom}(a_{i3}, a_{(i+2)3})] = 3(m - 2);$$

$$\text{edom}(a_{i1}, a_{(i+3)1}) = \text{edom}(a_{i2}, a_{(i+3)2}) = \text{edom}(a_{i3}, a_{(i+3)3}) = 1 \text{ for } i = 1 \text{ to } m - 3;$$

$$\text{Therefore } \sum_{i=1}^{m-3} [\text{edom}(a_{i1}, a_{(i+3)1}) + \text{edom}(a_{i2}, a_{(i+3)2}) + \text{edom}(a_{i3}, a_{(i+3)3})] = 3(m - 3);$$

$$\begin{aligned} \text{edom}(a_{i1}, a_{(i+1)2}) &= \text{edom}(a_{i1}, a_{(i+1)3}) = \text{edom}(a_{i2}, a_{(i+1)3}) = \text{edom}(a_{i2}, a_{(i+1)1}) \\ &= \text{edom}(a_{i3}, a_{(i+1)1}) = \text{edom}(a_{i3}, a_{(i+1)2}) = 3 \text{ for } i = 1 \text{ to } m - 1; \end{aligned}$$

$$\begin{aligned} \text{Therefore } \sum_{i=1}^{m-1} [\text{edom}(a_{i1}, a_{(i+1)2}) + \text{edom}(a_{i1}, a_{(i+1)3}) + \text{edom}(a_{i2}, a_{(i+1)3}) + \text{edom}(a_{i2}, a_{(i+1)1}) \\ + \text{edom}(a_{i3}, a_{(i+1)1}) + \text{edom}(a_{i3}, a_{(i+1)2})] = 18(m - 1); \end{aligned}$$

$$\begin{aligned} \text{edom}(a_{i1}, a_{(i+2)2}) &= \text{edom}(a_{i1}, a_{(i+2)3}) = \text{edom}(a_{i2}, a_{(i+2)3}) = \text{edom}(a_{i2}, a_{(i+2)1}) \\ &= \text{edom}(a_{i3}, a_{(i+2)1}) \\ &= \text{edom}(a_{i3}, a_{(i+2)2}) = 3 \text{ for } i = 1 \text{ to } m - 2; \end{aligned}$$

$$\begin{aligned} \text{Therefore } \sum_{i=1}^{m-2} [\text{edom}(a_{i1}, a_{(i+2)2}) + \text{edom}(a_{i1}, a_{(i+2)3}) + \text{edom}(a_{i2}, a_{(i+2)3}) + \text{edom}(a_{i2}, a_{(i+2)1}) + \\ \text{edom}(a_{i3}, a_{(i+2)1}) + \text{edom}(a_{i3}, a_{(i+2)2})] = 18(m - 2). \end{aligned}$$

$$\begin{aligned} \text{ETDV(G)} &= 27(m - 1) + 33(m - 2) + 3(m - 3) + 18 \\ &= 63m - 27 - 66 - 9 + 18 \\ &= 63m - 84 = 3(21m - 28) \end{aligned}$$

$$\text{EMD(G)} = \frac{\text{ETDV(G)}}{\binom{p}{2}} = \frac{3(21m - 28)}{\binom{3m}{2}}$$

3. CONCLUSION

For the purpose of interpreting with real life communication system so as make an effective solution, we need the extended medium domination number of some more special types of graphs. The recently introduced extended medium domination number also plays an vital role to avoid many conflicts. Hence in this paper, we obtained the extended medium domination number of mirror graph of Path, mirror graph of cycle, mirror graph of complete, m-nested triangle, firecracker, and Banana tree.

4. REFERENCES

1. D.Vargor, P.Dundar, The medium domination number of a graph, International Journal of Pure and Applied Mathematics, Vol.:70, 2011, pp.297-306.
2. M.Ramachandran, N. Parvathi, The medium domination number of Jahangir graph $J_{m, n}$, Indian Journal of Science Technology, Vol: 8(5), March 2015, pp.400-406.
3. F. Buckley, F. Harary, Distance in graphs, Addison Wesley Pub., California (1990).
4. DA. Mojdeh, AN. Ghamesholu, Domination in Jahangir graph $J_{2, m}$, International Journal of Math Sciences, Vol: 2(24), 2007, pp.1193-9.
5. G. Mahadevan, V. Vijayalakshmi, Akila, Some more results on the medium domination number of some special types of graphs- Proceedings of the National Conference on New Horizons in Computational Intelligence and Information Systems- December 2015
6. G. Mahadevan, V. Vijayalakshmi, A. Selvam Avadayappan, Akila, Exact values of the medium domination number of some specialized types of graphs- International Journal of Applied Engineering Research, Vol:11, No:1, 194-203.
7. G. Mahadevan, V. Vijayalakshmi, C. Sivagnanam, Investigation of the medium domination number of some special types of graphs, Aust. J. Basic & Appl. Sci., 9(35): 126-129, 2015.
8. G. Mahadevan, V. Vijayalakshmi, C. Sivagnanam, Medium domination number of $J(m, n)$, F_n and $S'(B_{k,k})$, Journal of Graph Labeling, Vol.2, 263-272, 2016.
9. G. Mahadevan, V. Vijayalakshmi, C. Sivagnanam, Extended Medium domination number of a graph, International Journal of Applied Engineering Research, Vol:10, No:92, 355-360, 2015.
10. G. Mahadevan, V. Vijayalakshmi, C. Sivagnanam, General result of Extended Medium domination number of a graph $B(n,n)$, $J(m,n)$ and $C_n Q K_{1,n}$, FACT -2016 - Proceedings of 4th National Conference on Frontiers in Applied Sciences and Computer Technology, National Institute of Technology, Trichy Vol.4, 269-273, 2016.
11. G. Mahadevan, V.G. Bhagavathi Ammal, MATLAB program to stimulate the graphs whose restrained triple connected domination number equals to two less than the number of vertices, International Journal of Applied Engineering Research, Research India Publication, Vol:9, No:24, 24505-24526, 2014.
12. G. Mahadevan, V.G. Bhagavathi Ammal, C. Sivagnanam, Investigation of the relationship between strong triple connected domination number with other graph theoretical parameter, International Journal of Applied Engineering Research, Research India Publication, Vol:10, No:69, 129-134, 2015.