IJCTA, 9(2) 2016, pp. 893-902

DOA by Antenna Array Plane Rotation Minimizes Antenna Elements and Phase Error

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ABSTRACT

Using smart antennas for transmission and reception in communication systems ensure efficient energy utilization. When used on the receive side, the receiver can focus only in the intended direction where the source is located. This can be achieved by extra capability to track the signal and interferer both using direction of arrival (DOA) estimation. This work investigates and compares DOA estimation for a smart antenna system and compares it with new approach of rotation of antenna array plane. It has been proved in the conclusion that, by using mechanical rotation of antenna array plane, DOA estimation can be done more accurately. Moreover, it also minimizes number of antenna elements required in an array for high resolution DOA estimation.

Keywords: DOA, MUSIC, ESPRIT, adaptive beam forming, smart antenna system

1. INTRODUCTION

Smart antenna systems are those which have additional capability of detecting the direction and location of the intended signal source. Such antenna system then is able to rotate its radiation pattern to the corresponding node's direction. This capability is gained by incorporating appropriate signal processing block into the system [1-3]. This type of antenna system helps to utilize transceiver power to be used more efficiently. Several null steering techniques are available [4-5] in which a null in the radiation pattern is rotated towards the interfering signal source. Accurate DOA estimation is achieved by processing all the signal impinged on an antenna array and determining their relative phase shifts and signal power values.

DOA estimation algorithms very broadly can be categorized in subspace based and quadratic algorithms [6]. Examples of quadratic algorithms are Bartlett and Capon [7]. Accuracy and angle resolution of these algorithms are function of array aperture and array factor [8]. Subspace based algorithms are based on Eigen value decomposition. MUSIC and ESPRIT are examples of this type [9]. MUSIC and ESPRIT algorithm is not limited to antenna array aperture and can be extended by novel approaches. However, MUSIC algorithm faces limitations in terms antenna array shape and size. It had been proved that MUSIC algorithm has limitations when used with uniform linear array (ULA). However, yields good results with uniform circular array (UCA).

Though individual antenna beam width is limited, a circular array can provide coverage of 360° in azimuth and elevation. Albeit the circular array is used, it poses certain restrictions on the antenna system. First, resolution of DOA is function of antenna array radius [10]; resolution and radius are directly proportional to each other. Second, higher resolution requires more number of antenna elements required. First and second restrictions make antenna array larger, which help in increasing the array aperture but such larger

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arrays can be impractical in some applications. Third important restriction is, theoretically, number of antenna elements in an array should be more than number of incident signals. This is basically aligned with second restriction. But when considered separately, in some situations it is very difficult to predict a number of possible incident signals. Rotation of antenna base-line can be one option to overcome these restrictions and obtain better DOA as proposed in [11].

In this paper, DOA estimation accuracy and resolution is compromised by keeping limited number of elements in antenna array and limiting the array radius. However, phase error is minimized by rotating the array axis with respect to array reference direction. Results of the proposed technique are compared with MUSIC and ESPRIT algorithms. Remaining paper is organized as follows. Section II gives system signal model and assumptions considered. Section III gives MUSIC and ESPRIT implementations for fixed uniform circular array. Cramer-Rao bound for the system is derived in Section IV. Section V provides proposed method of antenna array plane rotation and its implementation. Simulation results are compared here.

2. SYSTEM SIGNAL MODEL

Following assumptions are considered when the system signal model was derived:

1. All the elements in an antenna array are identical and equi-spaced in an array. All individual antennas have same gain.
2. Receiver receives all signal components during channel symbol period.
3. Signals emitted from the sources are narrowband and can be modeled by random processes.
4. All the signal sources and interferers are located in far field of antenna array.
5. Noise present in the medium is modeled by AWGN.

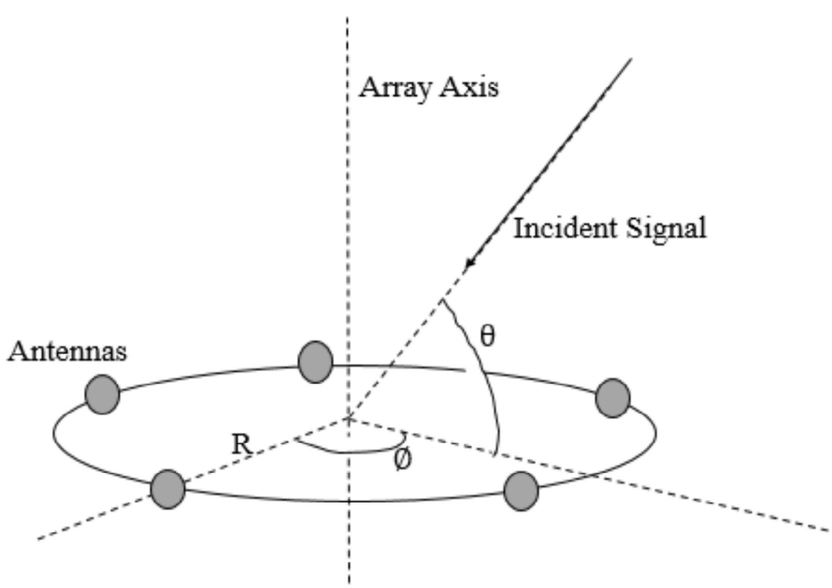

Figure 1: Antenna array diagram used for signal modelling of system

Also, complete system block diagram for DOA estimation is shown in figure 2.

Assume uniform circular array with N antenna elements is impinged by M signals at a given time. There are P number of sources transmitting signals at azimuth values ϕ_i and elevation values θ_i. Radius of antenna array is R and noise present can be modelled by AWGN. Assuming zero mean Gaussian noise array output is represented as,

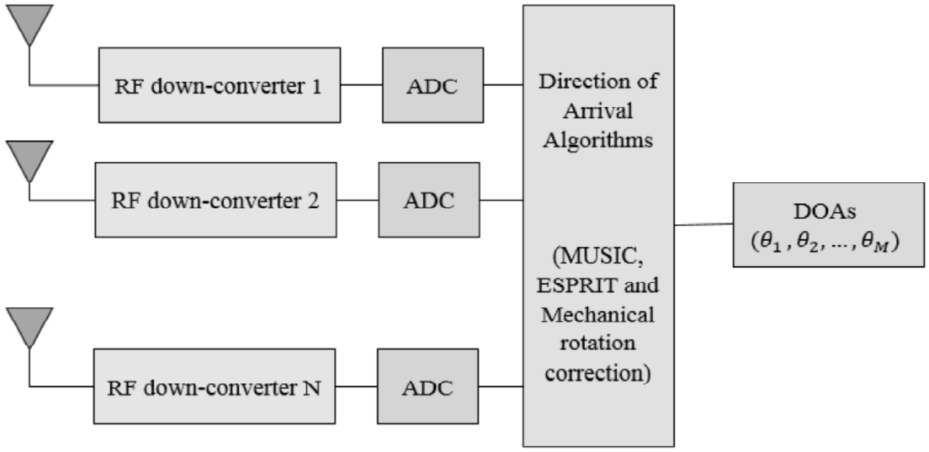

Figure 2: System block diagram considered

$$y(t) = \sum_{i=1}^{N} w_i x_i(t) \tag{1}$$

We have,

$$x(t) = a(\theta)\, s(t) + n(t) \tag{2}$$

Here terms can be given and defined as $x(t) = [x1(t), x2(t),\ldots, xN(t)]^T$ and $a(\theta) = [a(\theta_1),\ldots, a(\theta_N)]$ are received array data and array manifold matrix, respectively. Also, $s(t)$ is source waveform vector while $n(t)$ is noise vector. Array manifold vector relates to array beam steering vector as,

$$a(\theta_j) = [1, e^{\frac{j2\pi d}{\lambda}\sin\theta_i}, \ldots, e^{\frac{j2\pi(N-1)d}{\lambda}\sin\theta_i}]^T \tag{3}$$

In the above equation, d stands for antenna separation in the array and λ is wavelength of operation of array. Correlation matrix of order $N \times N$ can be obtained assuming that received signal as stationary random process,

$$R = E[(x(t) - m_x(t)).(x(t) - m_x(t))^H] \tag{4}$$

Here, $(\cdot)^H$ indicates conjugate matrix. While calculating these equations some important assumptions have been made, these are, first, signals and Gaussian noise are stationary processes and ergodic zero mean complex valued. Calculation of covariance matrix then done as,

$$\hat{R}_{xx} = \frac{1}{K}\sum_{i=1}^{K} XX^H \tag{5}$$

At this stage, we can apply DOA algorithms for accurate estimation.

3. MUSIC AND ESPRIT IMPLEMENTATIONS

3.1. MUSIC algorithm

MUSIC stands for Multiple Signal Classification. MUSIC algorithm basically transforms the problem of DOA into spectral estimation of signal [12]. MUSIC decomposes co-variance matrix into noise subspace and signal subspace. Major assumption done here is noise in sub-channels is uncorrelated. Covariance matrix for the source signals S can be written as,

$$S = E[s(t)\, s^H(t)] \tag{6}$$

MUSIC requires S to be non-singular matrix. However, this assumption exempts linear uniform array. From the given array structure, eigenvalues can be decomposed into matrix and we plot MUSIC pseudo-spectrum using,

$$P_{MUSIC} = \frac{1}{a(\theta)^H E_N E_N^H a(\theta)} \tag{7}$$

MUSIC algorithm is applied to uniform circular array with 8 elements for frequency components of 2.4 GHz and SNR variation between -25 to +35 dB. Snapshot values varied from 16 to 160. Accuracy of MUSIC for varying SNR is shown in Table 1. Table 2 gives DOA Estimation with variation of snapshots.

Table 1
Accuracy of MUSIC for SNR variation

SNR(dB)	*Estimated Frequency (MHz)*	*Actual DOA(°)*	*Estimated DOA (°)*
-25	2355	60	50.56
-20	2387	45	38.315
-15	2455	15	18.455
-10	2388	78	74.221
-5	2411	25	28.547
0	2391	35	32.524
5	2401	24	21.756
10	2430	88	84.525
15	2390	46	43.112
20	2405	72	68.624
25	2416	30	26.452
30	2441	42	40.214
35	2435	48	47.225

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Table 2
Accuracy of MUSIC for snapshot variation

Actual DOA(°)	*Number of Snapshots*			
	16	*80*	*128*	*160*
60	35.12	45.34	52.45	50.56
45	51.52	25.13	32.15	38.315
15	32.44	18.45	12.42	18.455
78	93.25	82.11	70.41	74.221
25	42.11	32.52	22.65	28.547
35	60.27	40.62	26.71	32.524
24	41.54	30.44	18.54	21.756
88	22.35	92.15	89.33	84.525
46	28.33	32.52	37.21	43.112
72	41.31	51.32	59.11	68.624
30	11.15	37.54	15.21	26.452
42	57.25	49.66	32.14	40.214
48	61.45	54.12	43.33	47.225

MUSIC algorithm is then applied to three dimensions for DOA estimation with the scenario of multiple signal sources and interferers. Results of the algorithm are shown in figure 3 where it was able to detect two signal sources accurately.

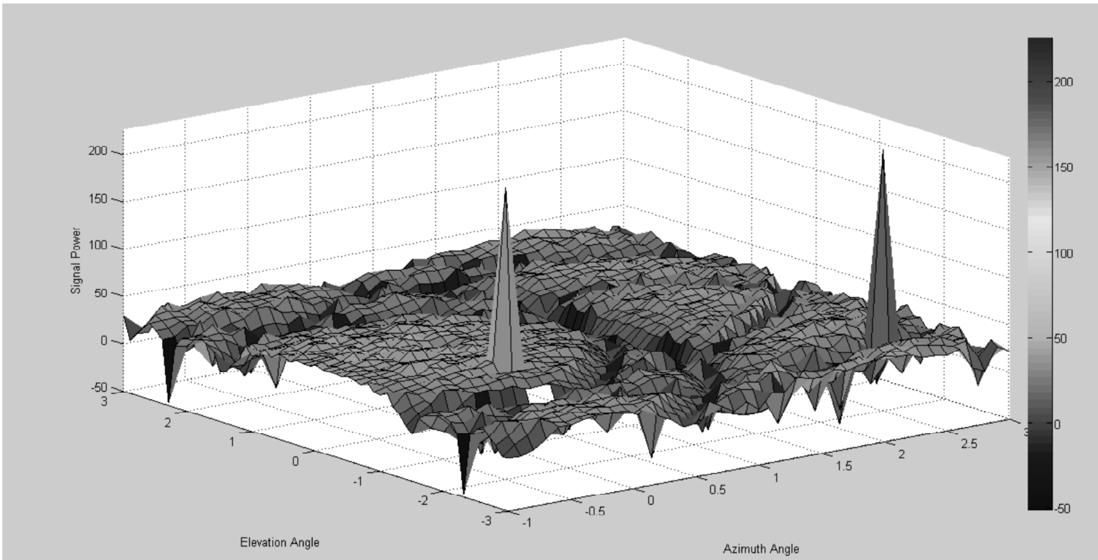

Figure 3: MUSIC algorithm results

3.2. ESPRIT algorithm

Advantage of using ESPRIT over MUSIC is that ESPRIT takes into account the antenna array geometry imperfections [13]. Rotational invariance characteristics of the signal subspace is the main feature used in ESPRIT. When array elements placed equidistant and form matched pairs with N array elements. Total number of doublets are, $n = N/2$. Signal subspace is computed for two sub-arrays, which in turn result into two vectors. Signal subspace is computed for two sub-arrays, which in turn result into two vectors. It also assumes existence of non-singular matrix which will help in eigenvalue decomposition. ESPRIT is based on the fact that between any two adjacent antennas in antenna array, phase shift is constant; whenever steering vector is taken into consideration. General steps applied in ESPRIT are [14]:

1. Estimate correlation matrix R and find it Eigen value decomposition.
2. Partition the matrix Q, so as to obtain matrix corresponding to N largest Eigen values which span the signal subspace.
3. Obtain estimate for order NxN.
4. Find Eigen values for NxN matrix. Diagonal elements of this matrix are the estimates for angles.
5. Calculate DOA using ESPRIT estimates.

Author has developed MATLAB code for ESPRIT algorithm, assuming same data as in MUSIC implementation. The result obtained for three dimensional simulation using ESPRIT are shown in Table 3 and Table 4. Figure 4 shows histograms of azimuth and elevation angles for four sources. It is clear that the algorithm can identify their azimuth and elevation angles distinctly.

Table 3 shows accuracy of ESPRIT algorithm for SNR variation; it is found that estimated angles of DOA are fairly accurate comparing MUSIC. When ESPRIT algorithm applied in three dimensions for multiple signal sources, it was able to distinguish four signal sources distinctly than the interfering sources.

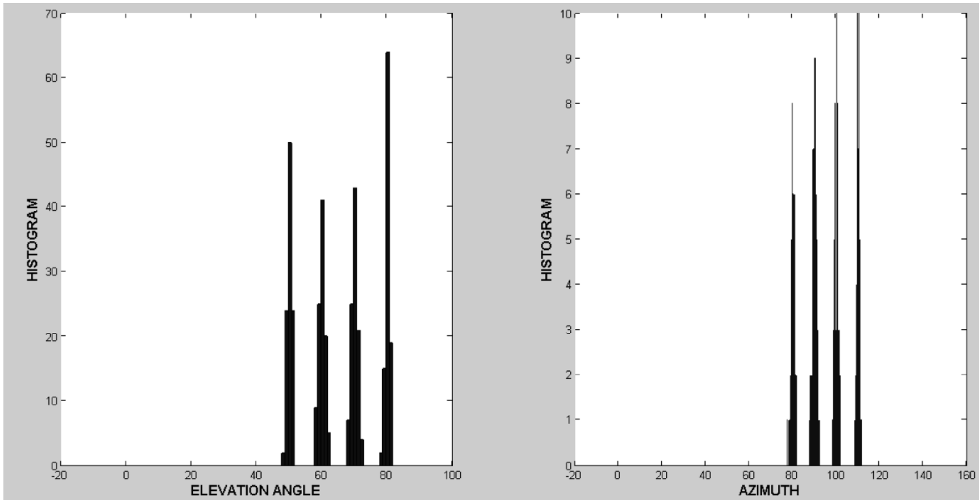

Figure 4: Histogram plots obtained from ESPRIT algorithm

Detailed results for the ESPRIT implementation are shown in Table 3.

Table 3
Accuracy of ESPRIT for SNR variation

SNR(dB)	*Estimated Frequency (MHz)*	*Actual DOA(°)*	*Estimated DOA (°)*
-25	2310	60	53.23
-20	2320	45	40.15
-15	2378	15	18.01
-10	2452	78	72.21
-5	2390	25	26.51
0	2391	35	31.74
5	2367	24	20.16
10	2442	88	86.52
15	2458	46	44.31
20	2461	72	69.56
25	2421	30	28.23
30	2410	42	44.52
35	2392	48	47.01

4. CRAMER-RAO BOUND

Cramer Rao bound gives lower bound for unbiased parameter estimation. Consider angle parameter vector **v** [15-16],

$$v = [\phi^T, \theta^T]^T \tag{8}$$

Where sub-matrices are given as,

$$\theta = [\theta_1, \theta_2, ..., \theta_M] \tag{9}$$

$$\phi = [\phi_1, \theta_2, ..., \phi_M] \tag{10}$$

Cramer-Radio bound for the angle parameters is given as,

$$\text{var (v)} \geq \text{CRB} \tag{11}$$

$$\text{var}(v) \geq E\{(\hat{v} - v)(v - \hat{v})^T\} \tag{12}$$

Fisher information matrix (FIS) for parameter v is given by,

$$F = \begin{bmatrix} F_{\theta Ø} & F_{Ø\theta} \\ F_{\theta Ø} & F_{\theta\theta} \end{bmatrix} \quad (13)$$

5. PROPOSED METHOD

5.1 Antenna array plane rotation

Mechanical rotation of antenna array method is proposed in [11] [17-19] which discusses DOA estimation by rotating the antenna array plane for azimuth direction. At any antenna element from the array, available signal is given by,

$$x_n = Ae^{j\frac{\omega n \cos\varphi}{\lambda}} \quad (14)$$

Equation (14) gives any time instance, A being the amplitude of signal. φ in the above equation gives DOA. Now, we want to mechanically rotate the antenna array plane, then φ becomes (φ + δφ). Equation (14) then modified in terms of (φ + δφ).

$$x_n' = Ae^{j\frac{\omega n \cos(\varphi+\delta\varphi)}{\lambda}} \quad (15)$$

Taking the frequency ratios from equations (14) and (15), we can obtain the constant value K, which is known from spectral analysis of the signals. Rearranging the equation in terms of φ.

$$\varphi = \tan^{-1}\left[\frac{\cos(\delta\varphi) - K}{\sin(\delta\varphi)}\right] \quad (16)$$

5.2. Proposed Antenna Rotator System

Proposed system consists of two major components: phase shifter lookup table and stepper motor controller. Digital phase shifter is 6 bit phase shifter and gives resolution of 5.625°. Command signal decides the phase shift for the antenna rotation angle by comparing the Phase Shifter Look-Up Table (LUT) and DOA feedback. DOA feedback is angle estimated by DOA algorithm with maximum possible resolution. The result of comparison is the angle by which stepper motor rotates the axis of the antenna array to compensate the DOA error and point exactly towards the intended signal source. DOA error between desired angle and

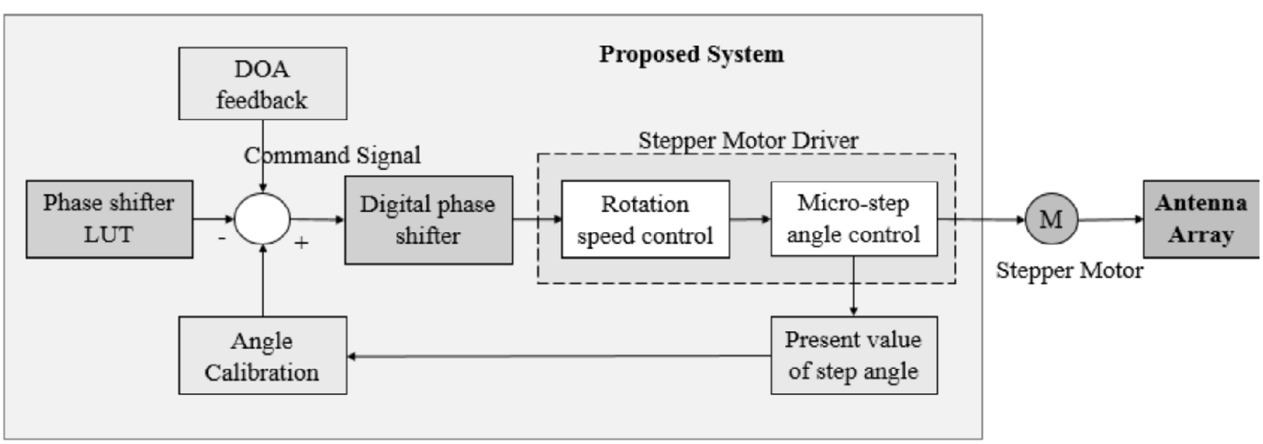

Figure 5: Proposed system for antenna array rotation

available angle from phase shifter without stepper motor rotation reflects in phase and gain error [20-22]. This is illustrated in Table 4 for DOA of 0° to 90°; but the table can be extrapolated till complete 360° DOA. Available azimuth of 360° is divided into 64 sectors and antenna HPBW is assumed to be 25°, 5.625° of resolution is available as mentioned earlier.

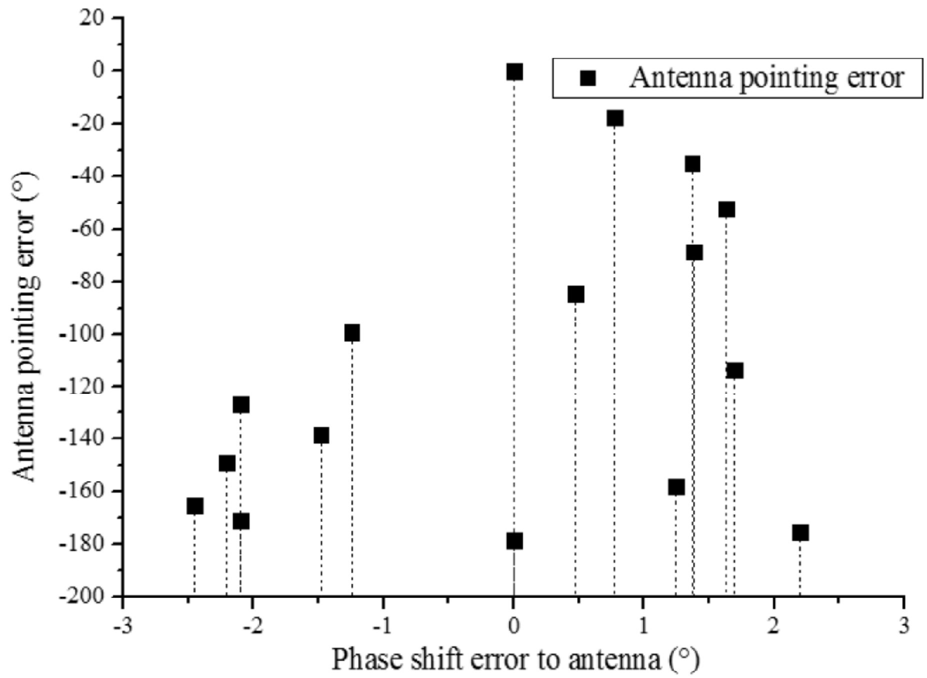

Figure 6: Antenna pointing error due to phase shift error

Table 4
DOA error calculation

Sr. No.	*Desired beam direction (°)*	*Required phase shift to antenna (°)*	*Phase shifter value (°)*	*Phase shift error to antenna (°)*	*Corresponding gain error (dB)*	*Antenna pointing error (°)*
1	0	0	0	0	1	0
2	5.625	17.6430	16.875	0.768	0.705	-17.5491
3	11.25	35.1162	33.75	1.3662	0.0666	-34.9276
4	16.875	52.2512	50.625	1.6262	-0.3223	-51.9661
5	22.5	68.883	67.5	1.383	0.0436	-68.4986
6	28.125	84.8514	84.375	0.4764	0.8865	-84.3635
7	33.75	100.0026	101.25	-1.2473	0.222	-99.4052
8	39.375	114.1907	112.5	1.6907	-0.4293	-113.5156
9	45	127.2792	129.375	-2.0957	-1.1961	-126.4772
10	50.625	139.1418	140.625	-1.4831	-0.0998	-138.2452
11	56.25	149.6645	151.875	-2.2104	-1.443	-148.6602
12	61.875	158.7458	157.5	1.2458	0.2239	-157.6803
13	67.5	166.2983	168.75	-2.4516	-2.0053	-165.0829
14	73.125	172.2492	174.345	-2.0957	-1.196	-170.9297
15	78.75	176.5413	174.345	2.1963	-1.4119	-175.2218
16	84.375	179.1332	180	-0.8667	0.6243	.-177.5625
17	90	180	180	0	1	-178.4292

This algorithm considers four place resolution after decimal point. The calculated error is fed into phase shifter. Phase shifter produces angle value through which antenna array to be rotated and fed to the stepper motor driver. Stepper motor driver is operates motor in micro-stepping mode with resolution of 0.45°. All the description here compensates the azimuth angle error providing single degree of freedom to the antenna array. Another stepper motor is used and command signal is fed to this motor for compensating the error in elevation angle. Set of two stepper motors (for elevation and azimuth) provide two degrees of freedom and system is able to minimize the error in azimuth as well as elevation.

As mentioned during introduction part, better resolution in DOA can be obtained increasing number antenna elements in an antenna array. However, with the proposed method, better resolution can be obtained keeping number of antenna elements same.

During start of the system and DOA estimation, residual phase in the phase shifter and in stepper motor driver need to be restored to default values (pointing the antenna array in 0° direction). A calibration need to be performed to nullify these residual values.

4. CONCLUSION

DOA estimation methods MUSIC, ESPRIT and mechanical rotation of antenna array are compared. By comparison it was concluded that mechanical rotation of antenna array elements gives combines features of MUSIC and ESPRIT. It provides accuracy and robustness. With the proposed method, better resolution can be obtained keeping number of antenna elements same.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their helpful comments and suggestions to improve the quality of the manuscript.

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