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The rate of surplus value and the structure of the tax system

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Abstract: The objective of this work is twofold. First, it aims to establish procedures for approximating the rate of surplus value using disaggregated data published by statistical agencies. Second, and perhaps more importantly, it highlights that the dynamics of the rate of surplus value should be regarded as a preferred indicator for assessing the effects of public policies, particularly taxation. This is because not all taxes impact the rate equally. To illustrate the potential of this analytical approach, we employ recent data from Spain, organized in a Social Accounting Matrix (SAM).

Keywords: The rate of surplus value, Social Accounting Matrix, Taxation and consumption, Policy indicators.

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INTRODUCTION

The surplus value (or exploitation) rate is a central concept in Marxian economics. In a quantitative sense, it measures the extent of the implicit unequal exchange for labour that occurs in the production of commodities (Roemer, 1985). In a theoretical and more profound sense, it attempts to provide a platform for a general critique of the capitalist organization of society. Whether or not this broad goal is achieved has been the object of debate.

Cohen (1979), for instance, claims that since exploitation is based on the labour theory of value, and this theory turns out to be unsound, the concept of exploitation becomes irrelevant. Cohen's position is that labour itself does not produce value; instead, labour produces objects that have

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value. Thus, it is the appropriation of part of the value in the object that gives rise to exploitation. Value, in Cohen's view, is market value, not labour value, and this displaces appropriation to market mechanisms rather than to any inherent unequal exchange of labour of the type indicated by Roemer (1985).

We agree with Cohen (and many other authors) that the labour theory of value does not provide a basis for the determination of market values, or prices, but we disagree regarding the alleged lack of usefulness of the determination of labour values. Firstly, labour values can, in fact, be quantitatively determined, and they provide an adequate accounting for total congealed labour (Morishima, 1973, ch. 1; Miller and Blair, 2009, ch. 6; Vegara, 1979, ch. 3). Secondly, labour value accounting proves useful in elucidating properties of the dynamics of capitalism, such as the trend of the rate of profit (Vegara, 1977) and the general concept of exploitation beyond labour proper, which argues that labour is not especial after all since all basic commodities can be seen to be equally exploited (Vegara, 1979, ch. 3; Roemer, 1982, ch. 6).

Roemer (1985) also concludes that exploitation theory does not provide a sufficient basis for a sensible critique of the evils of capitalism. From a purely theoretical economics perspective, as already mentioned, the exploitation of labour is not unique. Similarly, Roemer argues that the inequality of access to the ownership of the means of production is not accurately measured by the rate of exploitation. Finally, outside of economics proper, Roemer asserts that domination and alienation have little to do with the actual exploitation mechanism, even if they are key societal issues.

Be that as it may, the numerical value of the rate of surplus value at a given moment yields information about a piece of the state of socioeconomic affairs worth knowing, if only from the descriptive perspective provided by a numerical estimate. Surprisingly, there have been fewer attempts than expected to empirically evaluate the rate of surplus value. We report on some of them. Wolff (1975) used input-output data for Puerto Rico and evaluated the rate of surplus value at 0.97 (for 1948) and 0.92 (for 1963). His methodological approach is based on using input-output data in current prices, transforming them into value (i.e., labour value) using a proportionality scheme, and then using the transformed data to calculate the rate of surplus value. In a later paper with broader objectives, Wolff (1979) once again uses input-output data to estimate the rate of surplus value for the US economy in four years (1947, 1958, 1963, 1967). He finds values of the rate slightly above 1 in all four cases. The same author updates and expands his data in Wolff (1986), also for the US economy, but now for six years, including the previous four plus 1972 and 1976. The average rate of surplus value turns out to be 0.99, with the lowest value of 0.75 (in 1976) and the highest one of 1.08 (in 1963).

Moseley (1988) revises Wolff's results, arguing the necessity to distinguish between productive and unproductive capital. Moseley's numerical results are clearly above Wolff's. His average rate of surplus value for the period 1947 1976 is 1.58, with a minimum value of 1.35 in 1948 and a maximum of 1.73 in 1965. Unlike Wolff, Moseley's calculations are not based on input-output data. Instead, he uses time series records from the US National Income and Product Accounts data. However, in doing so, he omits the necessary transformation of market value data into labour value data to adequately account for the correct definition of the rate of surplus value in terms of labour values. Therefore, his indicators do not exactly measure the rate of surplus value as initially conceptualised by Marx.

More recently, Qi (2018) reports the rate of surplus value for the Chinese economy using aggregate monetary magnitudes extracted from this country's National Accounts. If we omit the estimated rates for the period 1956 77 and focus on the calculated rates from 1978 to 2015, the reported values oscillate around 2. Once again, Qi uses market value data instead of labour value data to approximate the true value of the rate of surplus value. Freitas (2021) also uses National Accounts data to calculate annual time series figures for the rate of surplus value for Brazil between 1996 and 2016. Similar to the approach of Qi, the used data is "money" data, as the author explains, or current market value data in the standard terminology. Calculated values oscillate close to and around 2.5. In both studies, the transformation problem between market and labour values is overstepped, and the calculated indicators—as with Moseley's values—do not correspond with the canonical definition of the rate of surplus value as first proposed by Marx. This discrepancy does not mean that this type of calculation is wrong or uninteresting. On the contrary, they are neither wrong nor uninteresting but, simply stated, they measure something related to but different than the rate of surplus value, whose definition is based on labour values.

Notice that calculations based on disaggregated input-output data (Wolff, 1975, 1979, 1985) seem to yield numerical values smaller than calculations based on aggregate market value data (Moseley, 1988; Qi, 2018; Freitas, 2021), which may be an empirically based clue that data aggregation and the transformation problem matters. Regarding this issue, Rieu (2008, 2009) discusses the so-called 'new interpretation' of the theory of value as posited by, among others, Foley (1982), as a justification for using market value data instead of labour value data for the calculation of the rate of surplus value. If prices and labour values happen to be proportional, then indeed there is no difference in using either of them for the calculations, and the transformation problem would not be an issue. This proportionality, however, cannot happen in general. Prices systematically deviate from labour values, and they do so in a non-proportional way, as Morishima (1973, ch. 6), Vegara (1979, ch. 3) and later Sotirchos and Stamatis (1999) demonstrate. Hence, the translation of market value magnitudes to labour values, assuming, for instance, that prices are proportional to values, is not in general correct.

Proportionality would occur only in two highly restrictive scenarios. The first scenario involves an economy in which there were no positive profits in any industry, an uncommon situation in the actual context of a modern capitalist economy. The second, more conceptually significant case, would require that all sectors in the economy have identical organic compositions of capital (Morishima, 1973, ch. 7). In these two cases, prices and values would be strictly proportional. However, such scenarios are highly unlikely to be observed empirically.

Thus, we are back to the two distinct and separate accounting systems of market values and labour values, and the framework for calculating the rate of surplus value, as originally defined by Marx, happens to be rooted in the second of these accounting systems. Results obtained from the first accounting system based on aggregate magnitudes at market prices are also interesting and undoubtedly valuable to know as descriptive of the underlying economic situation. However, these calculations do not correspond to the rate of surplus value as it was initially conceived.

Whatever the virtues (or flaws) of the aforementioned calculations regarding the rate of surplus value, they only capture a descriptive snapshot of the prevailing socioeconomic status quo as reflected in the compiled statistical data. This is adequate but somewhat limited in reach. The current *status quo* is just one that ended up materializing among the many possible alternative states. For example, imagine that the government, in using its political attributions, modifies its tax policies. In response, the economy would adjust, and a new state would come into being. Since there is a wide range of possible alternative tax policies, a new state could be attained for each one of them. Each of these hypothetical states would give rise to an alternative database similar in structure to the data collected in the known *status quo*. For each of these alternative states of the economy, the same technique used to calculate the rate of surplus value for the initial database state is equally useful for recalculating it in any other state. Therefore, we can compare and catalog the states according to the evolution of the rate, thereby elucidating the role exerted by fiscal policy in terms of impact (magnitude) and desirability (direction).

In terms of data availability, the most comprehensive accounting tool that reflects the prevalent flows of an economic state is the Social Accounting Matrix, or SAM for short. SAMs integrate input-output and national accounts data in a balanced way. They were first developed by Stone (1962) and have become the database of choice for multisectoral modelling (Pyatt and Round, 1985). In turn, Olsen (2011) discusses the accounting modifications and reclassifications that, if implemented, would yield a SAM with a Marxian flavour. Ideally, in Olsen's framework we should be able to distinguish between salaried workers, self-employed individuals, and non-workers (such as capitalists and rentiers). Workers should further be classified by skill level and disposable income. Additionally, in the distribution of gross capital income, pure profits should be differentiated from implicit payments for the use of services. Unfortunately, these classifications have not attracted interest from the public administrations responsible for data processing. The final result is that we must constrain ourselves to use socioeconomic data as they are prepared and published, while highlighting their limitations.

In using the SAM structure, we can distinguish between the available compiled SAM and the counterfactual ones that could be assembled under a different policy scenario. As a matter of fact, it is possible to show that for any parameter scenario, there will be a corresponding counterfactual SAM with the same structure and balanced properties of the initially given empirical SAM (Lima et al., 2017). Needless to say, in each of these counterfactual SAMs, we would have all the data needed to re-calculate the rate of surplus value. The same calculation technique used for the *status quo* data can also be used in any counterfactual scenario.

This approach opens a significant and novel avenue that allows us to glimpse the consequences of government decisions on tax policies and do so from a non-neoclassical perspective. The availability of empirical data, even if not under Olsen's ideal conditions, combined with the modeling of structural relationships between key variables and the tax structure, facilitates estimating how potential fiscal policies might affect the rate of surplus value. This, in turn, enables us to move beyond purely descriptive outcomes as it provides an effective means to estimate potential impacts to policy changes.

The rest of the text is organized as next: In the next Section, we outline a simple but novel accounting procedure to estimate, at first approximation, the role played by tax instruments in determining the rate of surplus value. For this purpose, the availability of a SAM is essential, as the type of statistical information on tax flows needed is not available in the input-output tables. The practical use of an empirical SAM is therefore an innovation within the field of heterodox economics and offers a promising path in terms of its potential. In the following Section we report some of the simulation results obtained and we are able to verify the economic logic connecting tax rates and the rate of surplus value. Finally, in the last Section, we conclude by discussing possibilities and limitations of the approach.

METHODOLOGY

Economic data representing the circular flow of income in a certain period is most thoroughly described in a Social Accounting Matrix. In a SAM, we have data on transactions between production sectors (input-output flows), data connecting income generation to expenditure (from value-added to final demand), data on fiscal instruments (taxes and public spending), and data on trade (imports and exports). All this data is organized in a square matrix of dimensions $N \times N$, with N being the number of distinct accounts disaggregated in the data base. It includes—in the standard denomination used in the National Income and Product Accounts—Production sectors, Households, Primary factors (Value-added), Savings and Investment, Government activities, and the Foreign sector. A SAM always satisfies the property that the sum of all entries in row j coincides with the sum of all entries in column j. In budget terms, for each category j in the accounts listed in N, total expenditure outlays equal total income.

With this in mind, we begin with the aggregate equality for Households between income generation and income disposition, typical of the National Accounts. From the income perspective, in a modern economy, Households receive most of their income from two main sources: labour income primarily, and other sources of income, which include a variety of flows such as rents, interests, distributed pure profits, capital services retribution, etc. For simplicity, we will refer to this aggregate as “capital” income. Finally, Households also receive transfers from the Government, such as unemployment payments, pensions, etc.

Turning to the expenditure side, Households use their income to finance their current consumption, pay their aggregate tax bill, and save a portion of their net income to finance future consumption. The tax bill includes direct taxes on taxable income, indirect taxes on consumption and personal labour taxes.

We represent this macroeconomic income/expenditure accounting identity thus:

$$w \cdot L + r \cdot K + T \equiv C + S + I + V + P \tag{1}$$

where we use the following notation:

w:	wage rate
r:	capital compensation
L:	labour used in the period
K:	capital used in the period
T:	government transfers
C:	consumption
S:	savings
I:	direct income tax collections
V:	indirect consumption tax collections
P:	personal labour tax collections

For instance, from the economic data in the SAM of Spain for 2015 we find (in millions of euros):

$$w \cdot L + r \cdot K + T = 410.583 + 453.464 + 170.583 = 1034.630$$

$$C + S + I + V + P = 538.086 + 270.387 + 117.488 + 67.657 + 41.012 = 1034.630$$

In expression (1), the key variable for calculating the rate of surplus value is consumption, denoted as *C*. From a Marxian perspective, *C* represents the necessary consumption for the reproduction of the labour capacity that fuels production within the current economic status quo. However, since the accounting identity (1) must always hold, changes in fiscal variables will necessarily have a direct impact on the value of *C*, the feasible consumption level. If *C* changes in response to possible changes in fiscal policy variables, the rate of surplus value will accordingly adjust to reflect the new policies, but always within the constraints imposed by the structure of the accounting identity (1). Let us attempt to unveil how this occurs.

Total savings *S* is a proportion of the net income level once the fiscal bill is fully paid:

$$\alpha = \frac{S}{w \cdot L + r \cdot K - I - V - P} \tag{2}$$

Notice that we exclude government transfers T since they are either in-kind transfers or correspond to basic income transfers out of which little or no savings are possible. They are essentially devoted to finance consumption. From expression (2) we write total savings as:

$$S = \alpha \cdot (w \cdot L + r \cdot K - I - V - P) \tag{3}$$

The personal labour tax rate is calculated as:

$$t_P = \frac{P}{w \cdot L} \tag{4}$$

with total tax payments P as a function of the tax rate and taxable labour income:

$$P = t_P \cdot \left[w \cdot L \right] \tag{5}$$

The income tax payments, in turn, are levied at a tax rate t_I defined by:

$$t_I = \frac{I}{w \cdot (1 - t_P) \cdot L + r \cdot K} \tag{6}$$

Observe that the income tax rate defined in expression (6) does not include the payments for the personal labour tax, as they are exempt in the (Spanish) legislation to avoid double taxation. Hence, personal labor tax payments are netted out from the income tax base. Total income tax payments can be calculated as follows:

$$I = t_I \cdot \left[w \cdot (1 - t_P) \cdot L + r \cdot K \right] \tag{7}$$

Finally, the indirect tax rate on consumption is calculated from:

$$t_C = \frac{V}{C} \tag{8}$$

with total indirect tax receipts being:

$$V = t_C \cdot C \tag{9}$$

By substituting (3), (5), (7), and (9) into expression (1) we can eliminate S, I, V, and P and obtain, with some algebra, an equivalent but simpler accounting expression. The reduced expression contains the level of affordable aggregate consumption along with labour and capital incomes and government transfers, which we consider fully exogenous as a policy variable:

$$\begin{aligned} & w \cdot L + r \cdot K + T = \\ & = C \cdot \left(1 + t_V \cdot (1-\alpha)\right) + \\ & + \left(\alpha \cdot (1-t_P) \cdot (1-t_I) + t_I + t_P \cdot (1-t_I)\right) \cdot w \cdot L + \\ & + \left(\alpha \cdot (1-t_I) + t_I\right) \cdot r \cdot K \end{aligned} \tag{10}$$

Rearranging terms:

$$\begin{aligned} & C \cdot \left(1 + t_V \cdot (1-\alpha)\right) = \\ & = \left[1 - \left(\alpha \cdot (1-t_P) \cdot (1-t_I) + t_I + t_P \cdot (1-t_I)\right)\right] \cdot w \cdot L + \\ & + \left[1 - \left(\alpha \cdot (1-t_I) + t_I\right)\right] \cdot r \cdot K + T \end{aligned} \tag{11}$$

Alternatively:

$$C = w_u \cdot L + r_u \cdot K + T \cdot \left(1 + t_V \cdot (1-\alpha)\right)^{-1} \tag{12}$$

In expression (12), the variables w_u and r_u capture what we term the "usable" levels of labour and capital retributions that eventually allow for the financing of consumption expenditures C:

$$\begin{aligned} & w_u = w \cdot \left[1 - \left(\alpha \cdot (1-t_P) \cdot (1-t_I) + t_I + t_P \cdot (1-t_I)\right)\right] \cdot \left(1 + t_V \cdot (1-\alpha)\right)^{-1} \\ & r_u = r \cdot \left[1 - \left(\alpha \cdot (1-t_I) + t_I\right)\right] \cdot \left(1 + t_V \cdot (1-\alpha)\right)^{-1} \end{aligned} \tag{13}$$

Observe that by construction, $w_u < w$ and $r_u < r$, and thus we can envision the detracting role on expendable income by origin that the taxation system plays. Note also that due to the presence of a positive personal labour tax t_P, the usable level of labour income is necessarily lower than that of capital, $w_u < r_u$.

In the accounting expressions (12) and (13), we have made explicit the mechanisms through which fiscal parameters intervene in the determination of affordable consumption, C. Any change in any of these fiscal parameters will therefore influence the usable levels of income and, as a result, the affordable consumption level C available to Households will scale up or down.

The surplus value, or exploitation rate, e is defined[1] as the ratio of surplus labour SL over necessary labour NL:

$$e = \frac{SL}{NL} \tag{14}$$

Surplus labour is the difference between supplied labour and the "value" of labour that fuels its supply. In Marxian economics, value is always defined as labour value, which is the total labour incorporated in the delivery of one unit of a good for final uses. Therefore, in Marxian terms both *SL* and *NL* are defined and measured in terms of labour value.

Consider now an input-output economy (Leontief, 1986; Miller and Blair, 2009) with *n* production sectors and goods. Let *A* be the $(n \times n)$ input-output technology matrix and l' be the $(1 \times n)$ row vector of direct technical labour coefficients for this economy. The coefficients in *A* and l' are derived using the standard normalization method in input-output analysis. This process involves taking the initial data in current prices from the database and redefining units such that each new unit has a value equivalent to one currency unit. This is achieved by treating the currency values as if they represented physical quantities. As a result, the data are converted into index numbers, expressed in these redefined units, which can function as physical indices and remain unaffected by price changes. These new units do not need to have any official metric classification. They just exist. For further details and examples of this transformation, see Miller and Blair (2009, ch. 2) and Cardenete et al. (2017, ch. 6).

The labour values $(1 \times n)$ row vector λ' incorporates all the direct and indirect labour necessary to produce and deliver units of final demand (Morishima, 1973, ch. 1; Vegara, 1979, ch. 3; Miller & Blair 2009, ch. 6):

$$\lambda' = \ell' + \lambda' \cdot A \tag{15}$$

Provided the technology matrix *A* is productive[2], we can non-negatively solve for vector *l'* and obtain:

$$\lambda' = \ell' \cdot (I - A)^{-1} \tag{16}$$

Equation (16) explains how labour values can be calculated within the framework of an economy with an input-output production structure. However, as argued by Morishima (1973), Vegara (1979), and Sotirchos and Stamatis (1999) it is not the basis for a theory of price formation for commodities since it does not incorporate market variables and only reflects the technical side of the economy. In what follows, to be specific, when we refer to "value" we implicitly mean labour value as accounted in (16).

Total consumption *C* is the aggregation of the consumptions of the *n* goods. Let *B* be the $(n \times 1)$ column vector of sectoral consumptions as reported in the data in the baseline SAM. Then, for a fixed set of prices[3], and from the data, the following aggregation is observed:

$$C = \sum_{j=1}^{n} B_j \tag{17}$$

Define the (n×1) column vector *b* of consumptions per unit of labour as:

$$\beta = (\beta_j) = (B_j / L) \tag{18}$$

The vector product $\lambda' \cdot \beta$ measures the labour value of the consumptions that yield one unit of labour, or necessary labour *NL*, whereas $1 - \lambda' \cdot \beta$ measures surplus labour *SL* over necessary labour. Therefore, the rate of surplus value *e* per unit of delivered labour is given by:

$$e = \frac{SL}{NL} = \frac{1 - \lambda' \cdot \beta}{\lambda' \cdot \beta} \tag{19}$$

All the required information to calculate the benchmark *e* is therefore contained in the SAM database. The available technology (A, ℓ') is all that is required to obtain the labour values λ', whereas the current socioeconomic conditions define affordable consumption expenditures *C*. Because of equations (17) and (18) we can derive the associated base unitary consumptions β.

The level of aggregate consumption *C* depends, on the other hand, on the tax rates, as expressed in equations (12) and (13). Policy shifts in tax rates will therefore affect *C*, causing it to increase or decrease. This in turn will scale up or down the vector β, consequently affecting the rate of surplus value *e*. Therefore, the calculation of the counterfactuals requires some additional modelling assumption regarding consumption behaviour. Let us assume that consumption behaviour follows a Leontief consumption function with strict complementarity, defined by positive shares α_j:

$$C = \underset{j=1,2,\ldots,n}{Min} \left\{ \frac{B_j}{\alpha_j} \right\} \tag{20}$$

We can now appreciate the transmission mechanism at work. Any tax policy induced change over aggregate *C* will give rise to scaling changes in sectoral consumption levels B_j through the behavioural rule in equation (20). The same scaling will translate through (18) into the vector components in *b* and finally will affect the rate of surplus value calculated in equation (19).

RESULTS

We illustrate using data from a Social Accounting Matrix of Spain for 2015. This SAM was constructed using the input-output framework, national accounts identities, and tax summaries published by the "Instituto Nacional de Estadística" of Spain. It includes the input-output data as a subset, incorporates all tax categories and their distribution among SAM accounts, and satisfies all well-known national accounts identities. The SAM for Spain contemplates a disaggregation of 30 production sectors, 2 primary factors, 6 tax categories, and 5 final demand items. Descriptive details of the accounts in this SAM appear in the Annex. It is a standard SAM that follows the European national products and income accounts methodology. The data in the SAM is "standard" in the narrow sense of the typology of statistical information that is usually presented in National Income and Product Accounts.

In Table 1, we enumerate some relevant benchmark values extracted from the 2015 SAM data for Spain. Notice first that more than half of the unitary labour and capital retributions are detracted because of the action of the tax system affecting earned incomes. This tax-induced reduction in disposable income acts to contract the affordable consumption level C and, in turn, impacts the rate of surplus value e. The estimated the rate of surplus value for the 2015 data is 1.186, slightly above 1. Observe that this rate is quite close to the rates estimated by Wolff (1975, 1979, 1986) using input-output data as well.

We now consider alternative tax rates and reconstruct the counterfactual dataset. Each of the three major tax categories that affect disposable income sees a 10 percent reduction in its tax rate. In Table 2, we report the subsequent implications of these three simulations resulting from the adjustment in the tax rates.

The first result is the positive relationship between the reduction in tax rates and the reduction in the exploitation rate. As more income is liberated from the budget constraint (12), the level of affordable consumption would increase (as shown in the first row of Table 2), and so would the consumption coefficients $\hat{a}$ per unit of labour, all of which would scale up. Consequently, the value of socially necessary labour would increase, whereas surplus labour per hour would fall. This would lead to a decrease in the rate of surplus values in all scenarios compared to the initial benchmark rate.

Table 1: Benchmark data for 2015 (Millions of euros when applicable)

Consumption	538.086
Income tax receipts	117.488
Consumption tax receipts	67.657
Personal labour tax receipts	41.012
Aggregate income tax rate	0.143
Aggregate consumption tax	0.126
Aggregate personal labour tax rate	0.099
Usable wage rate (in % of *w*)	0.415
Usable capital retribution (in % of *r*)	0.461
Surplus labour (per unit of labour time)	0.547
Necessary labour (per unit of labour time)	0.457
The rate of surplus value	1.186

Source: SAM 2015 of Spain and our model calculations

Since the 10 percent reduction in tax rates is, admittedly, somewhat arbitrary, we standardize the induced changes using an elasticity indicator (last row of Table 2). This enables us to observe how a 1 percent change in a tax rate affecting disposable income through equation (12) quantitatively impacts the system in various ways. The elasticity weight for the income tax rate is approximately twice the elasticity value for the consumption tax and almost three times that of the personal labour tax. Accordingly, the greater the elasticity, the larger the reduction in the rate of surplus value from the benchmark value. The conclusion is that similar percentage changes in tax rates would impact the rate of surplus value differently.

We also observe that, in general, as expression (13) captures, usable wage and usable capital retribution would increase as we lower the tax rates. The increased availability of usable income after a tax reduction provides us with another perspective on how consumption expenditures are financed, considering the origin of income and the tax structure.

Table 2: 10% reduction in tax rates (Millions of euros when applicable)

	benchmark	income tax	consumption tax	personal labour tax
Consumption	538.086	544.398	541.746	539.975
Income tax receipts	117.488	105.739	117.488	118.074
Consumption tax receipts	67.657	67.657	61.305	67.894
Personal labour tax receipts	41.012	41.012	41.012	36.911
Usable wage rate (in % of *w*)	0.415	0.421	0.417	0.419
Usable capital retribution (in % of *r*)	0.461	0.468	0.463	0.461
Surplus labour (per unit of labour time)	0.547	0.537	0.539	0.541
Necessary labour (per unit of labour time)	0.457	0.462	0.460	0.459
The rate of surplus value *e*	1.186	1.161	1.172	1.179
Elasticity of *e* to tax rate changes	—-	0.214	0.125	0.065

Source: SAM 2015 of Spain and our model calculations

CONCLUDING REMARKS

Regardless of whether the rate of surplus value occupies a central position in Marxian economics, it is evident that, at a minimum, an approximation of the rate can be calculated using data regularly published by statistical agencies. Even if this data does not align with the tenets of the theory—a limitation of the analysis—the mathematical methodology is well-defined and ready for use should adequate data become available. The calculation can be performed using the conventional definition of the rate, which relies on labour values, without the need to resort to calculations based on market value data. As we have observed, numerical estimates based on market values will differ from those based on labour values. In other words, the transformation problem between prices and labour values does indeed matter.

There is, nonetheless, an operational advantage that justifies the use of calculations based on aggregate market values. National accounting data are published annually and relatively quickly. In contrast, the elaboration of the data appearing in the input-output framework requires a substantially longer assembly period. These delays limit the usability of the input-output data. It is also common that annual series of input-output data are not available. These difficulties complicate the compilation of Social Accounting Matrices and make the use of annual market value data more attractive, even when the calculations do not conform to the conventional definition of the rate of surplus value. Related estimates, in our opinion, are better than no estimates.

Another conclusion worth noting pertains to the ability of economic modelling to estimate rates of surplus value in scenarios different from the baseline scenario reflected in the data. In models built upon the data integrated into a SAM, it becomes straightforward to estimate the rate of surplus value in response to changes in the economic environment. Using, in our case, a simple accounting procedure, we have seen some of the possible implications of broad changes in fiscal policy. It is worth recalling that mainstream economics utilizes (without apology) welfare indicators based on utility calculations for policy evaluation, even when such calculations are quite abstract and, arguably, somewhat ethereal. Given this observation, there is no compelling reason not to use the rate of surplus value as an additional, or even alternative, economic indicator to evaluate the effects of policies. By highlighting the dynamics of unequal labour exchange that the rate captures, we broaden the visualization of the effects of policy changes and enrich our conclusions.

We should also comment more on some of the limitations of the proposed approach. On the demand side, the type of behaviour used to calculate consumption adjustments may be somewhat simplistic. When using a Leontief consumption function, changes in aggregate disposable income yield proportional (homothetic) changes in all sectoral consumptions. In other words, the adjustments of all sectoral consumptions follow a linear pattern, regardless of the type of good in question. While acceptable as a first approximation for the evaluation of small-scale changes, larger-scale changes would possibly require a different procedural adjustment in consumption levels to capture potential non-linearities. One way to improve could be to define a new criterion that introduces the importance or necessity of different consumptions while retaining the practical functionality of a Leontief-type structure without excessively increasing the implementation costs of the modelling. An avenue to explore could be a Stone-Geary demand system with minimal consumptions (Geary 1950, Stone 1954), but subsumed within a Leontief structure to maintain operational simplicity.

On the supply side, the accounting procedure does not provide any transmission mechanism for production levels. Consequently, disposable income remains blind to the changes in labour and capital incomes that would result from a change in taxation. For instance, a reduction in taxes would also improve disposable income, up to a certain point, through an increase in the demand for labour necessary to accommodate the higher levels of production incentivized by the tax reduction. In this sense, our calculation offers a first approximation of the first-round effects of tax policies on the rate of surplus value. Capturing second and third-round effects requires additional modelling layers, which are certainly possible using input-output analysis, linear SAM models, and non-linear general equilibrium models.

Regarding data, SAMs with the alternative flavour described in Olsen (2011) are, unfortunately, lacking. The distinction between productive and unproductive labour, to mention just one case, would first require a disaggregation of input-output labour data in the value-added sub-matrix, distinguishing their differential contribution to the production activities in each sector. In turn, for the coherence of the input-output accounts, a second distinction should also be made in the final demand sub-matrix between the consumptions of workers who provide productive and unproductive labour. Sadly, this type of information will not be available unless there is a dramatic change in the priorities of the statistical agencies charged with the responsibility of compiling socioeconomic data.

The trade-off, as mentioned earlier, lies in choosing between using statistically accessible and usable data for calculations that bring us closer to unveiling a magnitude of interest, or refraining from any calculations due to the immense difficulty of accessing ideal information. In our case, we advocate for pragmatism and support the use and analysis of available statistical data as it is currently compiled and published. This approach should not prevent us from highlighting the desiderata for data availability to improve the descriptive quality of the estimates. At the same time, the modeling framework should be adapted to better capture elements of economic structure—on both the demand and supply sides—that more accurately reflect the rules governing a capitalist economy.

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Notes

1 Morishima (1973) shows that there are three common definitions for the exploitation rate in Marxian economics and he also shows they are all equivalent.

2 See Nikaido (1972), chapter 3, for the definition of technical productiveness.

3 We can always choose units so that all prices are 1 and do not have to make them explicit for aggregation purposes.

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ANNEX

The SAM of Spain is available upon request. The 30 industries are aggregated by sector affinities from the 63 industries in the raw input-output data compiled by the “Instituto Nacional de Estadística.” No distinction in skill levels for labour is reported in the published data. Consumption expenditures correspond to a representative Household, with no distinction reported by household types. Capital income include pure profits, rents, and capital services. No distinction by origin is made.

Sectors			**Factors**	
1	Agriculture		1	Labour
2	Mining		2	Capital
3	Foodstuffs			
4	Leather and textiles		**Taxes**	
5	Wood and paper products		1	Indirect taxes on production
6	Coke and petroleum		2	Indirect taxes on products
7	Chemical products		3	Indirect taxes on consumption (value-added tax)
8	Metal products		4	Social Security payments by employers
9	Electronic, electric and precision products		5	Social Security payments by employees
10	Machinery and vehicles		6	Income and wealth taxes
11	Other manufactures			
12	Repairs and maintenance		**Final demand**	
13	Electricity and gas		1	Private consumption by households
14	Water		2	Public consumption by government
15	Recycling		3	Gross investment
16	Construction		4	Exports to th European Union
17	Wholesale retail		5	Exports to the rest of the world
18	Transportation			
19	Telecommunications			
20	Hostelry			
21	Entertainment			
22	Financial services			
23	Real estate services			
24	Professional services			
25	Commercial services			
26	Public services			
27	Education			
28	Health services			
29	Recreational services			
30	Personal services			