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NEW GENERALIZED FRACTIONAL ELZAKI – TARIG AND OTHER FRACTIONAL INTEGRAL TRANSFORMS WITH ITS APPLICATIONS TO FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract: In this paper we have defined new generalized fractional Elzaki – Tarig Transformation with its properties and its relations to other fractional Integral Transformations, as an application we have solved fractional order differential equation and find analytic solution of it.

Keywords: Elzaki – Tarig Transform, fractional Derivatives

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1. INTRODUCTION

Though the idea of fractional transformations, fractional derivatives have long back history the idea of fractional partial differential equation in different spaces has been rediscovered mainly in quantum mechanics, fluid dynamics, and stochastic processes. In recent years many linear boundary value and initial value problems in applied mathematics, mathematical physics and engineering science are effectively solved by Laplace, Fourier and other Transforms. It has been studied by many researchers and contributed. The partial differential equations has applications in the field of mathematics as well in real life situations, such as Abel's integral equation, visco – elasticity, capacitor theory, conductance of biological systems [4, 5]

Transformations generally used for shifting the given problem into another domain which is simple to calculate rather than the previous domain and by applying inverse it gives back to the given situation . There are different kinds of Transformations having different kernels like Laplace, Fourier, Mellin, Hartley, Yang – Fourier Abel, Continuous Wavelet, Weirstrass, Hilbert, Randon, Whittakar, Hankel, L_2 – Transform, finite Legendre Transform, and many more to solve the real life problems [4, 5] mainly in signal processing, computational fluid dynamics, fractals, Bio – mathematics, railway Engineering and in fractional calculus [4]. The Elzaki – Tarig transform recently defined [9] having tremendous applications in the fractional calculus [10, 11]. The generalized versions [3, 7] have been defined with its properties and applications to fractional differential equations.

The paper mainly divided into four parts, in the first part we have defined some new definitions which are required for further results, in the second part there are some properties of generalized fractional Elzaki – Tarig Transform which we have already studied, third part consist of finding relations with other fractional Transformations. In the fourth part we have given an application of it to fractional differential equations conclusion part ends our manuscript.

2. DEFINITIONS

In this section we consider some new definition which we will be require for deriving further relations with other Transformation along with properties.

Definition 1: Tarig Transformation

The set $S = \{f(t): \exists\, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0,\infty)\}$ where M > 0 a finite number while k_1, k_2 may be infinite, then for given function which satisfies the condition of the set S, the Tarig Transformation of $f(t)$ is defined as [8],

$$T[f(t)](\xi) = F(\xi) = \int_0^\infty f(\xi t)e^{\frac{-t}{\xi}}dt, \quad \xi \neq 0 \tag{1}$$

Definition 2: Generalized Elzaki – Tarig Transform

We consider the definition of Generalized Elzaki – Tarig Transformation [6] by using the definition (1) as,

$$\Im_\varepsilon\{f(x); p\} = \int_0^\infty \Phi(p)\Phi_1(p)\varepsilon'(x)e^{-\Phi(p)\varepsilon(x)}f(x)dx, \quad p \neq 0 \tag{2}$$

Where $f(x) \in S = \{f(x): \exists\, k_1, k_2 > 0, |f(x)| < Me^{\frac{|x|}{k_j}}, x \in (-1)^j \times [0,\infty) and\ M > 0\}$ and $\Phi(p), \Phi_1(p)$ are invertible functions of p with $\varepsilon(x) = \int e^{-a(x)}dx$ an exponential function and $a(x)$ as invertible function, thus from the definitions above it can be seen that it is the generalization of Elzaki – Tarig transformation.

Definition 3: Generalized Fractional Elzaki – Tarig Transform

We consider the definition of Generalized Fractional Elzaki – Tarig Transformation of order α by using the definition (1), (2) as,

$$\Im_\varepsilon^\alpha\{f(x);\ p^\alpha\} = \int_0^\infty \Phi(p^\alpha)\Phi_1(p^\alpha)\varepsilon'(x)e^{-\Phi(p^\alpha)\varepsilon(x^\alpha)}f(x)dx, \quad p \neq 0 \tag{3}$$

Where $f(x) \in S = \left\{ f(x) \colon \exists\, k_1, k_2 > 0, |f(x)| < Me^{\frac{|x|}{k_j}}, x \in (-1)^j \times [0, \infty) and\ M > 0 \right\}$ and $\Phi(p^\alpha)\Phi_1(p^\alpha)$ are invertible functions of p^α with $\varepsilon(x^\alpha) = \int e^{-a(x^\alpha)}dx$ an exponential function and $a(x^\alpha)$ as invertible function, thus from the definitions above it can be seen that it is the generalization of Elzaki – Tarig transformation.

Definition 4: Generalized bilateral fractional Elzaki – Tarig Transform

We consider the definition of Generalized Bilateral Elzaki – Tarig Transformation by using the definition (1), (2) and (3) as,

$$\mathfrak{I}_\varepsilon\{f(x); p\} = \int_{-\infty}^{\infty} \Phi\left(\frac{1}{p^\alpha}\right)\Phi_1(p^\alpha)\varepsilon'(x)e^{-\Phi(p^\alpha)\varepsilon(x)}f(x)dx, \quad p \neq 0 \quad (4)$$

Where $f(x) \in S = \left\{ f(x) \colon \exists\, k_1, k_2 > 0, |f(x)| < Me^{\frac{|x|}{k_j}}, x \in (-1)^j \times [0, \infty) and\ M > 0 \right\}$ and $\Phi\left(\frac{1}{p^\alpha}\right), \Phi_1(p^\alpha)$ are invertible functions of p with $\varepsilon(x) = \int e^{-a(x)}dx$ an exponential function and $a(x)$ as invertible function, thus from the definitions above it can be seen that it is the generalization of Elzaki – Tarig transformation.

Definition 5: Generalized bilateral fractional Elzaki – Tarig Transform

We consider the definition of Generalized Bilateral Elzaki – Tarig Transformation by using the definition (1), (2), (3) and (4) as,

$$\mathfrak{I}_\varepsilon\{f(x); p^\alpha\} = \int_{-\infty}^{\infty} \Phi\left(\frac{1}{p^\alpha}\right)\Phi_1(p^\alpha)\varepsilon'(x^\alpha)e^{-\Phi(p^\alpha)\varepsilon(x^\alpha)}f(x)dx, p \neq 0 \ (5)$$

Where $f(x) \in S = \{ f(x) \colon \exists\, k_1, k_2 > 0, |f(x)| < Me^{\frac{|x|}{k_j}}, x \in (-1)^j \times [0, \infty) and\ M > 0\}$ and $\Phi\left(\frac{1}{p^\alpha}\right), \Phi_1(p^\alpha)$ are invertible functions of p^α with $\varepsilon(x^\alpha) = \int e^{-a(x^\alpha)}dx$ an exponential function and $a(x^\alpha)$ as invertible function, thus from the definitions above it can be seen that it is the generalization of bilateral Elzaki – Tarig transformation.

3. MAIN RESULT

In this section for various values of $\Phi\left(\frac{1}{p^\alpha}\right), \Phi_1(p^\alpha)$ and $\varepsilon(x^\alpha)$ we have obtain various fractional Integral Transformations like Elzaki – Tarig, Tarig, Laplace, Mellin, Hilbert, Abel, Wavelet and L_2 – Transform under different conditions by using generalized fractional Elzaki – Tarig Transformation [6] and Generalized fractional Bilateral Transformation definition as well as some properties [6] of Generalized fractional Elzaki – Tarig Transformations.

3.1 Properties of generalized fractional Elzaki – Tarig Transform

3.1.1 (Inversion formula)

The definition of generalized fractional Elzaki – Tarig transformation (3) tells us that

$$\mathfrak{I}_{\varepsilon}^{\alpha}\{f(x);\, p^{\alpha}\} = \int_0^{\infty} \Phi\left(\frac{1}{p^{\alpha}}\right) \Phi_1(p^{\alpha})\varepsilon'(x)e^{-\Phi(p^{\alpha})\varepsilon(x^{\alpha})}f(x)dx, \qquad p \neq 0$$

which satisfies the given conditions in definition (4)

Then the inverse fractional transformation to be defined as [6],

$$\mathfrak{I}^{-1}{}_{\varepsilon}\big(F(p^{\alpha})\big) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(\Phi^{-1}(p^{\alpha})\Phi_1^{-1}(p^{\alpha}))e^{\Phi(p^{\alpha})\varepsilon(x^{\alpha})}dp$$

Proof: By definition of generalized fractional Elzaki – Tarig transformation (3) with defining

$$\Phi\left(\frac{1}{p^{\alpha}}\right) = \frac{1}{r^{\alpha}} \text{ and } \Phi_1(p^{\alpha}) = r^{\alpha}. \Rightarrow F\left(\Phi^{-1}\left(\frac{1}{r^{\alpha}}\right)\Phi_1^{-1}(r^{\alpha})\right) = \int_0^{\infty} \varepsilon'(x^{\alpha})\, e^{-r^{\alpha}\varepsilon(x^{\alpha})}f(x)dx$$

Put $\varepsilon(x^{\alpha}) = t^{\alpha}$ in the above equation

$$\Rightarrow F\left(\Phi^{-1}\left(\frac{1}{r^{\alpha}}\right)\Phi_1^{-1}(r^{\alpha})\right) = \int_0^{\infty} e^{-r^{\alpha}t^{\alpha}} f(\varepsilon^{-1}(t^{\alpha}))dt = \mathfrak{I}_{\varepsilon}\{f(\varepsilon^{-1}(t^{\alpha})); r^{\alpha}\}$$

Whenever, $\Phi\left(\frac{1}{p^{\alpha}}\right)$ *and* $\Phi_1(p^{\alpha})$ are inverses of each other so that by complex inversion formula for the Laplace transform with $\varepsilon^{-1}(t^{\alpha}) = x^{\alpha}$ *and* $r^{\alpha} = p^{\alpha}$

$$\Rightarrow f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F\big(\Phi^{-1}(p^{\alpha})\Phi_1^{-1}(p^{\alpha})\big)e^{p^{\alpha}\varepsilon(x^{\alpha})}dp^{\alpha}$$

3.1.2 (Generalized fractional Elzaki – Tarig transform of derivative)

Let $f(x)$ satisfies all the conditions in definition (3) with $f^{\alpha}(x)$ has piecewise continuous derivative then [6] where, fractional derivative is in ABR fractional derivative sense

$$\Im_{\varepsilon}^{\alpha}\{ f^{\alpha}(x); p^{\alpha}\} = \frac{1}{\Phi(p^{\alpha})}[\Im_{\varepsilon}^{\alpha}\{f(x); p^{\alpha}\}] - \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})f(0^{+})$$

Proof: By definition of generalized fractional Elzaki – Tarig transformation (3), we have

$$\Im_{\varepsilon}^{\alpha}\{f^{\alpha}(x); p\} = \int_0^{\infty} \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\varepsilon'(x^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(x^{\alpha})}f^{\alpha}(x)dx, \quad p \neq 0$$

$$= \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\int_0^{\infty} \varepsilon'(x^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(x^{\alpha})}f^{\alpha}(x)dx$$

Applying integration by part to the above

$$\Rightarrow \Im_{\varepsilon}^{\alpha}\{f^{\alpha}(x); p^{\alpha}\} = \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\left\{\left[\varepsilon'(x^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(x^{\alpha})}f(x)\right]_0^{\infty} + \frac{1}{\Phi(p^{\alpha})}\int_0^{\infty}[\varepsilon'(x)e^{-\Phi(p)\varepsilon(x)}]f(x)dx\right\}$$

$\Rightarrow \frac{1}{\Phi(p^{\alpha})}[\Im_{\varepsilon}^{\alpha}\{f^{\alpha}(x); p^{\alpha}\}] - \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})f(0^{+})$ --------- as $f(x)$ is of exponential order

3.1.3 (Convolution Theorem)

If $F(p^{\alpha})$ *and* $G(p^{\alpha})$ are the generalized fractional Elzaki – Tarig transformations of two functions $f(x)$ *and* $g(x)$ respectively then the generalized convolution theorem [6] for definition (3) is calculated as follows,

$$\Im_{\varepsilon}^{\alpha}\{f * g; p^{\alpha}\} = \left(\int_0^{\infty} \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\varepsilon'(y^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(y^{\alpha})}f(y)dy\right)\left(\int_0^{\infty} \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\varepsilon'(t^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(t^{\alpha})}g(t)dt\right)$$

$$= \iint_0^{\infty} \Phi^2\left(\frac{1}{p^{\alpha}}\right)\Phi^2(p)\, \varepsilon'(y)\varepsilon'(t)e^{-\Phi(p^{\alpha})[\varepsilon(y^{\alpha})+\varepsilon(t^{\alpha})]}f(y)g(t)dydt$$

Substitute $\varepsilon(y^{\alpha}) + \varepsilon(t^{\alpha}) = \varepsilon(x^{\alpha})$ and changing the order of integration in the double integral we get,

$$F(p^{\alpha})G(p^{\alpha}) = \int_0^{\infty} \Phi^2\left(\frac{1}{p^{\alpha}}\right)\Phi_1^2(p^{\alpha})\varepsilon'(x^{\alpha})e^{-\Phi(p^{\alpha})\varepsilon(x^{\alpha})}dx \int_0^{x} \varepsilon'(t^{\alpha})g(t^{\alpha})f(\varepsilon^{-1}(\varepsilon(x^{\alpha}) - \varepsilon(t^{\alpha})))dt$$

$$= \Im_{\varepsilon}^{\alpha}\left\{\left(\int_0^{x} \Phi\left(\frac{1}{p^{\alpha}}\right)\Phi_1(p^{\alpha})\varepsilon'(t^{\alpha})g(t^{\alpha})f\left(\varepsilon^{-1}\left(\varepsilon(x^{\alpha}) - \varepsilon(t^{\alpha})\right)\right)dt\right); p\right\}$$

3.2 Relation of generalized fractional Elzaki – Tarig Transform with other Integral Transform

3.2.1) Fractional Elzaki – Tarig Transform:

The fractional Elzaki – Tarig Transformations of a function $f(x)$ can be obtained by taking $\Phi(p^\alpha) = \frac{1}{p^\alpha}\ and\ \Phi_1(p^\alpha) = 1\ with\ \varepsilon(x^\alpha) = x^\alpha$ in definition (3)

$$\mathfrak{I}_x^\alpha\{f(x); p^\alpha\} = \int_0^\infty p^\alpha e^{-\frac{x^\alpha}{p^\alpha}} f(x)dx, \quad p \neq 0$$

3.2.2) Fractional Tarig Transformation:

The fractional Tarig Transformation of a function $f(x)$ can be obtained by taking $\Phi(p^\alpha) = \frac{1}{p^{2\alpha}}\ and\ \Phi_1(p^\alpha) = \frac{1}{p^{3\alpha}}\ with\ \varepsilon(x^\alpha) = x^\alpha$ in definition (3)

$$\mathfrak{I}_x^\alpha\{f(x); p^\alpha\} = \int_0^\infty p^{2\alpha} \frac{1}{p^{3\alpha}} e^{-\frac{x^\alpha}{p^{2\alpha}}} f(x)dx, \quad p \neq 0$$

$$\Rightarrow \mathfrak{I}_x^\alpha\{f(x); p^\alpha\} = \int_0^\infty \frac{1}{p^\alpha} e^{-\frac{x^\alpha}{p^{2\alpha}}} f(x)dx, \quad p \neq 0$$

3.2.3) Fractional Laplace Transformation:

The fractional Laplace Transformation of a function $f(x)$ can be obtained by taking $\Phi(p^\alpha) = p^\alpha\ and\ \Phi_1(p^\alpha) = p^\alpha\ with\ \varepsilon(x^\alpha) = x^\alpha$ in definition (3) with Re.$(p^\alpha) > 0$

$$\mathfrak{I}_x^\alpha\{f(x); p^\alpha\} = \int_0^\infty \frac{1}{p^\alpha} p^\alpha e^{-x^\alpha p^\alpha} f(x)dx, \quad Re.(p^\alpha) > 0$$

$$\Rightarrow \mathfrak{I}_x^\alpha\{f(x); p^\alpha\} = \int_0^\infty e^{-x^\alpha p^\alpha} f(x)dx, \quad Re.(p^\alpha) > 0$$

3.2.4) Fractional Mellin Transform

The Mellin Transformation of a function $f(x)$ can be obtained by taking $\Phi(p^\alpha) = -p^\alpha\ and\ \Phi_1(p^\alpha) = -p^\alpha\ with\ \varepsilon(x^\alpha) = \ln(x)$ in definition (3) with $p^\alpha > 0$ and $x \in (0, \infty)$

$$\mathfrak{I}_{\ln(x)}^\alpha\{f(x); p^\alpha\} = \int_0^\infty (-\frac{1}{p^\alpha})(-p^\alpha) \frac{1}{x^\alpha} e^{p^\alpha ln(x)} f(x)dx, \quad p > 0$$

$$\Rightarrow \mathfrak{I}_{\ln(x)}^\alpha\{f(x); p^\alpha\} = \int_0^\infty \frac{1}{x^\alpha} e^{p^\alpha ln(x)} f(x)dx, \quad p > 0$$

$$\Rightarrow \Im_{\ln(x)}^{\alpha}\{f(x);p\} = \int_0^\infty \frac{1}{x} x^{p^\alpha} f(x)dx, \qquad p > 0$$

$$\Rightarrow \Im_{\ln(x)}^{\alpha}\{f(x);p\} = \int_0^\infty x^{p^\alpha - 1} f(x)dx, \qquad p > 0$$

3.2.5) Fractional $\mathcal{L}_2$ Transform

As a particular case of fractional $\mathcal{L}_\varepsilon -$ transform; the fractional $\mathcal{L}_2$ transform [2] of a function $f(x)$ at a point p^α can be obtained from (3) by substitution of $\Phi(p^\alpha) = p^{2\alpha}$ *and* $\Phi_1(p^\alpha) = p^{2\alpha}$ *with* $\varepsilon(x^\alpha) = x^{2\alpha}$

$$\Im_{x^{2\alpha}}^{\alpha}\{f(x);p^\alpha\} = \int_0^\infty \frac{1}{p^{2\alpha}} p^{2\alpha} e^{-p^{2\alpha}x^{2\alpha}} f(x)dx, \qquad Re.(p^{2\alpha}) > 0$$

3.2.6) Fractional Abel's Transform

The fractional Abel's Transformation of a function $f(x)$ at a point p^α can be obtained from (3) by substitution of $\Phi(p^\alpha) = -1$ *and* $\Phi_1(p^\alpha) = \frac{-1}{2}$ *with* $\varepsilon(x) = \begin{cases} \ln\left(\sqrt{x^2 - y^2}\right), & x > y \\ 0, & x \le y \end{cases}$ then we get

$$\Im_{\ln(\sqrt{x^2+y^2})}^{\alpha}\{f(x);\ p^\alpha\} = \int_y^\infty (-1)\left(\frac{-1}{2}\right)\left(\frac{2x}{x^2-y^2}\right) e^{\ln\left(\sqrt{x^2-y^2}\right)} f(x)dx$$

$$\Rightarrow \quad \Im_{\ln(\sqrt{x^2+y^2})}^{\alpha}\{f(x);\ p^\alpha\} = \int_y^\infty \left(\frac{x}{x^2 - y^2}\right)\left(\sqrt{x^2 - y^2}\right) f(x)dx$$

$$\Rightarrow \quad \Im_{\ln(\sqrt{x^2+y^2})}^{\alpha}\{f(x);\ p^\alpha\} = \int_y^\infty \left(\frac{x}{\sqrt{x^2 - y^2}}\right) f(x)dx = 2\,F(y)$$

Thus we have the relation, $\Im_{\ln(\sqrt{x^2+y^2})}^{\alpha}\{f(x);\ p^\alpha\} = 2F(y)$

3.2.7) Fractional Hilbert Transform

The Fractional Hilbert Transformation of a function $f(x)$ at a point p^α can be obtained from (5) by substitution of

$\Phi(p^\alpha) = -1$ *and* $\Phi_1(p^\alpha) = \frac{1}{\pi}$ *with* $\varepsilon(x^\alpha) = \ln(\ln(p^\alpha - x)), 0 \le x < p^\alpha$

then we get

$\mathfrak{I}^{\alpha}_{ln(\ln(p-x))}\{f(x);\, p^{\alpha}\} = \int_{-\infty}^{\infty}(-1)(\frac{1}{\pi})(\frac{1}{\ln(p^{\alpha}-x)})(\frac{-1}{p^{\alpha}-x})e^{\ln(\ln(p^{\alpha}-x))}f(x)dx$ $, 0 \leq x < p^{\alpha}$

$\Rightarrow \mathfrak{I}^{\alpha}_{ln(\ln(p^{\alpha}-x))}\{f(x);\; p^{\alpha}\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(x)}{(p^{\alpha}-x)}dx \;, 0 \leq x < p^{\alpha}$

$\Rightarrow \mathfrak{I}^{\alpha}_{ln(\ln(p^{\alpha}-x))}\{f(x);\; p^{\alpha}\} = F(p)\;, 0 \leq x < p^{\alpha}$

3.2.8) Fractional Weirstrass Transform

The Fractional Weirstrass Transformation of a function $f(x)$ at a point p^{α} denoted by $F(p^{\alpha})$ can be obtained from (5) by substitution of $\Phi(p^{\alpha}) = 1\; and\; \Phi_1(p^{\alpha}) = \frac{e^{-\frac{p^{2\alpha}}{4}}}{\sqrt{4\pi}}\; with\; \varepsilon(x) = \ln\left(\frac{1}{\int e^{-\frac{(p^{\alpha}-x^{\alpha})^2}{4}}}\right)$ then we get

$$\mathfrak{I}^{\alpha}_{\varepsilon(x)}\{f(x);\; p^{\alpha}\} = \int_{-\infty}^{\infty}(1)\frac{1}{\sqrt{4\pi}}e^{-(\frac{(p^{\alpha}-x^{\alpha})^2}{4})}f(x)dx$$

$$\Rightarrow \mathfrak{I}^{\alpha}_{\varepsilon(x)}\{f(x);\; p^{\alpha}\} = \int_{-\infty}^{\infty}\frac{1}{\sqrt{4\pi}}e^{-\left(\frac{(p^{\alpha}-x^{\alpha})^2}{4}\right)}f(x)dx$$

$$\Rightarrow \mathfrak{I}^{\alpha}_{\varepsilon(x)}\{f(x);\; p^{\alpha}\} = F(p)$$

3.2.9) Fractional Fourier Transform

The Fractional Fourier Transformation of a function $f(x)$ at a point p^{α} denoted by $\hat{f}(p^{\alpha})$ can be obtained from (5) by substitution of $\Phi(p) = p^{\alpha}\; and\; \Phi_1(p) = \frac{p^{\alpha}}{\sqrt{2\pi}}\; with\; \varepsilon(x^{\alpha}) = i\,x^{\alpha}$ then we get,

$$\mathfrak{I}^{\alpha}_{i\,x^{\alpha}}\{f(x);\; p^{\alpha}\} = \int_{-\infty}^{\infty}(\frac{1}{p^{\alpha}})(\frac{p^{\alpha}}{\sqrt{2\pi}})(i)e^{-i\,p^{\alpha}\,x^{\alpha}}f(x)dx$$

$\Rightarrow \mathfrak{I}^{\alpha}_{i\,x^{\alpha}}\{f(x);\; p^{\alpha}\} = \int_{-\infty}^{\infty}\frac{i}{\sqrt{2\pi}}e^{-i\,p^{\alpha}\,x^{\alpha}}f(x)dx$

$\Rightarrow \mathfrak{I}^{\alpha}_{i\,x^{\alpha}}\{f(x);\; p^{\alpha}\} = i\hat{f}(p^{\alpha})$

3.2.10) Fractional Laplace – Carson TransformThe Fractional Laplace - Carson Transformation of a function $f(x)$ at a point p^{α} denoted by $F(p^{\alpha})$ can be obtained from (3) by substitution of

$\Phi(p^{\alpha}) = p^{\alpha}$ *and* $\Phi_1(p^{\alpha}) = p^{2\alpha}$ *with* $\varepsilon(x^{\alpha}) = x$ **then we get,**

$$\mathfrak{I}_x^{\alpha}\{f(x);\ p^{\alpha}\} = \int_0^{\infty}\left(\frac{1}{p^{\alpha}}\right) p^{2\alpha} e^{-p^{\alpha}x} f(x)dx$$

$$\Rightarrow \mathfrak{I}_x^{\alpha}\{f(x);\ p^{\alpha}\} = \int_0^{\infty} p^{\alpha} e^{-px} f(x)dx$$

$$\Rightarrow \mathfrak{I}_x^{\alpha}\{f(x);\ p^{\alpha}\} = F(p^{\alpha})$$

3.2.11) Fractional Laplace – Stieltjes Transform

We know that, if f has derivative f' then Laplace – Stieltjes transform of f is Laplace Transform of f' i.e. $(\mathcal{L}^* f)(p) = (\mathcal{L} f')(p)$. Thus from the relation (3.3.2) and definition (3) one can have the relation,

$$\mathfrak{I}_x^{\alpha}\{f'(x);\ p^{\alpha}\} = \int_0^{\infty} \frac{1}{p^{\alpha}} p^{\alpha} e^{-x^{\alpha} p^{\alpha}} f'(x)dx, \qquad Re.(p^{\alpha}) > 0$$

$$\Rightarrow \mathfrak{I}_x^{\alpha}\{f'(x);\ p^{\alpha}\} = \int_0^{\infty} e^{-x^{\alpha} p^{\alpha}} f^{\alpha}(x)dx, \qquad Re.(p^{\alpha}) > 0$$

$$\Rightarrow \mathfrak{I}_x^{\alpha}\{f'(x);\ p^{\alpha}\} = (\mathcal{L}^* f)(p^{\alpha}), \qquad Re.(p^{\alpha}) > 0$$

4. Application of Generalized fractional Elzaki – Tarig Transform

Consider the space fractional differential equation with Cauchy type initial and boundary conditions as,

$${}^{RL}_{t}D_{0+}^{\alpha}\, u(x,t) = -(x+1)\frac{\partial u(x,t)}{\partial x} \text{ ------------------------------ (A)}$$

$0 < \alpha \leq 1$ and ${}^{RL}_{t}D_{0+}^{\alpha-1}\, u(x,0) = f(x), u(0,t) = 0$ *with* $x, t \in \mathcal{R}^{+}$

Since the equation (A) Linear fractional partial differential equation with non – constant coefficient to solve it by the method of generalized fractional Elzaki – Tarig transform method. We consider the following substitutions;

$\varepsilon(x) = \ln(x+1)\,, \phi\left(\frac{1}{p}\right) = \frac{1}{p}, \phi_1(p) = p$ and $p \neq 0$, $(x+1) > 0$

Then from definition (3) we have,

$$\mathfrak{J}_{\ln(x+1)}\{(x+1)u(x,t);p\}$$
$$=\int_0^\infty (\frac{1}{p})\,p\ \ (\frac{1}{(x+1)})(x+1)e^{-p\ln(x+1)}u(x,t)dx$$

Thus from above property one can write

$$\mathfrak{J}_{\ln(x+1)}\{(x+1)u_x(x,t);p\} = p\hat{u}(p,t) - u(0,t)$$

Hence by using initial condition we have, $\mathfrak{J}_{\ln(x+1)}\{(x+1)u_x(x,t);p\} = p\hat{u}(p,t)$

Applying Laplace Transform on LHS of the equation (A) and using the relation between Laplace and Tarig Transform

$$\mathfrak{J}\{{}_{t}^{RL}D_{0+}^{\alpha}\,u(x,t);s\} = s^{\alpha-1}\tilde{u}\left(x,\frac{1}{s^2}\right) - {}_{t}^{RL}D_{0+}^{\alpha-1}\,u(x,0)$$

Thus by applying Tarig Transform on both sides we get,

$$\Rightarrow s^{\alpha-1}\hat{\tilde{u}}(p,s) - F(p) = p\hat{\tilde{u}}(p,s)$$

Where, $F(p)$ is nothing but $\mathfrak{J}_{\ln(x+1)}$ - Transform of *f(x)*

$$\Rightarrow \hat{\tilde{u}}(p,s) = \frac{1}{p+s^{\alpha-1}}F(p)$$

The solution can be found by writing it into convolution form and then by applying Inverse Tarig Transform.

4. CONCLUSION

The new definition of Generalized fractional Elzaki – Tarig Transform seems to be very interesting as one find relation between various fractional Transforms by proper choice of $\Phi\left(\frac{1}{p^\alpha}\right), \Phi_1(p^\alpha)$ and $\varepsilon(x^\alpha)$. Right now we are at very first stage, so that the definition itself has limit to the defined set with most beautiful property gives us at $\alpha = 1$. One can modified it and found new interesting results, also for solving various types of linear fractional partial differential equation it can be used by extending the definition to higher dimension.

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