**IJCTA,** 9(28), 2016, pp. 539-547


# Further Result On Skolem Mean Labeling

**A.Manshath* V. Balaji** P. Sekar*** and M. Elakkiya******

***Abstract :*** In this paper, we prove If $a \le b < c,$ the six star $K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ is a skolem mean graph if $|b-c| < 4+4a$ for $a = 2,3,4,\ldots$ $b = 2,3,4,\ldots$ and $4a+b-3 \le c \le 4a+b+3.$
***Keywords :*** Skolem mean graph and star.
**2010 Mathematical Subject Classification Number :** 05C78.

## 1. INTRODUCTION

In **[3],** we proved the following theorems to study the existence of skolem mean graphs. We proved the three star $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m–n| = 4 + l$ for $l = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $l \le m < n.$ The three star $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m–n| > 4 + l$ for $l = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$ and $l \le m < n.$ The four star $K_{1,l} \cup K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4 + 2l$ for $l = 2, 3, 4, \ldots\ldots$; $m = 2, 3, 4, \ldots$ and $l \le m < n.$ The four star $K_{1,l} \cup K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m–n| > 4 + 2l$ for $l = 2, 3, 4, \ldots\ldots$; $m = 2, 3, 4, \ldots$ and $l \le m < n.$ The five star $K_{1,l} \cup K_{1,l} \cup K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m – n| = 4 + 3l$ for $l = 2, 3, 4, \ldots\ldots$; $m = 2, 3, 4, \ldots$ and $l \le m < n$ and the five star $K_{1,l} \cup K_{1,l} \cup K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m–n| > 4 + 3l$ for $l = 2, 3, 4, \ldots$; $m = 2, 3, 4, \ldots$ and $l \le m < n$. Further, we prove the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m–n| = 7$ for $m = 1, 2,3, \ldots$ and $1 \le m < n$; The four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m–n| > 7$ for $m = 1, 2, 3, \ldots$ and $1 \le m < n$; The five star $K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m–n| = 8$ for $m = 1, 2, 3, \ldots$ and $1 \le m < n$ and the five star $K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m–n| > 8$ for $m = 1, 2, 3, \ldots$ and $1 \le m < n$.

## 2. [2] SKOLEM MEAN LABELING

**Definition 2.1.**

The six star is the disjoint union of $K_{1,a}, K_{1,b}$, $K_{1,c}$, $K_{1,d}$, $K_{1,e}$ and $K_{1,f}$. It is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e} \cup K_{1,f}$.

* Department of Mathematics, Sriram Engineering College, Thiruvallur-602024, India.
** Department of Mathematics, Sacred Heart College, Tirupattur - 635 601, India.
*** Department of Mathematics, C. Kandasamy Naidu College, Chennai - 600102 india. E-mail: pulibala70@gmail.com
**** Department of Mathematics, Sacred Heart College, Tirupattur - 635 601, India.

**Definition 2.2**

A graph $G=(V, E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function *f* from the vertex set of G to $\{1,2,3,\ldots,p\}$ such that the induced map *f** from the edge set of G to $\{2,3,4,\ldots,p\}$ defined by

$$f^*(e=uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd, then} \end{cases}$$

the resulting edges get distinct labels from the set $\{2,3,4,\ldots,p\}$.

**Note 2.3.**

In a skolem mean graph, $p \geq q+1$.

**Theorem 2.4**

If $a \leq b < c$, the six star $K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ is a skolem mean graph if $|b-c| < 4+4a$ for $a=2,3,4,\ldots$ $b=2,3,4,\ldots$ and $4a+b-3 \leq c \leq 4a+b+3$.

**Proof :**

**Case($a$)** Consider the graph $G=K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \leq b < c$, where $c=4a+b+3$ for $a=2,3,4,\ldots$ and $b=2,3,4,\ldots$

Let us take the case that $|b-c| < 4+4a$ for $a=2,3,4,\ldots$ $b=2,3,4,\ldots$ and $c=4a+b+3$

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \leq i \leq a\} \cup \{u\} \cup \{u_j : 1 \leq j \leq a\} \cup \{v\} \cup \{v_k : 1 \leq k \leq a\} \cup \{w\} \cup \{w_h : 1 \leq h \leq a\} \cup \{x\} \cup \{x_s : 1 \leq s \leq b\} \cup \{y\} \cup \{y_r : 1 \leq r \leq c\}.$$

$$E(G) = \{tt_i : 1 \leq i \leq a\} \cup \{uu_i : 1 \leq j \leq a\} \cup \{vv_k : 1 \leq k \leq a\} \cup \{ww_h : 1 \leq h \leq a\} \cup \{xx_s : 1 \leq s \leq b\} \cup \{yy_r : 1 \leq r \leq c\}.$$

Then G has $4a+b+c+6$ vertices and $4a+b+c$ edges.

The required vertex labeling $f : V(G) \to \{1,2,3,4,\ldots 4a+b+c+6\}$ is defined as follow

$$f(t) = 1;\ f(u)=2;\ f(v)=3;\ f(w)=4;\ f(x)=5;$$

$$f(y) = 4a+b+c+5;$$

$$f(t_i) = 2i+5 \qquad \text{for } 1 \leq i \leq a;$$

$$f(u_j) = 2a+2j+5 \qquad \text{for } 1 \leq j \leq a;$$

$$f(v_k) = 4a+2k+5 \qquad \text{for } 1 \leq k \leq a;$$

$$f(w_h) = 6a+2h+5 \qquad \text{for } 1 \leq h \leq a;$$

$$f(x_s) = 8a+2s+5 \qquad \text{for } 1 \leq s \leq b;$$

$$f(y_r) = 2r + 4 \quad \text{for } 1 \le r \le c-2;$$

$$f(y_{c-1}) = 4a + b + c + 4 \text{ and}$$

$$f(y_c) = 4a + b + c + 6$$

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i + 3$ for $1 \le i \le a$; $uu_j$ is $a + j + 4$ for $1 \le j \le a$; $VV_k$ is $2a + k + 4$ for $1 \le k \le a$; $WW_h$ is $3a+h+5$ for $1 \le h \le a$; $XX_s$ is $4a+s+5$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r+4a+b+c+9}{2}$ for $1 \le r \le c-2$. Also, the edge label of $yy_{c-1}$ is $4a + b + c + 5$ and $yy_c$ is $4a + b + c + 6$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case(*b*)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a + b + 2$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$.

Let us take the case that $|b-c| < 4 + 4a$ for $a = 2,3,4,\ldots$ $b = 2,3,4,\ldots$. and $c = 4a + b + 2$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup \{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$

$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_i : 1 \le i \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\} \cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a + b + c + 6$ vertices and $4a + b + c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4a + b + c + 6\}$ is defined as follows:

$$f(t) = 1; f(u)=2; f(v)=3; f(w)=4; f(x)=6$$

$$f(y) = 4a + b + c + 6;$$

$$f(t_i) = 2i + 6 \quad \text{for } 1 \le i \le a;$$

$$f(u_j) = 2a + 2j + 6 \quad \text{for } 1 \le j \le a;$$

$$f(v_k) = 4a + 2k + 6 \quad \text{for } 1 \le k \le a;$$

$$f(w_h) = 6a + 2h + 6 \quad \text{for } 1 \le h \le a;$$

$$f(x_s) = 8a + 2s + 6 \quad \text{for } 1 \le s \le b;$$

$$f(y_r) = 2r + 3 \quad \text{for } 1 \le r \le c.$$

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i + 4$ for $1 \le i \le a$; $uu_j$ is $a + j + 4$ for $1 \le j \le a$; $VV_k$ is $2a + k + 5$ for $1 \le k \le a$; $WW_h$ is $3a+h+5$ for $1 \le h \le a$; $XX_s$ is $4a+s+6$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r+4a+b+c+9}{2}$ for $1 \le r \le c$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case(*c*)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a + b + 1$ for $a = 2, 3, 4, \ldots$ and $b = 2, 3, 4, \ldots$ .

Let us take the case that $|b - c| < 4 + 4a$ for $a = 2, 3, 4, \ldots$ and $b = 2, 3, 4, \ldots$ . and $c = 4a + b + 1$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup \{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$

$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_i : 1 \le j \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\} \cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a + b + c + 6$ vertices and $4a + b + c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4a + b + c + 6\}$ is defined as follows:

$$f(t) = 1;\ f(u) = 2;\ f(v) = 3;\ f(w) = 4;\ f(x) = 5$$

$$f(y) = 4a + b + c + 6;$$

$$f(t_i) = 2i + 5 \quad \text{for } 1 \le i \le a;$$

$$f(u_j) = 2a + 2j + 5 \quad \text{for } 1 \le j \le a;$$

$$f(v_k) = 4a + 2k + 5 \quad \text{for } 1 \le k \le a;$$

$$f(w_h) = 6a + 2h + 5 \quad \text{for } 1 \le h \le a;$$

$$f(x_s) = 8a + 2s + 5 \quad \text{for } 1 \le s \le b;$$

$$f(y_r) = 2r + 4 \quad \text{for } 1 \le r \le c.$$

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i + 3$ for $1 \le i \le a$; $uu_j$ is $a + j + 4$ for $1 \le j \le a$; $VV_k$ is $2a + k + 4$ for $1 \le k \le a$; $WW_h$ is $3a + h + 5$ for $1 \le h \le a$; $XX_s$ is $4a + s + 5$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r + 4a + b + c + 10}{2}$ for $1 \le r \le c$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case(*d*)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a + b$ for $a = 2, 3, 4, \ldots$ and $b = 2, 3, 4, \ldots$ .

Let us take the case that $|b - c| < 4 + 4a$ for $a = 2, 3, 4, \ldots$ and $b = 2, 3, 4, \ldots$ . and $c = 4a + b$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup \{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$

$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_j : 1 \le j \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\} \cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a + b + c + 6$ vertices and $4a + b + c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4a + b + c + 6\}$ is defined as follows:

$$f(t) = 1; f(u)=2; f(v)=3; f(w)=4; f(x)=5$$
$$f(y) = 4a + b + c + 6;$$
$$f(t_i) = 2i+4 \quad \text{for } 1 \le i \le a;$$
$$f(u_j) = 2a + 2j + 4 \quad \text{for } 1 \le j \le a;$$
$$f(v_k) = 4a + 2k + 4 \quad \text{for } 1 \le k \le a;$$
$$f(w_h) = 6a + 2h + 4 \quad \text{for } 1 \le h \le a;$$
$$f(x_s) = 8a + 2s + 4 \quad \text{for } 1 \le s \le b;$$
$$f(y_r) = 2r + 5 \quad \text{for } 1 \le r \le c.$$

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i + 3$ for $1 \le i \le a$; $uu_j$ is $a + j + 3$ for $1 \le j \le a$; $VV_k$ is $2a + k + 4$ for $1 \le k \le a$; $WW_h$ is $3a+h+4$ for $1 \le h \le a$; $XX_s$ is $4a+s+5$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r+4a + b + c +11}{2}$ for $1 \le r \le c$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case(*e*)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a + b - 1$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$.

Let us take the case that $|b-c| < 4 + 4a$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$. and $c = 4a + b - 1$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup$$
$$\{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$
$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_j : 1 \le j \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\}$$
$$\cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a + b + c + 6$ vertices and $4a + b + c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4\ell + m + n + 6\}$ is defined as follows:

$$f(t) = 1; f(u)=2; f(v)=4; f(w)=6; f(x)=8;$$
$$f(y) = 4a + b + c + 5;$$
$$f(t_i) = 2i+1 \quad \text{for } 1 \le i \le a;$$
$$f(u_j) = 2a + 2j + 1 \quad \text{for } 1 \le j \le a;$$
$$f(v_k) = 4a + 2k + 1 \quad \text{for } 1 \le k \le a;$$
$$f(w_h) = 6a + 2h + 2 \quad \text{for } 1 \le h \le a;$$
$$f(x_s) = 8a + 2s + 1 \quad \text{for } 1 \le s \le b;$$

$f(y_r) = 2r + 8$ for $1 \le r \le c-2$;

$f(y_c) = 4a+b+c+4$;

$f(y_c) = 4a+b+c+6$.

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i+1$ for $1 \le i \le a$; $uu_j$ is $a+j+2$ for $1 \le j \le a$; $VV_k$ is $2a+k+3$ for $1 \le k \le a$; $WW_h$ is $3a+h+4$ for $1 \le h \le a$; $XX_s$ is $4a+s+5$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r+4a+b+c+13}{2}$ for $1 \le r \le c-2$. Also, the edge label of $yy_{c-1}$ is $4a+b+c+5$ and the edge level of $yy_c$ is $4a+b+c+6$

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case(*f*)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a+b-2$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$.

Let us take the case that $|b-c| < 4+4a$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$. and $c = 4a+b-2$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup \{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$

$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_j : 1 \le j \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\} \cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a+b+c+6$ vertices and $4a+b+c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4\ell + m + n + 6\}$ is defined as follows:

$f(t) = 1$; $f(u)=2$; $f(v)=4$; $f(w)=6$; $f(x)=8$;

$f(y) = 4a+b+c+6$;

$f(t_i) = 2i+1$ for $1 \le i \le a$;

$f(u_j) = 2a+2j+1$ for $1 \le j \le a$;

$f(v_k) = 4a+2k+1$ for $1 \le k \le a$;

$f(w_h) = 6a+2h+1$ for $1 \le h \le a$;

$f(x_s) = 8a+2s+1$ for $1 \le s \le b$;

$f(y_r) = 2r+8$ for $1 \le r \le c$.

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i+1$ for $1 \le i \le a$; $uu_j$ is $a+j+2$ for $1 \le j \le a$; $VV_k$ is $2a+k+3$ for $1 \le k \le a$; $WW_h$ is $3a+h+4$ for $1 \le h \le a$; $XX_s$ is $4a+s+5$ for $1 \le s \le b$; and $yy_r$ is $\frac{2r+4a+b+c+14}{2}$ for $1 \le r \le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

**Case($g$)** Consider the graph $G = K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,a} \cup K_{1,b} \cup K_{1,c}$ if $a \le b < c$, where $c = 4a + b - 3$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$.

Let us take the case that $|b - c| < 4 + 4a$ for $a = 2,3,4,\ldots$ and $b = 2,3,4,\ldots$. and $c = 4a + b - 3$.

We have to prove that G is a skolem mean graph.

The vertex and edge set of G is given as follows

$$V(G) = \{t\} \cup \{t_i : 1 \le i \le a\} \cup \{u\} \cup \{u_j : 1 \le j \le a\} \cup \{v\} \cup \{v_k : 1 \le k \le a\} \cup \{w\} \cup \{w_h : 1 \le h \le a\} \cup \{x\} \cup \{x_s : 1 \le s \le b\} \cup \{y\} \cup \{y_r : 1 \le r \le c\}.$$

$$E(G) = \{tt_i : 1 \le i \le a\} \cup \{uu_j : 1 \le j \le a\} \cup \{vv_k : 1 \le k \le a\} \cup \{ww_h : 1 \le h \le a\} \cup \{xx_s : 1 \le s \le b\} \cup \{yy_r : 1 \le r \le c\}.$$

Then G has $4a + b + c + 6$ vertices and $4a + b + c$ edges.

The required vertex labeling $f : V(G) \to \{1, 2, 3, 4, \ldots 4a + b + c + 6\}$ is defined as follows:

$f(t) = 1;\ f(u) = 3;\ f(v) = 5;\ f(w) = 7;\ f(x) = 9;$

$f(y) = 4a + b + c + 6;$

$f(t_i) = 2i$ for $1 \le i \le a$;

$f(u_j) = 2a + 2j$ for $1 \le j \le a$;

$f(v_k) = 4a + 2k$ for $1 \le k \le a$;

$f(w_h) = 6a + 2h$ for $1 \le h \le a$;

$f(x_s) = 8a + 2s$ for $1 \le s \le b$;

$f(y_r) = 2r + 9$ for $1 \le r \le c - 1$ and

$f(y_c) = 4a + b + c + 5$

**The corresponding edge labels are as follows:**

The edge label of $tt_i$ is $i + 1$ for $1 \le i \le a$; $uu_j$ is $a + j + 2$ for $1 \le j \le a$; $VV_k$ is $2a + k + 3$ for $1 \le k \le a$; $WW_h$ is $3a + h + 4$ for $1 \le h \le a$; $XX_s$ is $4a + s + 5$ for $1 \le s \le b$; and $yy_r$ is $\dfrac{2r + 4a + b + c + 15}{2}$ for $1 \le r \le c - 1$. Also the edge label of $yy_c$ is $4a + b + c + 6$.

Hence the induced edge labels of G are distinct.

Hence the graph G is a skolem mean graph.

## 3. APPLICATION OF GRAPH LABELING IN COMMUNICATION NETWORKS

The Graph Theory plays a vital role in various fields. One of the important area is Graph (Skolem mean) Labeling, used in many applications like coding theory, X- ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management. Applications of labeling (Skolem Mean) of graphs extends to heterogeneous fields but here we mainly focus on the communication networks. Communication network is of two types 'Wired Communication' and 'Wireless Communication'. Day by day wireless networks have been developed to ease communication between any two systems, results more efficient communication. To explore the role of labeling in expanding the utility of this channel assignment process in communication networks. Also, graph labeling has been observed and identified its usage towards communication networks. We address how the concept of graph labeling can be applied to network security, network addressing, channel assignment process and social networks.

Network representations play an important role in many domains of computer science, ranging from data structures and graph algorithms, to parallel and communication networks.

Geometric representation of the graph structure imposed on these data sets provides a powerful aid to visualizing and understanding the data.The graph labeling is one of the most widely used labeling methods of graphs. While the labeling of graphs is perceived to be a primarily theoretical subject in the field of graph theory and discrete mathematics, it serves as models in a wide range of applications as listed below.

**The coding theory.**

**The x-ray crystallography.**

**The communication network addressing.**

**Fast Communication in Sensor Networks Using Graph Labeling.**

**Automatic Channel Allocation for Small Wireless Local Area Network.**

**Graph Labeling in Communication Relevant to Adhoc Networks.**

**Effective Communication in Social Networks by Using Graphs.**

**Secure Communication in Graphs.**

## 4. CONCLUSION

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

## 5. ACKNOWLEDGEMENT

One of the authors (Dr. V. Balaji) acknowledges University Grants Commission, SERO and Hyderabad, India for financial assistance (No. F MRP 5766 / 15 (SERO / UGC)).

## 6. REFERENCES

1. J. C. Bermond, **Graceful graphs, radio antennae and French windmills,** Graph theory and Combinatorics, Pitman, London, (1979), 13 – 37.

2. V. Balaji, D. S. T. Ramesh and A. Subramanian**, Skolem Mean Labeling,** Bulletin of Pure and Applied Sciences, vol. 26E No. 2, 2007, 245 – 248.

3. V. Balaji, D. S. T. Ramesh and A. Subramanian,**Some Results on Skolem Mean Graphs**,Bulletin of Pure and Applied Sciences, vol. 27E No. 1, 2008, 67 – 74.

4. V.Balaji, D.S.T. Ramesh and A.Subramanian, **Some Results On Relaxed Skolem Mean Graphs**, Bulletin of Kerala Mathematics Association, Vol.5(2), December 2009, 33-44.

5. V. Balaji, D.S.T. Ramesh and A.Subramanian, **Relaxed Skolem Mean Labeling**, Advances and Applications in Discrete Mathematics, Vol.5 (1), January 2010, 11-22.

6. V. Balaji, **Solution of a Conjecture on Skolem Mean Graph of stars** $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, International Journal of Mathematical Combinatorics, vol.4, 2011, 115 – 117.

7. V. Balaji, D. S. T. Ramesh and V. Maheswari, **Solution of a Conjecture on Skolem Mean Graph of Stars** $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, International Journal of Scientific & Engineering Research, 3(11) 2012, 125 - 128.

8. V. Balaji, D. S. T. Ramesh and V. Maheswari, **Solution of a Conjecture on Skolem Mean Graph of Stars** $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$, Sacred Heart Journal of Science & Humanities, Volume 3, July 2013.

9. V.Balaji, D.S.T. Ramesh and S. Sudhakar, **Further results on relaxed mean labeling**,International Journal of Advances in Applied Mathematics and Mechanics, Vol.3 No.3, 2016, 92-99.

10. V.Balaji, D.S.T. Ramesh and S. Ramarao, **Skolem Mean Labeling For Four Star**, International Research Journal of Pure Algebra, Vol.6 No.1, Jan. – 2016, 221-226.

11. V.Balaji, D.S.T. Ramesh and K. Valarmathy, **On Relaxed Skolem Mean Labeling For Three Star**, International Journal of Mathematical Archive, Vol.7 No.2, Feb. – 2016, 1-7.

12. J. A. Gallian, **A dynamic survey of graph labeling**, The Electronic Journal of Combinatorics, 6 (2010), # DS6.

13. F. Harary, **Graph Theory**, Addison – Wesley, Reading, 1969.

14. V. Maheswari, D. S. T. Ramesh and V. Balaji, **On Skolem MeanLabeling**, Bulletin of Kerala Mathematics Association, Vol. 10, No.1, 2013, 89 – 94.

15. A. Manshath, V. Balaji, P. Sekar and M. Elakkiya, **Further Result on Skolem Mean Labeling For Five Star**, Bulletin of Kerala Mathematics Association (Communicated).

16. S. Somasundaram and R. Ponraj, **Mean labeling of graphs**, National Academy Science letters, 26(2003), 210-213.

17. S. Somasundaram and R. Ponraj, **Non – Existence of mean labeling for a wheel**, Bulletin of Pure and Applied Sciences (Section E: Mathematics & Statistics), 22E (2003), 103 – 111.

18. S. Somasundaram and R. Ponraj, **Some results on mean graphs,** Pure & Applied Mathematika Sciences 58 (2003), 29 - 35.