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DYNAMICAL PROBLEM OF GENERALIZED MAGNETO MICROPOLAR THERMOELASTIC MEDIUM IN HALF SPACE

Ranjit Singh[*] and Varun Kumar[**]

Abstract: A two dimensional plain strain problem in magneto micropolar thermoelastic medium has been studied with the help of integral transformations. The fundamental equations with two relaxation times for generalized magneto micropolar thermoelastic media have been considered by taking heat sources in half space in the absence of body forces. Integral transforms have been used to obtain the solution in transformed domain. A numerical approach has been implemented to invert the obtained solution in the physical domain. Results are presented graphically for a particular model.

Key Words: *Magneto elasticity; thermoelasticity; micropolar elasticity; thermal source; two temperature theory;*

1. INTRODUCTION

Term Micropolar coined by Eringen (1966) is used to describe all materials having an internal structure, contrary to the assumption of classical theory where material is regarded as a continuous medium. So far Classical theory, which is based on Hooke's law was being used to analyze the behaviour of commonly used materials in civil, mechanical and aeronautical etc. engineering fields. This theory is based upon the principal that each point on the continuum possesses three degrees of freedom due to presence of three displacement components in three mutually orthogonal directions. But discrepancies arise in this theory when certain special materials lacking symmetry in their microstructure are modelled using the classic theory. This discrepancy arises due to presence of additional mechanisms which resists deformation. Micropolar theory of elasticity which is part of Solid Mechanics came into existence aftermath of failure of classical theory to explain the behaviour of material possessing internal structure and all such materials with fibrous, polycrystalline materials, coarse grain, polymeric materials, microcracks and microfractures. Also such materials are capable of producing couple stress in addition to usual force stress. Transmission of load across a differential element of the surface of a micropolar elastic solid is described by a couple stress vector along with force vector. As small-scale effects become paramount in the prediction of the overall mechanical behavior of modern day advanced construction materials, importance of Micropolar theory can be greatly felt in the manufacturing and design of these materials.

The theory of thermoelasticity deals with the effect of mechanical and thermal disturbances on an elastic body. This theory was suffering from two drawbacks; Foremost is the fact that the

[*] S.G.A.D. Govt. College, Tarn Taran, Punjab, India, 143401
Email: kalsi_ranjit@yahoo.com,
[**] Lovely Professional University, Phagwara, Punjab, India, 144411
varun.kumar@hotmail.co.in

temperature had no impact on the mechanical state of the elastic body. Second drawback was due to the parabolic nature of the heat equation resulting in to letting infinite speed of thermal wave propagation defying the physical experiments. In 1967 the theory of generalized thermoelasticity [1] with one relaxation time for the special case of an isotropic body was introduced. In this theory, a modified law of heat conduction, including both the heat flux and its time derivative, replaces the conventional Fourier law. The second shortcoming was addressed by introduction of temperature-rate-dependent thermoelastic theories of two relaxation times [2]. It does not violate Fourier's law of heat conduction when the body under consideration has a center of symmetry, and it is valid for both isotropic and anisotropic bodies. The theory of micropolar thermoelasticity has been a subject of intensive study. A comprehensive review of works on the subject is available in texts [3] and [4].

The theories of magneto-elasticity and magneto-thermo-elasticity have been developed to study the interacting effects of an externally applied magnetic field on the elastic and thermo-elastic deformations. These theories are being rapidly developed in recent years because of the possibilities of their extensive practical applications in diverse fields such as geophysics, optics, acoustics, damping of acoustic waves in the magnetic field and so on. This theory can find applications in analysis of propagation of seismic waves from the earth's mantle to its core, It also [5] suggests that the existence of the earth's magnetic field may be taken into consideration for explaining certain phenomena concerning these waves. Basic equations of magneto micro thermoelasticity were derived by Kaliski [6]. Later on, Knopoff [7] attempted to determine the effects of the magnetic field on the propagation of elastic waves on a geophysical scale. In recent years number of authors [8], [9] have contributed to the development of this field. A comparative study between one-temperature theory and two temperature theory in generalized magneto thermoelastic medium in perfectly conducting medium was made by Ezzat and Bary [10] by using state space approach and found that two-temperature generalized theory describes the behavior of the particles of an elastic body more accurately than the one-temperature theory. Ezzat and Awad [11] introduced the theories of modified Ohm's law which also took the temperature gradient and charge density effects in to account with generalized Fourier's law taking current density effect in to considerations by using the linearized equations of micropolar generalized magneto thermoelasticity. Normal mode analysis is used to obtain the solution. He and Cao [12] in the context of L-S theory investigated the problem involving thin slim strip placed in the magneto thermoelastic medium which is subjected to a moving heat source and proved that magnetic field significantly influences the variations of non-dimensional displacement and stress but has no effect on the non-dimensional temperature. Singh and Kumar [13] studied the interaction of electromagnetic field with elastic field in the presence of temperature by applying Mechanical force and thermal source by using modified Fourier and Ohm's law.

In the present study a two dimensional model has been used to analyze the magneto micropolar thermoelastic model with two temperature parameter subjected to concentrated force. Solution has been obtained in frequency domain by employing Laplace and Fourier transform and inversion has been done numerically.

2. FORMULATION OF THE PROBLEM

Considering the region $x_3 \geq 0$ to be occupied with linear homogenous isotropic micropolar thermoelastic medium with two temperatures which is perfectly conducting. We use Cartesian coordinate system (x_1, x_2, x_3) with x_3-axis pointing vertically into the medium. A magnetic field with intensity $\vec{H} = (0, H_0, 0)$ is acting parallel to the boundary plane. Following Nowacki [14] and Lotfy [15], we take linearized equations of electro-dynamics of slowly moving medium as

$$curl\vec{h} = \vec{J} + \dot{\vec{D}},\ curl\vec{E} = -\dot{\vec{B}}, \tag{1}$$

$$\vec{E} = -\mu_0(\dot{\vec{u}} \times \vec{H}_0), \tag{2}$$

$$div\vec{H} = 0, \quad \vec{B} = \mu_0\vec{H}, \tag{3}$$

$$\vec{h} = -H_0(0, e, 0), \tag{4}$$

where $\vec{D} = \epsilon_0\vec{E},\ \ \vec{H} = \vec{H}_0 + \vec{h}$.

The equations of balance of linear as well as angular momentum taking Lorentz force without body couples and taking into account with the electromagnetic couples can be written as follows

$$\sigma_{ji,j} + \in_{ijk} J_j B_k = \rho \ddot{u}_i, \tag{5}$$

$$\in_{ijk} \sigma_{jk} + u_{ji,j} + \in_{ijk} x_j (\in_{klm} J_l B_m) = \rho j \ddot{\phi}_i, \tag{6}$$

$$K^* \nabla^2 \varphi = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \nu T_0 \nabla.\vec{u}), \tag{7}$$

Following Youssef [16] relation between the heat conduction and the dynamical heat with $a > 0$ as two-temperature can be written as

$$\varphi - T = a\nabla^2\varphi, \tag{8}$$

Constitutive relations are

$$\sigma_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + \kappa(u_{j,i} - \epsilon_{ijr}\phi_r) - \nu\left(1 + \tau_1 \frac{\partial}{\partial t}\right)T\delta_{ij}, \tag{9}$$

$$\mu_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{10}$$

3. SOLUTION OF THE PROBLEM

As we are considering a two dimensional plain strain problem, we take

$$\vec{u} = (u_1, 0, u_3), \vec{\phi} = (0, \phi, 0), \vec{h} = (0, h, 0) \tag{11}$$

Introducing the displacement potential functions $\psi(x_1, x_3, t)$ and $\vec{\zeta}(x_1, x_3, t)$ and taking

$$\vec{u} = \nabla\psi + \nabla \times \vec{\zeta}, \nabla.\vec{\zeta} = 0, \vec{\zeta} = (o, \zeta, 0) \tag{12}$$

For convenience following dimensionless quantities have been used

$$(x_1', x_3', u_1', u_3') = c_0\eta(x_1, x_3, u_1, u_3), \qquad (t', \tau_0', \tau_1') = c_0^2\eta(t, \tau_0, \tau_1), \qquad \sigma_{ij}' = \frac{\sigma_{ij}}{\lambda + 2\mu + \kappa},$$

$$\mu_{ij}' = \frac{\kappa}{c_0\eta(\mu+\kappa)}\mu_{ij}, \qquad J_i' = \frac{\eta}{\sigma_0^2\mu_0^2 c_0 H_0}J_i, \qquad \phi' = \frac{\kappa}{\mu+\kappa}\phi, \tag{13}$$

$$\mathbf{h}' = \frac{\boldsymbol{\eta}}{\boldsymbol{\sigma_0\mu_0}\mathbf{H_0}}\mathbf{h}, \qquad (\boldsymbol{\varphi}', \boldsymbol{\theta}') = \frac{(\boldsymbol{\varphi}, \mathbf{T}) - \mathbf{T_0}}{\mathbf{T_0}}, \qquad \mathbf{E_i'} = \frac{\boldsymbol{\eta}}{\boldsymbol{\sigma_0\mu_0^2}\mathbf{c_0H_0}}\mathbf{E_i}.$$

Where

$$\eta = \frac{\rho C_E}{K^*}, \qquad c_0^2 = \frac{\lambda + 2\mu + \kappa}{\rho}.$$

After some simplification and using (11)-(13) in equations in equations (1)-(10), we obtain (dropping dashes for convenience)

$$a_1\nabla^2\psi - a_2\ddot{\psi} - a_3\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta - a_4\ddot{\psi} = 0, \tag{14}$$

$$a_5\nabla^2\zeta - a_5\phi - a_2\ddot{\zeta} - a_4\ddot{\zeta} = 0, \tag{15}$$

$$\nabla^2\phi - a_6\nabla^2\zeta - a_7\phi - a_8\ddot{\phi} = 0, \tag{16}$$

$$\nabla^2\varphi - a_9\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\theta - a_{10}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{17}$$

$$\varphi - \theta = a_{11}\nabla^2\varphi, \tag{18}$$

$$\sigma_{33} = a_{12}\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta, \tag{19}$$

$$\sigma_{11} = \frac{\partial u_1}{\partial x_1} + a_{12}\frac{\partial u_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1\frac{\partial}{\partial t}\right)\theta, \tag{20}$$

$$\sigma_{31} = a_{13}\frac{\partial u_3}{\partial x_1} + a_{14}\frac{\partial u_1}{\partial x_3} + a_{14}\phi, \tag{21}$$

$$m_{32} = \gamma\frac{\partial\phi}{\partial x_3}, \tag{22}$$

Where

$$\boldsymbol{\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \quad a_1 = \lambda + 2\mu + \kappa, \quad a_2 = c_0^2 \epsilon_0 \mu_0 H_0^2, \quad a_3 = \nu T_0, \quad a_4 = \rho c_0^2, \quad a_5 = \mu + \kappa,}$$

$$\boldsymbol{a_6 = \frac{\kappa^2}{\gamma \eta^2 c_0^2 (\mu + \kappa)}, \quad a_7 = \frac{2\kappa}{c_0^2 \eta^2 \gamma}, \quad a_8 = \frac{\rho j c_0^2 \eta}{\gamma}, \quad a_9 = \frac{\rho C_E}{K^* \eta}, \quad a_{10} = \frac{\nu}{K^* \eta}, \quad a_{11} = a c_0^2 \eta^2,}$$

$$\boldsymbol{a_{12} = \frac{\lambda}{\lambda + 2\mu + \kappa}, \quad a_{13} = \frac{\mu}{\lambda + 2\mu + \kappa}, \quad a_{14} = \frac{\mu + \kappa}{\lambda + 2\mu + \kappa}, \quad a_{15} = \frac{\eta}{\sigma_0 \mu_0}, \quad a_{16} = \frac{\eta}{\sigma_0 \mu_0 c_0},}$$

$$a_{17} = \frac{c_0^2 \varepsilon_0 \eta^2}{\sigma_0^2 \mu_0} \tag{23}$$

We define the Laplace and Fourier transforms as

$$L\{f(x_1, x_3, t)\} = \int_0^\infty e^{-st} f(x_1, x_3, t) dt = \bar{f}(x_1, x_3, s), \tag{23}$$

$$F\{\bar{f}(x_1, x_3, s)\} = \int_{-\infty}^\infty e^{-\iota \xi x_1} \bar{f}(x_1, x_3, s) dx_1 = \tilde{f}(\xi, x_3, s), \tag{25}$$

Equations (14)- (24) after application of transformations defined in Error! Reference source not found.***-(23) give***

$$(D^2 - \alpha_1)\tilde{\psi} - \alpha_2 \tilde{\theta} = 0, \tag{25}$$

$$(D^2 - \alpha_3)\tilde{\zeta} - \tilde{\phi} = 0, \tag{26}$$

$$(D^2 - \alpha_4)\tilde{\psi} - a_6 (D^2 - \xi^2)\tilde{\zeta} = 0, \tag{27}$$

$$(D^2 - \xi^2)\tilde{\varphi} - \alpha_5 \tilde{\theta} - \alpha_6 (D^2 - \xi^2)\tilde{\psi} = 0, \tag{28}$$

$$a_{11}(D^2 - \alpha_7)\tilde{\varphi} + \tilde{\theta} = 0, \tag{29}$$

$$\tilde{\sigma}_{33} = a_{12} \frac{\partial \tilde{u}_1}{\partial x_1} + \frac{\partial \tilde{u}_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\tilde{\theta}, \tag{30}$$

$$\tilde{\sigma}_{11} = \frac{\partial \tilde{u}_1}{\partial x_1} + a_{12} \frac{\partial \tilde{u}_3}{\partial x_3} - \frac{a_3}{a_1}\left(1 + \tau_1 \frac{\partial}{\partial t}\right)\tilde{\theta}, \tag{31}$$

$$\tilde{\sigma}_{31} = a_{13}\frac{\partial \tilde{u}_3}{\partial x_1} + a_{14}\frac{\partial \tilde{u}_1}{\partial x_3} + a_{14}\tilde{\phi}, \tag{32}$$

$$\tilde{m}_{32} = \gamma \frac{\partial \tilde{\phi}}{\partial x_3}, \tag{33}$$

Equations (25)-(29) can be re-written as

$$(D^4 - \alpha_{13}D^2 + \alpha_{14})(\tilde{\zeta}, \tilde{\phi}) = 0, \tag{34}$$

$$(D^4 - \alpha_{15}D^2 + \alpha_{16})(\tilde{\psi}, \tilde{\theta}, \tilde{\varphi}) = 0, \tag{35}$$

Where

$$\mathbf{D} = \frac{\mathbf{d}}{\mathbf{dz}}, \quad \boldsymbol{\alpha_1} = \boldsymbol{\xi}^2 + \frac{\mathbf{a_2} + \mathbf{a_4}}{\mathbf{a_1}}\mathbf{s}^2, \quad \boldsymbol{\alpha_2} = \frac{\mathbf{a_3}}{\mathbf{a_1}}(\mathbf{1} + \boldsymbol{\tau_1}\mathbf{s}), \quad \boldsymbol{\alpha_3} = \boldsymbol{\xi}^2 + \frac{\mathbf{a_2} + \mathbf{a_4}}{\mathbf{a_5}}\mathbf{s}^2,$$

$$\boldsymbol{\alpha_4} = \boldsymbol{\xi}^2 + \mathbf{a_7} + \mathbf{a_8}\mathbf{s}^2, \quad \boldsymbol{\alpha_5} = \mathbf{a_9}(\mathbf{s} + \boldsymbol{\tau_0}\mathbf{s}^2), \quad \boldsymbol{\alpha_6} = \mathbf{a_{10}}(\mathbf{s} + \boldsymbol{\tau_0}\mathbf{s}^2), \quad \boldsymbol{\alpha_7} = \boldsymbol{\xi}^2 + \frac{\mathbf{1}}{\mathbf{a_{11}}},$$

$$\boldsymbol{\alpha_8} = \mathbf{1} + \mathbf{a_{11}}\boldsymbol{\alpha_5}, \quad \boldsymbol{\alpha_9} = \boldsymbol{\xi}^2 + \mathbf{a_{11}}\boldsymbol{\alpha_5}\boldsymbol{\alpha_7}, \quad \boldsymbol{\alpha_{10}} = \mathbf{a_{11}}\boldsymbol{\alpha_2}\boldsymbol{\alpha_6} + \boldsymbol{\alpha_2},$$

$$\boldsymbol{\alpha_{11}} = \mathbf{a_{11}}\boldsymbol{\alpha_2}\boldsymbol{\alpha_6}(\boldsymbol{\xi}^2 + \boldsymbol{\alpha_7}) + \boldsymbol{\alpha_1}\boldsymbol{\alpha_2} + \boldsymbol{\alpha_9}, \quad \boldsymbol{\alpha_{12}} = \mathbf{a_{11}}\boldsymbol{\alpha_2}\boldsymbol{\alpha_6}\boldsymbol{\alpha_7}\boldsymbol{\xi}^2 + \boldsymbol{\alpha_9}\boldsymbol{\alpha_{11}},$$

$$\alpha_{13} = \alpha_3 + \alpha_4 + a_6, \quad \alpha_{14} = \alpha_3\alpha_4 + a_6\xi^2, \quad \alpha_{15} = \frac{\alpha_{11}}{\alpha_{10}}, \quad \alpha_{16} = \frac{\alpha_{12}}{\alpha_{10}}.$$

Solutions of equation (34)-(35) satisfying the radiation condition $Re(m_j) \geq 0$ are of the form,

$$\tilde{\zeta}(\xi, x_3, s) = A_1(\xi, s)e^{-m_1 x_3} + A_2(\xi, s)e^{-m_2 x_3}, \tag{36}$$

$$\tilde{\psi}(\xi, x_3, s) = A_3(\xi, s)e^{-m_3 x_3} + A_4(\xi, s)e^{-m_4 x_3}, \tag{37}$$

Where

$$m_1^2 = \frac{\alpha_{13} + \sqrt{\alpha_{13}^2 - 4\alpha_{14}}}{2}, m_2^2 = \frac{\alpha_{13} - \sqrt{\alpha_{13}^2 - 4\alpha_{14}}}{2},$$

$$m_3^2 = \frac{\alpha_{15} + \sqrt{\alpha_{15}^2 - 4\alpha_{16}}}{2}, m_4^2 = \frac{\alpha_{15} - \sqrt{\alpha_{15}^2 - 4\alpha_{16}}}{2},$$

Using (36)-(37) in equations (25)-(33), we obtain

$$\tilde{\phi}(\xi, x_3, s) = (m_1^2 - \alpha_3)A_1 e^{-m_1 x_3} + (m_2^2 - \alpha_3)A_2 e^{-m_2 x_3}, \tag{38}$$

$$\tilde{\theta}(\xi, x_3, s) = \frac{1}{\alpha_2}[(m_3^2 - \alpha_1)A_3 e^{-m_3 x_3} + (m_4^2 - \alpha_1)A_4 e^{-m_4 x_3}], \tag{39}$$

$$\tilde{\varphi}(\xi, x_3, s) = -\frac{1}{a_{11}\alpha_2}\left[\frac{(m_3^2-\alpha_1)}{(m_3^2-\alpha_7)}A_3 e^{-m_3 x_3} + \frac{(m_4^2-\alpha_1)}{(m_4^2-\alpha_7)}A_4 e^{-m_4 x_3}\right], \tag{40}$$

$$\tilde{u}_1 = (m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) + \iota\xi(A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3}), \tag{41}$$

$$\tilde{u}_3 = \iota\xi(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}) - (m_3 A_3 e^{-m_3 x_3} + m_4 A_4 e^{-m_4 x_3}), \tag{42}$$

$$\tilde{\sigma}_{33} = -\iota\xi(1 - a_{12})(m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) + (\alpha_1 - \xi^2 a_{12})(A_3 e^{-m_3 x_3} + A_4 e^{-m_4 x_3}), \tag{43}$$

$$\begin{aligned}\tilde{\sigma}_{11} = \iota\xi(1 - a_{12})(m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}) + (\alpha_1 - \xi^2 - m_3^2 + a_{12} m_3^2)A_3 e^{-m_3 x_3} \\ + (\alpha_1 - \xi^2 - m_4^2 + a_{12} m_4^2)A_4 e^{-m_4 x_3},\end{aligned} \tag{44}$$

$$\tilde{\sigma}_{31} = -(\xi^2 a_{13} + a_{14}\alpha_3)(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}) - \iota\xi(a_{13} + a_{14})(m_3 A_3 e^{-m_3 x_3} + m_4 A_4 e^{-m_4 x_3}), \tag{45}$$

$$\tilde{m}_{32} = -\gamma[m_1(m_1^2 - \alpha_3)A_1 e^{-m_1 x_3} + m_2(m_2^2 - \alpha_3)A_2 e^{-m_2 x_3}], \tag{46}$$

4. APPLICATION

When the plane boundary is stress free and subjected to an instantaneous thermal point source, the boundary conditions at the plane $x_3 = 0$ are:

$$\theta(x_1, 0, t) = \theta_0 \delta(x_1)\delta(t), \tag{47}$$

$$\sigma_{33}(x_1, 0, t) = 0, \tag{48}$$

$$\sigma_{31}(x_1, 0, t) = 0, \tag{49}$$

$$m_{32}(x_1, 0, t) = 0. \tag{50}$$

where θ_0 is the maximum constant temperature applied on the boundary.

After making use of transformations as defined in **Error! Reference source not found.**-(23) on equations (47)-(50), and using equations (39), (43), (45) and (46), we obtain

$$\frac{1}{\alpha_2}[(m_3^2-\alpha_1)A_3+(m_4^2-\alpha_1)A_4]=\tilde{\theta}_0, \tag{51}$$

$$-\iota\xi(1-a_{12})[m_1A_1+m_2A_2]+(\alpha_1-\xi^2a_{12})[A_3+A_4]=0, \tag{52}$$

$$-(\xi^2a_{13}+\alpha_3a_{14})[A_1+A_2]-\iota\xi(a_{14}+a_{13})[m_3A_3+m_4A_4]=0, \tag{53}$$

$$m_1(m_1^2-\alpha_3)A_1+m_2(m_2^2-\alpha_3)A_2=0. \tag{54}$$

Equations (51)-(54), after simplification give,

$$A_1=\frac{\iota\alpha_2(\alpha_1-\xi^2a_{12})(m_2^2-\alpha_3)(\Delta_1+\Delta_2)}{\xi m_1(1-a_{12})(m_1^2-m_2^2)[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{55}$$

$$A_2=-\frac{\iota\alpha_2(\alpha_1-\xi^2a_{12})(m_1^2-\alpha_3)(\Delta_1+\Delta_2)}{\xi m_2(1-a_{12})(m_1^2-m_2^2)[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{56}$$

$$A_3=\frac{\alpha_2\Delta_2}{[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0, \tag{57}$$

$$A_4=\frac{\alpha_2\Delta_1}{[(m_3^2-\alpha_1)\Delta_2+(m_4^2-\alpha_1)\Delta_1]}\tilde{\theta}_0. \tag{58}$$

Where

$\Delta_1=[\alpha_{17}(m_2^2-\alpha_3)-\alpha_{18}m_1m_3(m_1^2-m_2^2)],$

$\Delta_2=[\alpha_{17}(m_1^2-\alpha_3)+\alpha_{18}m_2m_4(m_1^2-m_2^2)],$

$$\boldsymbol{\alpha_{17}=-\frac{\iota(\xi^2a_{13}+\alpha_3a_{14})(\alpha_1-\xi^2a_{12})}{\xi(1-a_{12})}},\qquad \boldsymbol{\alpha_{18}=\iota\xi(a_{14}+a_{13})}.$$

5. INVERSION OF THE TRANSFORMS

The transformed components of displacement, microrotation, tangential and normal stress, couple stress, induced electric field and magnetic field are dependent on ξ, x_3 and s. To obtain them in the physical domain in the form of $f(r,x_3,t)$, we invert integral transforms by using the inversion technique as used by Singh et al [13].

6. NUMERICAL DISCUSSION AND ANALYSIS

The analysis is conducted for a magnesium crystal. Using the physical parameters as given in reference [17] are taken as

$\lambda = 9.4\times10^{10}\,Nm^{-2},\ \mu = 4.0\times10^{10}\,Nm^{-2},\ \kappa = 1.0\times10^{10}\,Nm^{-2}, \rho = 1.74\times10^{3}\ Kg/m^{3},$
$J = 0.2\times10^{-19}m^{2},\ \varepsilon_0 = 10^{-9}/36\pi\ col^{2}/Nm^{2},\ \beta = 0.98\times10^{-5}\ N/m^{2},$
$\sigma_0 = 2.2356\times10^{7}/col^{2}/J\ m\ \sec, \mu_0 = 4\pi\times10^{-7}N\sec^{2}/col^{2},$
$H_o = 10^{7}col/m\ \sec,\ \eta = 0.0168, K^{*} = 386Wm^{-1}K^{-1},\ C_E = 383.1JKg^{-1}K^{-1},\ T_0 = 293K, \theta_0 = 1.$

Graphical analysis of variation in temperature distribution, displacement, normal stress and tangential couple stress has been done. Results have been compared in the absence and presence of magnetic field for two different t values. In the following figures 1-4, the solid line represents the magneto micropolar thermoelastic medium (MMT1) at t=0.1; small dashed line magneto micropolar thermoelastic medium (MMT2) at t=0.5; solid line with circles micropolar thermoelastic medium (MT1) at t=0.1 and small dashed line with circles represents micropolar thermoelastic medium (MT2) at t=0.5 under the application of thermal source.

Fig.1 shows the variation in normal displacement component (u_3)with changes in value of x_1.It is observed that the presence of magnetic field leads to higher values of u_3 as compared to its values in the absence of magnetic field. Also its nature is oscillatory for MT1 and MT2 and amplitude keeps on decreasing with increase in x_1 .

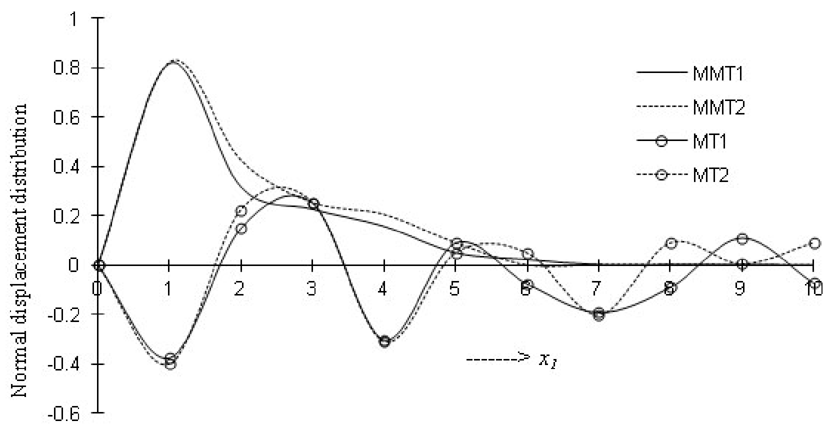

Figure 1. Variation of normal displacement

Fig.2 shows vartions in temperature distribution (θ) with x_1. Initially starting with same values for both mediums MMT and MT, value of θ has higher amplitude in the the absence of magnetic effect. For large values of x_1 ($4 \le x_1 \le 5$), its values start coinciding for both theories and tend to zero.

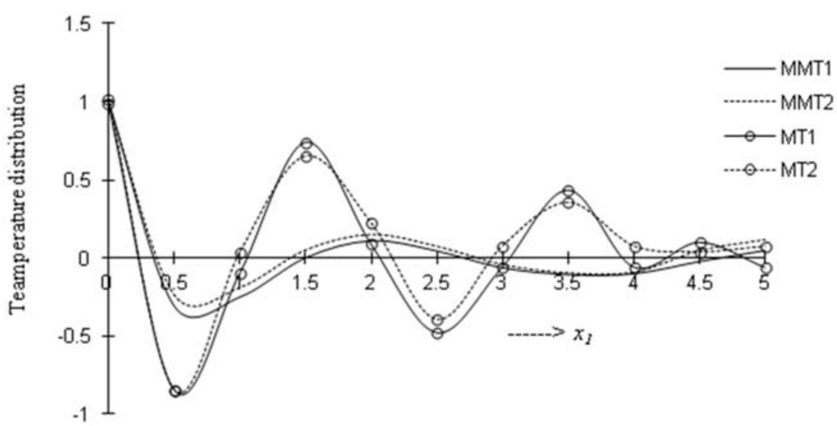

Figure 2. Variation of temperature distribution

Variation in Normal force stress (σ_{33}) is being shown in fig.3. Here it is observed that σ_{33} behaves in opposite manner under both theories for range $0 \leq x_1 < 9$ but this difference tends to cease in the range $x_1 \geq 9$.

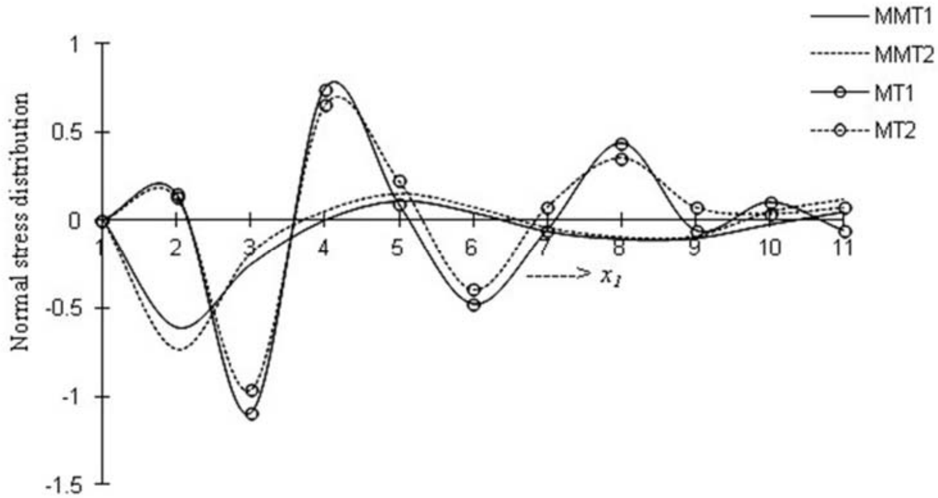

Figure 3. Variation of Normal stress distribution

Finally in the fig.4 behaviour of tangential couple stress (σ_{31}) is depicted. Values have been plotted after multiplying it with 10. σ_{31} again shows oscillatory nature in the absence of magnetic field (MT1 and MT2) but for large values of x_1, this behaviour tends to diminish and shows linear nature.

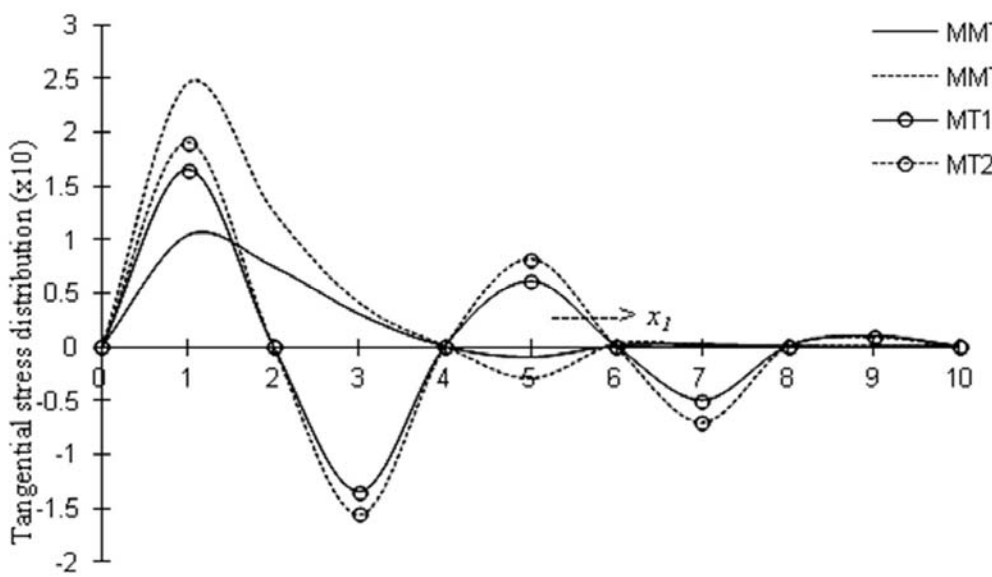

Figure 4. Variation of tangential stress distribution

6. CONCLUSION

This study highlights a simplified technique of obtaining the stress and strain components in the case of micropolar isotropic solid subjected to thermal field in the presence of magnetic field. The trend of variations of the considered components are different in the presence and absence of magnetic field which confirms that magnetic field has significant impact on the normal displacement component u_3, temperature distribution θ, normal force stress σ_{33} and tangential couple stress σ_{31} along with application of thermal source. This study can be useful in analysing stress-strain behaviour of earth like model which is subjected to both thermal and magnetic fields.

NOMENCLATURE

$\vec{h}$	induced magnetic field,
$\vec{E}$	induced electric field,
$\vec{J}$	current density vector,
μ_0	magnetic permeability,
ϵ_0	electric permeability,
$\vec{D}$	electric displacement vector,
e	cubical dilatation,
λ, μ	Lame's constants,
u_i	components of displacement vector,
t	time,
σ_{ij}	components of stress tensor,
$\vec{\phi}$	rotation vector,
j	micro inertia,
$\alpha, \beta, \gamma, \kappa$	micropolar constants,
δ_{ij}	Kronecker delta,
ϵ_{ijk}	alternating tensor,
ρ	mass density,
C_E	specific heat at constant strain,
K^*	thermal conductivity,
τ_0, τ_1	relaxation times,
θ	$\mathrm{T} - \mathrm{T}_0$,
T	temperature,
T_0	$\left\lvert\frac{\mathrm{T}-\mathrm{T}_0}{\mathrm{T}_0}\right\rvert$.

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