IJCTA, 9(20), 2016, pp. 149-158

Stability Result of Iterative Procedure in Normed Space

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ABSTRACT

The intent of this paper is to study the stability of Jungck-Noor iteration schemes for maps satisfying a general contractive condition in normed space. Our result contains some of the results of Berinde [2-3], [5], Bosede and Rhoades [6], Bosede [7], Imoru and Olatinwo [12], Olatinwo et al. [18].

Keywords: Jungck-Mann iteration, Jungck-Noor iteration, Stability of iterations, Fixed point iteration, Stability results in normed space, (*S*, *T*) stability.

1. INTRODUCTION AND PRILIMINIRIES

Let (X, d) be a complete metric space and $T: X \to X$. Let $\{x_n\}_{n=0}^{\infty} \subset X$ be the sequence generated by iteration procedure involving the operator *T*, if

$$x_{n+1} = f(T, x_n) = Tx_n, n = 0, 1, \ldots \quad (1.1)$$

then it is called Picard iteration process. The Picard iteration can be used to approximate the unique fixed point for strict type contractive operator. There was a need of some other iterative procedures for slightly weaker contractive conditions.

If for $x_0 \in X$, the sequence $\{x_n\}_{n=0}^{\infty}$ is defined by,

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n, n = 0, 1, \ldots \quad (1.2)$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0,1]$ is called Mann iteration process [16].

And

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tz_n,$$

if

$$z_n = (1-\beta_n)x_n + \beta_n Tx_n, n = 0, 1, \ldots \quad (1.3)$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are the real sequences in [0, 1], then it is called Ishikawa iteration process [13].

The sequence is defined by,

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n,$$

$$y_n = (1-\beta_n)x_n + \beta_n Tz_n,$$

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$$z_n = (1-\gamma_n)x_n + \gamma_n Tx_n, \ n = 0, 1, \ldots \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are the real sequences in [0, 1], then it is called Jungck-Noor iterative scheme.

On putting $\{\alpha_n\} = 1$ in (1.2), it becomes Picard iterative process. Similarly, if $\beta_n = 0$ for each '*n*' in (1.3), then it reduces to (1.2). If we put $\gamma_n = 0$ for each '*n*' in (1.4), then it becomes (1.3).

Definition 1.1 [14]. Let Y be an arbitrary non empty set and (X, d) be a metric space. Let $S, T : Y \to X$ and $T(Y) \subset S(Y)$ for some $x_0 \in Y$, consider

$$Sx_{n+1} = Tx_n, \ n = 0, 1, 2\ldots \tag{1.5}$$

If

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n, \ n = 0, 1, 2\ldots, \tag{1.6}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ is a sequence in [0, 1], then it is called Junck-Mann iteration process [36].

Olatinwo and Imoru [19] defined $\{Sx_n\}_{n=0}^{\infty}$ as

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Tz_n, \\ Sz_n &= (1-\beta_n)Sx_n + \beta_n Tx_n, n = 0, 1, \ldots \end{aligned} \tag{1.7}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are the real sequences in [0, 1], this scheme is called Jungck-Ishikawa iteration.

Further, Olatinwo [20] defined $\{Sx_n\}_{n=0}^{\infty}$ for three step iteration procedure as follows.

Definition 1.2 [20]. Let $S, T : T \to X$ and $T(X) \subseteq S(X)$. Define

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Tz_n, \\ Sz_n &= (1-\beta_n)Sx_n + \beta_n Tr_n, \\ Sr_n &= (1-\gamma_n)Sx_n + \gamma_n Tx_n \end{aligned} \tag{1.8}$$

where $n = 0, 1, \ldots$ and $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ satisfy

(i) $\alpha_0 = 1$

(ii) $0 \le \alpha_n, \beta_n, \gamma_n \le 1, n > 0$

(iii) $\sum \alpha_n = \infty$

(iv) $\sum_{j=0}^{n} \alpha_j \prod_{i=j+1}^{n} (1 - \alpha_i + a\alpha_i)$ converges.

This is called Jungck-Noor iteration scheme [20].

The first result on the stability is due to Ostrowoski [22]. However Harder and Hick [10-11] defined *T*-stability as follows:

Definition 1.3 [10-11]. The iterative procedure $x_{n+1} = f(T, x_n)$ is said to be *T*-stable with respect to *T* if $\{x_n\}$ converges to a fixed point *q* of *T* and whenever $\{y_n\}$ is a sequence in *X* with $\lim_{n\to\infty} d(y_{n+1}, f(T, y_n)) = 0$, we have $\lim_{n\to\infty} y_n = q$.

The (*S*, *T*) stability mapping is defined by Singh et al. [36] in the following manner.

Definition 1.4 [36]. Let $S, T : Y \to X$, $T(Y) \subset S(Y)$ and "*z*" a coincidence point of *T* and *S* that is $Sz = Tz = p$ (say), for any $x_0 \in Y$, let the sequence $\{Sx_n\}$, generated by iterative procedure (1.4), converges to '*p*'. Let $\{Sy_n\} \subset X$ be an arbitrary sequence, and set $\varepsilon_n = d(Sy_{n+1}, f(T, y_n))$, $n = 0, 1, 2...$ then the iterative procedure $f(T, x_n)$ will be called (S, T) stable if and only if $\lim_{n\to\infty} \varepsilon_n = 0 \Rightarrow \lim_{n\to\infty} Sy_n = p$.

Harder and Hick [10-11] obtained stability results for Zamfirescu operator (*Z*-operator) for Picard and Mann iterative procedures.

Suppose *X* is a Banach space and *Y* a nonempty set such that $T(Y) \subseteq S(Y)$. Then $S, T : Y \to X$ is called Zamfirescu operator if for $x, y \in Y$ and $h \in (0, 1)$,

$$\|Tx - Ty\| \le h \max\{\|Sx - Sy\|, \frac{\|Sx - Tx\| + \|Sy - Ty\|}{2}, \frac{\|Sx - Ty\| + \|Sy - Tx\|}{2}\}. \tag{1.9}$$

Rhoades [34-35] obtained fixed point results for Mann and Ishikawa iteration procedures in uniformly Banach space. Berinde [4] used these iterative procedures for approximating the fixed point of *Z*-operator in arbitrary Banach space. Several authors used *Z*-operator for different iterative procedures in the setting of different spaces. Motivated by rich literature of *Z*-operator, Osilike [21] established stability results for Picard, Mann and Ishikawa iterative procedures for a large class of mappings and introduced the following contractive condition.

$$\|Tx - Ty\| \le \delta \|Sx - Sy\| + L\|Sx - Tx\|, \ L > 0, \ 0 < \delta < 1. \tag{1.10}$$

It cn be seen that $(1.9) \Rightarrow (1.10)$. It can be understood it better by taking cases one by one.

Case I: On putting $\delta = h$ and $L = 0$ in (1.10), we get first part.

Case II: $\delta = \dfrac{h}{2-h}$ and $L = \dfrac{2h}{2-h}$ gives second part of (1.9).

Case III: $\|Tx - Ty\| \le h \dfrac{\|Sx - Ty\| + \|Sy - Tx\|}{2} \le \|Sy - Tx\| \le h\|Sy - Sx\| + h\|Sx - Tx\|$.

Hence $\delta = h$ and $L = h$ completes the proof.

Olantinwo [20] generalized the above contractive condition as follows.

$$\|Tx - Ty\| \le \delta \|Sx - Sy\| + \psi(\|Sx - Tx\|), \ 0 \le \delta < 1, \tag{1.11}$$

where $\psi : R_+ \to R_+$ is a monotone decreasing sequence with $\psi(0) = 0$. If we take $\psi(u) = Lu$ in (1.11), we get (1.10) which shows $(1.10) \Rightarrow (1.11)$. We see $(1.9) \Rightarrow (1.10) \Rightarrow (1.11)$. Thus the theory of stability of fixed point iteration has been widely studied in the literature and interesting fixed point results are obtained by a number of authors in various settings, see for instance [1-6], [10-12], [18-32] and several reference thereof.

The following lemma of Berinde [4] is required for the sequel.

Lemma 1.1 [4]. If δ is a real number such that $0 \le \delta < 1$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ is a sequence of positive number such that $\lim_{n\to\infty} \varepsilon_n = 0,$ then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1} \le \delta u_n + \varepsilon_n, n = 0,\ 1,\ 2...,$ we have $\lim_{n\to\infty} u_n = 0.$

2. MAIN RESULT

Theorem 2.1. Let $(X, \|\cdot\|)$ be a normed space and $S,\ T : Y \to X$ be non-self maps on an arbitrary set Y such that $T(Y) \subset S(Y),$ where $S(Y)$ is a complete subspace of X and S an injective operator. Let z be a coincidence point of S and T i.e; $Sz = Tz = p$ (say). Suppose S and T satisfy,

$$\|Tx - Ty\|) \le \psi(\|Sx - Tx\|) + \delta d\,(Sx, Sy),\ \ \delta \in [0,1),\ \ \psi(0) = 0. \tag{2.1}$$

for $x_0 \in Y.$ Let $\{Sx_n\}_{n=0}^{\infty}$ be Jungck-Noor iterative scheme (1.8) converging to p, where $\{\alpha_n\},\{\beta_n\},\{\gamma_n\}$ are sequences of positive number in [0, 1] with $\{\alpha_n\}$ satisfying $0 < \alpha \le \alpha_n\ \ \forall n.$ Then the Jungck-Noor iterative scheme is (S, T) stable.

Proof. Suppose that $\{Sy_n\}_{n=0}^{\infty} \subset X,\quad \varepsilon_n = \|Sy_{n+1} - (1-\alpha_n)Sy_n - \alpha_n Ts_n\|, n = 0,\ 1,\ 2,\ 3..,$

where

$$Ss_n = (1-\beta_n)Sy_n + \beta_n Tq_n,$$
$$Sq_n = (1-\gamma_n)Sy_n + \gamma_n Ty_n.$$

and let $\lim_{n\to\infty} \varepsilon_n = 0.$ Then it follows from (1.8) and (2.1) that

$$\begin{aligned}
\|Sy_{n+1} - p\| &\le \|Sy_{n+1} - (1-\alpha_n)Sy_n - \alpha_n Ts_n\| + \|(1-\alpha_n)Sy_n + \alpha_n Ts_n - (1-\alpha_n + \alpha_n)p\| \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \alpha_n\|Ts_n - p\| \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \alpha_n\|Ts_n - Tz\| \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \alpha_n[\delta\|Sz - Ss_n\| + \psi(\|Sz - Tz\|)] \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \delta\alpha_n\|p - Ss_n\| + \alpha_n\psi(0) \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \delta\alpha_n\|p - Ss_n\| + \alpha_n.0 \\
&\le \varepsilon_n + (1-\alpha_n)\|Sy_n - p\| + \delta\alpha_n\|p - Ss_n\|.
\end{aligned} \tag{2.2}$$

Now we have the following equation

$$\begin{aligned}
\|p - Ss_n\| &= \|(1-\beta_n + \beta_n)p - (1-\beta_n)Sy_n - \beta_n Tq_n\| \\
&\le (1-\beta_n)\|p - Sy_n\| + \beta_n\|p - Tq_n\| \\
&\le (1-\beta_n)\|p - Sy_n\| + \beta_n\|Tz - Tq_n\| \\
&\le (1-\beta_n)\|p - Sy_n\| + \beta_n[\delta\|Sz - Sq_n\| + \psi(\|Sz - Tz\|)] \\
&\le (1-\beta_n)\|p - Sy_n\| + \delta\beta_n\|Sz - Sq_n\| + \beta_n\psi(0)
\end{aligned}$$

$$\leq (1-\beta_n)\|p - Sy_n\| + \delta\beta_n \|Sz - Sq_n\| + \beta_n.0$$
$$\leq (1-\beta_n)\|p - Sy_n\| + \delta\beta_n \|p - Sq_n\|. \tag{2.3}$$

Also we have

$$\|p - Sq_n\| = \|(1-\gamma_n + \gamma_n)p - (1-\gamma_n)Sy_n - \gamma_n Ty_n\|$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \gamma_n \|p - Ty_n\|$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \gamma_n \|Tz - Ty_n\|$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \gamma_n[\delta\|Sz - Sy_n\| + \psi(\|Sz - Tz\|)]$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \delta\gamma_n \|Sz - Sy_n\| + \gamma_n\psi(0)$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \delta\gamma_n \|Sz - Sy_n\| + \gamma_n.0$$
$$\leq (1-\gamma_n)\|p - Sy_n\| + \delta\gamma_n \|p - Sy_n\|. \tag{2.4}$$

It follows from (2.2), (2.3) and (2.4) that

$$\|Sy_{n+1} - p\| \leq \varepsilon_n + [1-\alpha_n + \delta\alpha_n\{1-\beta_n + \delta\beta_n(1-\gamma_n + \delta\gamma_n)\}]\|p - Sy_n\|. \tag{2.5}$$

Using $0 < \alpha \leq \alpha_n$ and $\delta \in [0, 1)$, we have

$$[1-\alpha_n + \delta\alpha_n\{1-\beta_n + \delta\beta_n(1-\gamma_n + \delta\gamma_n)\}] < 1.$$

Hence, using lemma (2.1), (2.5) yields $\lim_{n\to\infty} Sy_{n+1} = p.$

Conversely, let $\lim_{n\to\infty} Sy_{n+1} = p.$ Then using contractive condition (2.1) and triangle inequality, we have

$$\varepsilon_n = \|Sy_{n+1} - (1-\alpha_n)Sy_n - \alpha_n Ts_n\|$$
$$\leq \|Sy_{n+1} - p\| + \|(1-\alpha_n + \alpha_n)p - (1-\alpha_n)Sy_n - \alpha_n Ts_n\|$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \alpha_n \|p - Ts_n\|$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \alpha_n \|Tz - Ts_n\|$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \alpha_n[\delta\|Sz - Ss_n\| + \psi(\|Sz - Tz\|)]$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \delta\alpha_n \|Sz - Ss_n\| + \alpha_n\psi(0)$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \delta\alpha_n \|Sz - Ss_n\| + \alpha_n.0$$
$$\leq \|Sy_{n+1} - p\| + (1-\alpha_n)\|p - Sy_n\| + \delta\alpha_n \|p - Ss_n\|. \tag{2.6}$$

Again using (2.3) and (2.4), it yields

$$\varepsilon_n \leq \|Sy_{n+1} - p\| + [1-\alpha_n + \delta\alpha_n\{1-\beta_n + \delta\beta_n(1-\gamma_n + \delta\gamma_n)\}]\|Sy_n - p\| \to 0 \text{ as } n \to \infty.$$

Hence, the iterative procedure defined in (1.8) is stable with respect to pair (S, T).

Example 2.1. Let $X = R_+$. Define $S,T : X \to X$ by $Sx = \frac{x}{2}$, $Tx = \frac{x}{4}$ and $\psi(x) = \frac{x}{3}$, where $\psi : R_+ \to R_+$ with $\psi(0)=0$ and (X, d) has the usual metric. Then T satisfies contractive condition (2.1) and $F(T) = 0$. Also Jungck-Noor iterative scheme (1.8) is stable.

Proof: Now $p = 0$ is the coincidence point. Taking $\alpha_n = \beta_n = \gamma_n = \frac{1}{2}$ for each $n \geq 1$.

Let
$$y_n = \frac{2}{2+n}.$$

Then,
$$\lim_{n\to\infty} y_n = p = 0$$

$$\varepsilon_n = \left\| Sy_{n+1} - (1-\alpha_n)Sy_n - \alpha_n Ts_n \right\|, n = 0,1,2,3..,$$

where

$$Ss_n = (1-\beta_n)Sy_n + \beta_n Tq_n,$$
$$Sq_n = (1-\gamma_n)Sy_n + \gamma_n Ty_n.$$

$$Sq_n = (1-\frac{1}{2})\frac{y_n}{2} + \frac{1}{2}\times\frac{y_n}{4} = \frac{3}{8}y_n,$$

$$\therefore q_n = 2(\frac{3}{8}y_n) = \frac{3}{4}y_n.$$

$$Ss_n = (1-\frac{1}{2})\frac{y_n}{2} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}y_n = \frac{11}{32}y_n,$$

$$\therefore s_n = 2(\frac{11}{32}y_n) = \frac{11}{16}y_n.$$

$$Sy_{n+1} = (1-\frac{1}{2})\frac{y_n}{2} + \frac{1}{2}\times\frac{1}{4}\times\frac{11}{16}y_n = \frac{33}{128}y_n,$$

$$\therefore s_n = 2(\frac{33}{128}y_n) = \frac{33}{64}y_n.$$

$$\varepsilon_n = \left\| Sy_{n+1} - (1-\alpha_n)Sy_n - \alpha_n Ts_n \right\|, n = 0,1,2,3..$$
$$= \left\| \frac{1}{2}(\frac{2}{2+(n+1)}) - \frac{33}{64}\times\frac{2}{2+n} \right\| \to 0 \text{ as } n\to\infty$$

Hence,
$$\lim_{n\to\infty}\varepsilon_n = 0.$$

Therefore, Jungck-Noor iteration is stable.

On putting $Y = X = E$ and $S = id$, the identity map on X, $\delta = a$ and considering p, a fixed point of T, that is, $p = Tx = x$ in Theorem 2.1, we get Theorem 3.1of Bosede [7].

Corollary 2.1 [7]. Let $(E,\|\cdot\|)$ be a Banach space, $T: E \to E$ be a selfmap of E with a fixed point p, satisfying the contractive condition, $\|p - Ty\| = a\|p - y\|$ such that for each $y \in E$ and $0 \leq a < 1$ where p is a fixed point. For $x_0 \in E$, let $\{x_n\}_{n=0}^{\infty}$ be the Noor iterative process defined as,

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tq_n,$$
$$q_n = (1-\beta_n)x_n + \beta_n Tr_n,$$
$$r_n = (1-\gamma_n)x_n + \gamma_n Tx_n.$$

converging to *p*, (i.e; $Tp = p$), where $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are sequences of real numbers in [0, 1] such that $0 < \alpha \le \alpha_n$, $0 < \beta \le \beta_n$ and $0 < \gamma \le \gamma_n$ for all *n*. Then, the Noor iteration process is *T*-stable.

In the same way, on putting $Y = X = E$ and $S = id$, the identity map on *X*, $\gamma_n = 0, \delta = a$ and considering $p = Tx = x$ in Theorem 2.1, we get Theorem 3.2 of Bosede [7].

On putting $Y = X = E$, $S = id$ and $\gamma_n = \beta_n = 0$, $\delta = a$ in Theorem 2.1, we get Theorem 2.2 of Bosede and Rhoades [6].

Corollary 2.2 [6]. Let *E* be a Banach space, *T* a selfmap of *E* with a fixed point *p* and satisfying

$$\|p - Ty\| \le a\|p - y\| \text{ for some } 0 \le a < 1 \text{ and for each } y \in X.$$

The Mann iteration with $0 < \alpha \le \alpha_n$ for all n, is *T*-stable.

If we put $\alpha_n = \lambda$ in Corollary 2.2, we get *T*-stability for Kransnoselskij iterative procedure where $0 < \lambda < 1$.

And if we put $\alpha_n = 1$ in Corollary 2.2, we get *T*-stability for Picard iterative procedure.

On putting $Y = X = E$, $S = id$, $\delta = b$, $\gamma_n = \beta_n = 0$ and $\alpha_n = \frac{1}{2}$, we get Theorem 1 of Olatinwo et al. [18].

Corollary 2.3 [18]. Let $\{y_n\}_{n=0}^{\infty} \subset E$ and $\varepsilon_n = \left\| y_{n+1} - \frac{1}{2}(y_n + Ty_n) \right\|$. Let $(E, \|\cdot\|)$ be a normed linear space and $T : E \to E$ a selfmap of *E* satisfying

$$\|Tx - Ty\| \le \psi(\|x - Tx\|) + b\|x - y\|, \ 0 \le \text{b}<1.$$

Suppose *T* has a fixed point *p*. For arbitrary $x_0 \in E$, define sequence $\{x_n\}_{n=0}^{\infty}$ iteratively by;

$$x_{n+1} = f(T, x_n) = \frac{1}{2}(x_n + Tx_n), \ n \ge 0.$$

Let $\psi : R_+ \to R_+$ be monotonic increasing with $\psi(0) = 0$. Then, the Krasnolseskij process is *T*-stable.

On putting $Y = X = E$, $S = id$, $\delta = b$, $\gamma_n = \beta_n = 0$ and $\alpha_n = a$, we get Theorem 2 of Olatinwo [18].

On putting $Y = X = E$, $S = id$, $\delta = b$, $\gamma_n = 0$, we get Theorem 3 of Olatinwo [18].

On putting $Y = X = E$, $S = id$, $\delta = b$, $\gamma_n = \beta_n = 0$, we get Theorem 3.2 of Imoru and Olatinwo [12].

Corollary 2.4 [12]. Let $(E, \|\cdot\|)$ be a normed linear space and let $T : E \to E$ be a selfmap of *E* satisfying

$$\|Tx - Ty\| \le \psi(\|x - Tx\|) + b\|x - y\|, \ 0 \le \text{b}<1.$$

Suppose *T* has a fixed point p^*. Let $x_0 \in E$ and suppose that $x_{n+1} = f(T, x_n) = (1 - \alpha_n)x_n + \alpha_n Tx_n$, $n \ge 0$, where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0, 1] such that $0 < \alpha \le \alpha_n$, $n = 0, 1, 2, ...$ Suppose also that $\psi : R_+ \to R_+$ be monotonic increasing with $\psi(0) = 0$. Then, the Mann iteration is *T*-stable.

We have already proved that $(13) \Rightarrow (15)$ and on putting $Y = X$, $S = id$ and $\gamma_n = \beta_n = 0$, we get Theorem 3 of Berinde [2].

Corollary 2.5 [2]. Let $(X, \|\cdot\|)$ be a normed linear space and $T : X \to X$ be a Zamfirescu contraction. Suppose there exists $p \in F(T)$ such that the Mann iteration $\{x_n\}_{n=0}^{\infty}$ with $x_0 \in X$ and $\{\alpha_n\}_{n=0}^{\infty}$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$, converges to p. Then the Mann iteration procedure is T-stable.

On putting $Y = X = E$, $S = id$, $\gamma_n = 0$ and $\psi(u) = L(u)$ where $u = d(x, Tx)$, we get Theorem 1 of Berinde [5].

Corollary 2.6 [5]. Let E be a normed linear space, K a closed convex subset of E, and $T : K \to K$ an operator with $F(T) \neq \phi$, satisfying (14). Let $\{x_n\}_{n=0}^{\infty}$ be the Ishikawa iteration and $x_0 \in K$, arbitrary, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in [0, 1] with $\{\alpha_n\}$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}$ converges strongly to the unique fixed point of T.

We have already proved that $(13) \Rightarrow (15)$ and on putting $Y = X = E$, $S = id$ and $\gamma_n = 0$, we get Corollary 2 of Berinde [5].

Similarly using $(13) \Rightarrow (15)$, $Y = X = E$, $S = id$ and $\gamma_n = \beta_n = 0$, we get Corollary 2 of Berinde [5].

On putting $Y = X$ and $S = id$, the identity map on X, $\alpha_n = 1$, $\beta_n = \gamma_n = 0$ and $\psi(u) = L(u)$ where $u = d(x, Tx)$ in Theorem 2.1, we get that $\{x_n\}$ is stable with respect to T. From remark and example 1 in [2], it is clear that any stable iteration procedure is also almost stable and it is obvious that any almost stable iteration procedure is also summably almost stable, since $\sum_{n=0}^{\infty} d(y_n, p) < \infty \Rightarrow \lim_{n\to\infty} y_n = p$. Hence, we get result Theorem 1 of Berinde [3].

Corollary 2.7 [3]. Let (X, d) be a metric space and $T : X \to X$ a mapping satisfying contractive condition

$$d(Tx, Ty) \leq ad(x, y) + Ld(x, Tx) \text{ for } a \in [0,1),\ L \geq 0\ \forall x, y \in X .$$

Suppose T has a fixed point p. Let $x_0 \in X$ and $x_{n+1} = Tx_n$, $n \geq 0$, then $\{x_n\}$ converges strongly to to p and is summable almost stable with respect to T.

Similar to above reason, On putting $Y = X$ and $S = id$, the identity map on X, $\gamma_n = 0$ and $\psi(u) = L(u)$ where $u = d(x, Tx)$ in Theorem 2.1, we get Theorem 2 of Berinde [3].

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