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Adaptive Control, Synchronization and LabVIEW Implementation of Rucklidge Chaotic System for Nonlinear Double Convection

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Abstract: In the recent decades, there is great interest shown in the literature in the discovery of chaotic motion and oscillations in nonlinear dynamical systems arising in physics, chemistry, biology and engineering. Chaotic systems have many important applications in science and engineering. This paper discusses the Rucklidge chaotic system (1992) for nonlinear double convection. When the convection takes place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modelled by Rucklidge's three-dimensional system of ordinary differential equations, which produces chaotic solutions. This paper starts with a detailed description of the Rucklidge's nonlinear double convection system and the parameter values for which the Rucklidge system exhibits chaotic behaviour. Next, an adaptive feedback controller is designed for the global chaos control of the Rucklidge chaotic system with unknown parameters. Furthermore, an adaptive feedback controller is designed for the global chaos synchronization of the identical Rucklidge chaotic system with unknown parameters. All the main results derived in this work are illustrated with MATLAB simulations. Finally, a circuit design of the novel 3-D chaotic system is implemented in LABVIEW to validate the theoretical chaotic model.

Keywords: Chaos, chaotic systems, dissipative systems, chaos control, chaos synchronization, Rucklidge system, double convection, fluid mechanics, circuit simulation, LABVIEW implementation.

1. INTRODUCTION

Chaos theory describes the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1]. Chaos theory has applications in several areas in Science and Engineering.

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler found a 3-D chaotic system [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

In fluid mechanics modelling, cases of two-dimensional convection in a horizontal lawyer of Boussinesq fluid with lateral constraints were studied by Rucklidge [82]. When the convection takes

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place in a fluid layer rotating uniformly about a vertical axis and in the limit of tall thin rolls, convection in an imposed vertical magnetic field and convection in a rotating fluid layer are both modelled by a new three-dimensional system of ordinary differential equations, which produces chaotic solutions like the Lorenz system [2].

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering. Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-132], adaptive control method [133-149], backstepping method [150-161], sliding mode control method [162-170], etc.

In this paper, we first discuss the Rucklidge chaotic system [171] for nonlinear double convection and detail its qualitative properties.

Next, we derive an adaptive control law that stabilizes the Rucklidge chaotic system with unknown system parameters. Furthermore, we also derive an adaptive control law that achieves global chaos synchronization of identical Rucklidge chaotic systems with unknown parameters. Our main adaptive control results for global chaos stabilization and synchronization are established using Lyapunov stability theory.

This paper is organized as follows. In Section 2, we describe the 3-D Rucklidge chaotic system for nonlinear double convection. In Section 3, we describe the qualitative properties of the Rucklidge chaotic system. In Section 4, we detail the adaptive control design for the global chaos stabilization of the 3-D Rucklidge chaotic system with unknown parameters. In Section 5, we detail the adaptive control design for the global and exponential synchronization of the identical novel Rucklidge chaotic systems. In Section 6, we describe the LABVIEW implementation of the Rucklidge chaotic system and the control results for the Rucklidge chaotic system.

2. RUCKLIDGE CHAOTIC SYSTEM FOR DOUBLE CONVECTION

In this section, we describe the Rucklidge chaotic system [171] for nonlinear double convection. Rucklidge chaotic system is modelled by the 3-D nonlinear dynamics

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b are constant, positive, parameters of the Rucklidge system.

The system (1) is *chaotic* when the parameter values are taken as

$$a = 2 \quad b = 6.7 \tag{2}$$

For numerical simulations, we take the initial conditions of the Rucklidge system (1) as

$$x_1(0) = 1.2, \quad x_2(0) = 0.8, \quad x_3(0) = 1.4 \tag{3}$$

The Lyapunov exponents of the 3-D chaotic system (1) for the parameter values (2) and the initial conditions (3) are numerically calculated as

$$L_1 = 0.1877, \quad L_2 = 0, \quad L_3 = -3.1893 \tag{4}$$

We note that the sum of the Lyapunov exponents of the Rucklidge chaotic system (1) is negative. Thus, the Rucklidge chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the Rucklidge chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0589 \tag{5}$$

Figure 1 shows the 3-D phase portrait of the Rucklidge 3-D chaotic system (1). Figures 2-4 show the 2-D projection of the Rucklidge chaotic system (1) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, respectively.

3. PROPERTIES OF THE RUCKLIDGE CHAOTIC SYSTEM

In this section, we discuss the qualitative properties of the Rucklidge chaotic system (1). We suppose that the parameter values of the Rucklidge system (1) are as in the chaotic case (2), *i.e.* $a = 2$ and $b = 6.7$.

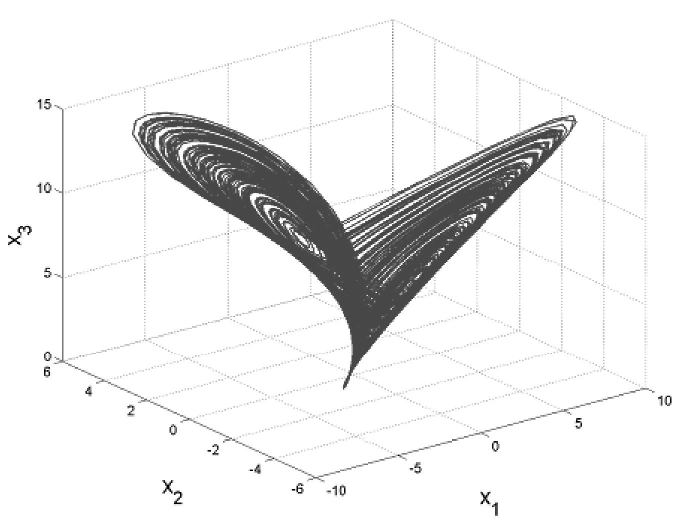

Figure 1: Phase portrait of the Rucklidge 3-D chaotic system

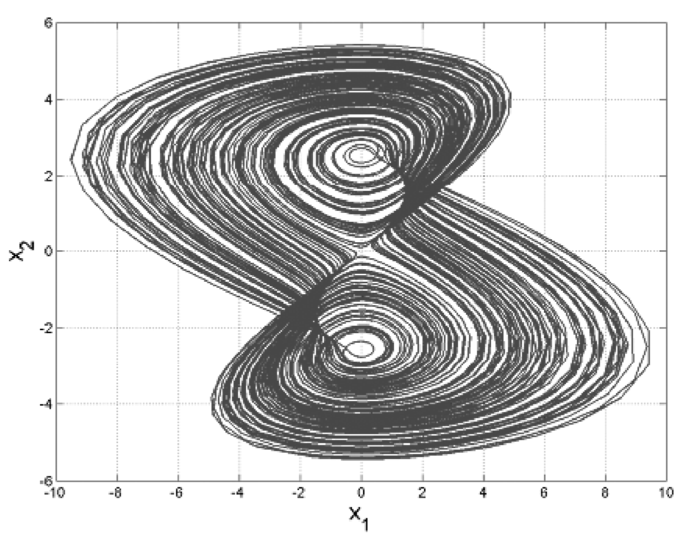

Figure 2: 2-D projection of the Rucklidge chaotic system on the (x_1, x_2) plane

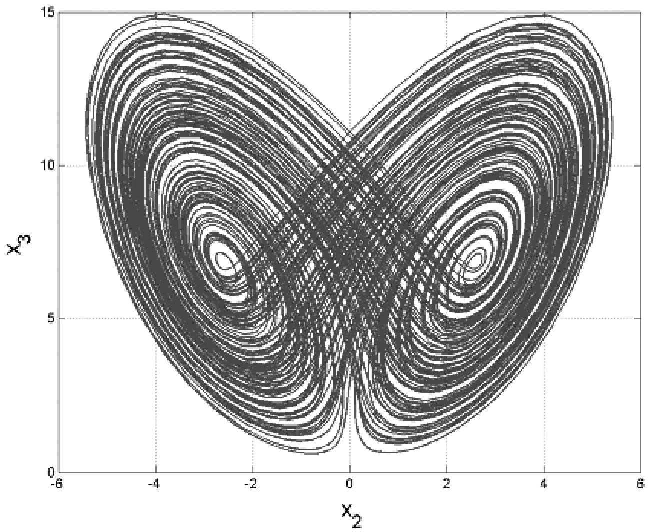

Figure 3: 2-D projection of the Rucklidge chaotic system on the (x_2, x_3) plane

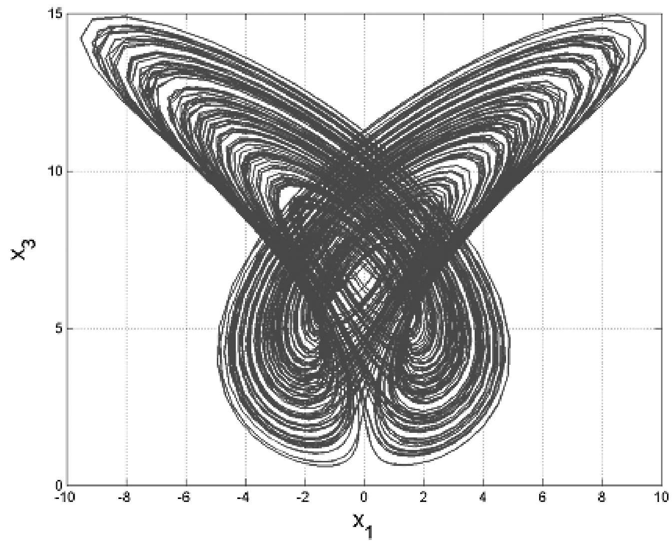

Figure 4: 2-D projection of the Rucklidge chaotic system on the (x_1, x_3) plane

3.1. Dissipativity

In vector notation, we may express the system (1) as

$$\dot{x} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{6}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = -ax_1 + bx_2 - x_2x_3 \\ f_2(x_1, x_2, x_3) = x_1 \\ f_3(x_1, x_2, x_3) = -x_3 + x_2^2 \end{cases} \tag{7}$$

Let Ω be any region in R^3 with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of the vector field f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\dot{V} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3 \tag{8}$$

The divergence of the novel chaotic system (1) is easily found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 = -\mu \tag{9}$$

where $\mu = a + 1 = 3 > 0$.

Substituting (9) into (8), we obtain the first order ODE

$$\dot{V} = -\mu V \tag{10}$$

Integrating (10), we obtain the unique solution as

$$V(t) = \exp(-\mu t)\ V(0) \text{ for all } t \geq 0 \tag{11}$$

Since $\mu > 0$, it follows that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the 3-D novel chaotic system (1) is dissipative. Thus, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry

It is easy to see that the system (1) is invariant under the coordinates transformation

$$(x_1, x_2, x_3) = (-x_1, -x_2, -x_3) \tag{12}$$

Thus, the system (1) exhibits *point reflection symmetry* about the origin in R^3.

3.3. Equilibrium Points

The equilibrium points of the system (1) are obtained by solving the system of equations

$$\begin{cases} -ax_1 + bx_2 - x_2 x_3 = 0 \\ x_1 = 0 \\ -x_3 + x_2^2 = 0 \end{cases} \tag{13}$$

Solving the system (13) with the values of the parameters as given in (2), we obtain three equilibrium points

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ E_1 = \begin{bmatrix} 0 \\ 2.5884 \\ 6.7000 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 \\ -2.5884 \\ 6.7000 \end{bmatrix} \tag{14}$$

The Jacobian of the system (1) at any point $x \in R^3$ is given by

$$J(x) = \begin{bmatrix} -a & b - x_3 & -x_2 \\ 1 & 0 & 0 \\ 0 & 2x_2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 6.7 - x_3 & -x_2 \\ 1 & 0 & 0 \\ 0 & 2x_2 & -1 \end{bmatrix} \tag{15}$$

We find that

$$J_0 = J(E_0) = \begin{bmatrix} -2 & 6.7 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{16}$$

The eigenvalues of J_0 are numerically determined using MATLAB as

$$\lambda_1 = -1,\ \ \lambda_2 = -3.7749,\ \ \lambda_3 = 1.7749 \tag{17}$$

This shows that the equilibrium E_0 is a saddle-point, which is unstable.

Next, we find that

$$J_1 = J(E_1) = \begin{bmatrix} -2 & 0 & -2.5884 \\ 1 & 0 & 0 \\ 0 & 5.1768 & -1 \end{bmatrix} \tag{18}$$

The eigenvalues of J_1 are numerically determined using MATLAB as

$$\lambda_1 = -3.5154, \quad \lambda_{2,3} = 0.2577 \pm 1.9353i \tag{19}$$

This shows that the equilibrium point E_1 is a saddle-focus, which is unstable.

We also find that

$$J_2 = J(E_2) = \begin{bmatrix} -2 & 0 & 2.5884 \\ 1 & 0 & 0 \\ 0 & -5.1768 & -1 \end{bmatrix} \tag{20}$$

The eigenvalues of J_2 are numerically determined as

$$\lambda_1 = -3.5154, \quad \lambda_{2,3} = 0.2577 \pm 1.9353i \tag{21}$$

This shows that the equilibrium point E_2 is a saddle-focus, which is unstable.

3.4. Lyapunov Exponents And Kaplan-yorke Dimension

We take the parameter values of the Rucklidge system (1) as in the chaotic case (2), i.e. a = 2 and b = 6.7.

We choose the initial values of the state as $x_1(0) = 1.2$, $x_2(0) = 0.8$ and $x_3(0) = 1.4$.

Then we obtain the Lyapunov exponents of the system (1) using MATLAB as

$$L_1 = 0.1877, \quad L_2 = 0, \quad L_3 = -3.1893. \tag{22}$$

Figure 5 shows the Lyapunov exponents of the system (1) as determined by MATLAB.

The Kaplan-Yorke dimension of the Rucklidge chaotic system (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0589. \tag{23}$$

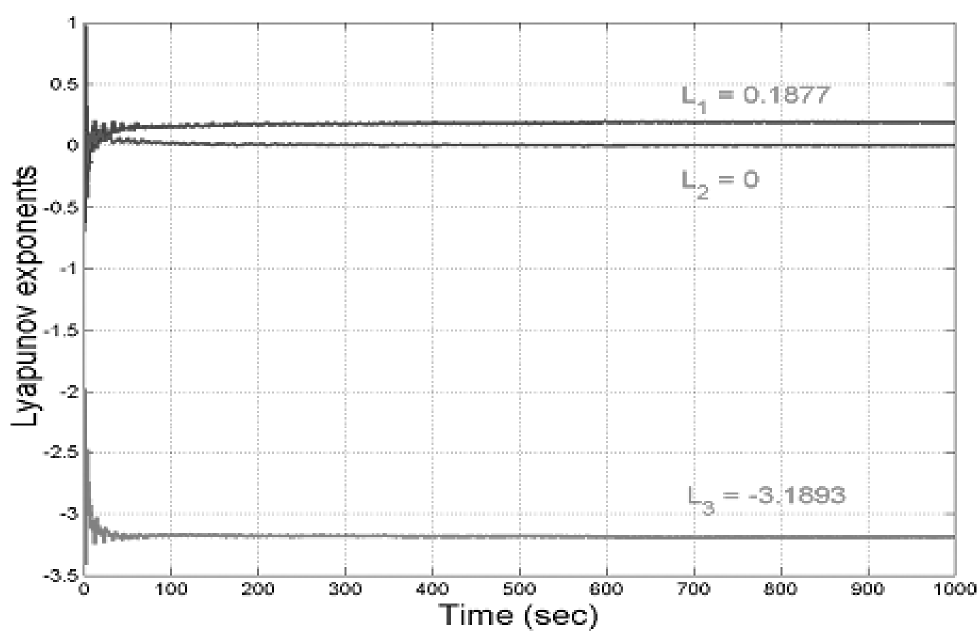

Figure 5: Lyapunov exponents of the Rucklidge chaotic system

4. ADAPTIVE CONTROL DESIGN FOR THE GLOBAL STABILIZATION OF THE RUCKLIDGE CHAOTIC SYSTEM

In this section, we use adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the Rucklidge chaotic system with unknown parameters.

Thus, we consider the Rucklidge chaotic system with controls given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 + u_1 \\ \dot{x}_2 = x_1 + u_2 \\ \dot{x}_3 = -x_3 + x_2^2 + u_3 \end{cases} \tag{24}$$

In (24), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for the unknown parameters a and b, respectively.

We consider the adaptive control law defined by

$$\begin{cases} u_1(t) = \hat{a}(t)x_1 - \hat{b}(t)x_2 + x_2x_3 - k_1x_1 \\ u_2(t) = -x_1 - k_2x_2 \\ u_3(t) = x_3 - x_2^2 - k_3x_3 \end{cases} \tag{25}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (25) into (24), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = -[a - \hat{a}(t)]x_1 + [b - \hat{b}(t)]x_2 - k_1x_1 \\ \dot{x}_2 = -x_1 - k_2x_2 \\ \dot{x}_3 = x_3 - x_2^2 - k_3x_3 \end{cases} \tag{26}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \end{cases} \tag{27}$$

Using (27), we can simplify the plant dynamics (26) as

$$\begin{cases} \dot{x}_1 = -e_ax_1 + e_bx_2 - k_1x_1 \\ \dot{x}_2 = -x_1 - k_2x_2 \\ \dot{x}_3 = x_3 - x_2^2 - k_3x_3 \end{cases} \tag{28}$$

Differentiating (27) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \end{cases} \tag{29}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{x}, e_a, e_b) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2\right) \tag{30}$$

Clearly, V is a positive definite function on R^5.

Differentiating V along the trajectories of (28) and (29), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[-x_1^2 - \dot{\hat{a}}\right] + e_b\left[x_1 x_2 - \dot{\hat{b}}\right] \tag{31}$$

In view of (31), we take the parameter update law as follows:

$$\begin{cases} \dot{\hat{a}} = -x_1^2 \\ \dot{\hat{b}} = x_1 x_2 \end{cases} \tag{32}$$

Theorem 1. The Rucklidge 3-D chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions $x(0) \in R^3$ by the adaptive control law (25) and the parameter update law (32), where k_1, k_2, k_3 are positive gain constants.

Proof. We prove this result by using Lyapunov stability theory [172].

We consider the quadratic Lyapunov function defined by (30), which is positive definite on R^5.

By substituting the parameter update law (32) into (31), we obtain the time derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{33}$$

From (33), it is clear that $\dot{V}$ is a negative semi-definite function on R^5.

Thus, we conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[\boldsymbol{x}(t) \quad e_a(t) \quad e_b(t)\right]^T \in L_\infty$$

We define $k = \min\{k_1, k_2, k_3\}$. Thus, it follows from (33) that

$$\dot{V} \le -k\left\|\boldsymbol{x}(t)\right\|^2 \tag{34}$$

Thus, we have

$$k\left\|\boldsymbol{x}(t)\right\|^2 \le -\dot{V} \tag{35}$$

Integrating the inequality (35) from 0 to t, we get

$$k\int_0^t \left\|\boldsymbol{x}(\tau)\right\|^2 d\tau \le V(0) - V(t) \tag{36}$$

From (36), it follows that $x \in L_2$.Using (28), we can conclude that $\dot{x} \in L_\infty$.

Using Barbalat's lemma [172], we can conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$. This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (24) and (32), when the adaptive control law (25) is applied.

The parameter values of the novel chaotic system (24) are taken as in the chaotic case (2), i.e.

$$a = 2, \quad b = 6.7 \tag{37}$$

We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the Rucklidge chaotic system (24), we take

$$x_1(0) = 12.3, \quad x_2(0) = -22.7, \quad x_3(0) = 16.4 \tag{38}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15.8, \quad \hat{b}(0) = 24.9 \tag{39}$$

Figure 6 shows the exponential convergence of the controlled state trajectories of the Rucklidge chaotic system (24).

5. ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL RUCKLIDGE CHAOTIC SYSTEMS

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical 3-D Rucklidge chaotic systems with unknown parameters.

As the master system, we consider the Rucklidge chaotic system given by

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2 - x_2x_3 \\ \dot{x}_2 = x_1 \\ \dot{x}_3 = -x_3 + x_2^2 \end{cases} \tag{40}$$

where x_1, x_2, x_3 are the states and a, b are unknown system parameters.

As the slave system, we consider the controlled Rucklidge chaotic system given by

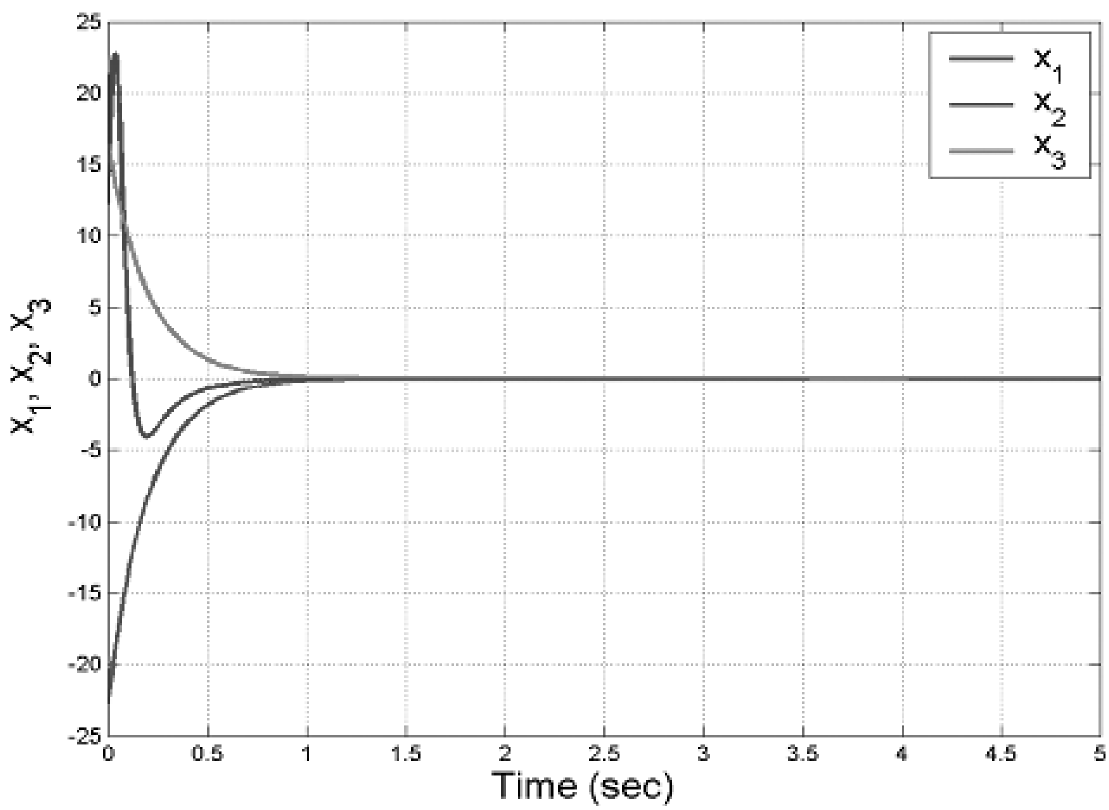

Figure 6: Time-history of the controlled state trajectories of the novel chaotic system

$$\begin{cases} \dot{y}_1 = -ay_1 + by_2 - y_2 y_3 + u_1 \\ \dot{y}_2 = y_1 + u_2 \\ \dot{y}_3 = -y_3 + y_2^2 + u_3 \end{cases} \tag{41}$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t),\ \hat{b}(t)$ for the unknown system parameters a, b, respectively.

The synchronization error between the novel chaotic systems (40) and (41) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{42}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = -ae_1 + be_2 - y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 = e_1 + u_2 \\ \dot{e}_3 = -e_3 + y_2^2 - x_2^2 + u_3 \end{cases} \tag{43}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = \hat{a}(t)e_1 - \hat{b}(t)e_2 + y_2 y_3 - x_2 x_3 - k_1 e_1 \\ u_2 = -e_1 - k_2 e_2 \\ u_3 = e_3 - y_2^2 + x_2^2 - k_3 e_3 \end{cases} \tag{44}$$

where k_1, k_2, k_3 are positive constants and $\hat{a}(t),\ \hat{b}(t)$ are estimates of the unknown parameters a, b, respectively.

Substituting (44) into (43), we can simplify the error dynamics (43) as

$$\begin{cases} \dot{e}_1 = -[a - \hat{a}(t)]e_1 + [b - \hat{b}(t)]e_2 - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{45}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a = a - \hat{a}(t) \\ e_b = b - \hat{b}(t) \end{cases} \tag{46}$$

Substituting (46) into (45), the error dynamics is simplified as

$$\begin{cases} \dot{e}_1 = -e_a e_1 + e_b e_2 - k_1 e_1 \\ \dot{e}_2 = -k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \tag{47}$$

Differentiating (43) with respect to t, we obtain

$$\begin{cases} \dot{e}_a = -\dot{\hat{a}}(t) \\ \dot{e}_b = -\dot{\hat{b}}(t) \end{cases} \tag{48}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right) \tag{49}$$

Differentiating along the trajectories of (47) and (48), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[-e_1^2 - \dot{\hat{a}}\right] + e_b\left[e_1 e_2 - \dot{\hat{b}}\right] \tag{50}$$

In view of (50), we take the parameter update law as follows.

$$\begin{cases} \dot{\hat{a}} = -e_1^2 \\ \dot{\hat{b}} = e_1 e_2 \end{cases} \tag{51}$$

Next, we state and prove the main result of this section.

Theorem 2. The Rucklidge chaotic systems (40) and (41) with unknown system parameters are globally and exponentially synchronized for all initial conditions $x(0)$, $y(0) \in R^3$ by the adaptive control law (44) and the parameter update law (51), where k_1, k_2, k_3 are positive constants.

Proof. We prove this result by applying Lyapunov stability theory [172].

We consider the quadratic Lyapunov function defined by (49), which is positive definite on R^5.

By substituting the parameter update law (51) into (50), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{52}$$

From (52), it is clear that $\dot{V}$ is a negative semi-definite function on R^5.

Thus, we can conclude that the synchronization error vector $e(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\left[\boldsymbol{e}(t) \quad e_a(t) \quad e_b(t)\right]^T \in L_\infty \tag{53}$$

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (52) that

$$\dot{V} \le -k\left\|\boldsymbol{e}(t)\right\|^2 \tag{54}$$

Thus, we have

$$k\left\|\boldsymbol{e}(t)\right\|^2 \le -\dot{V} \tag{55}$$

Integrating the inequality (55) from 0 to t, we get

$$\int_0^t k\left\|\boldsymbol{e}(\tau)\right\|^2 d\tau \le V(0) - V(t) \tag{56}$$

From (56), it follows that $e \in L_2$. Using (47), we can conclude that $\dot{e} \in L_\infty$.

Using Barbalat's lemma [172], we conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$. This completes the proof. ■

For numerical simulations, we take the parameter values of the chaotic systems (40) and (41) as in the chaotic case (2), *i.e.* $a = 2$ and $b = 6.7$.

We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

As initial conditions of the master system (40), we take

$$x_1(0) = 12.4, \quad x_2(0) = -3.1, \quad x_3(0) = -17.8 \tag{57}$$

As initial conditions of the slave system (41), we take

$$y_1(0) = 5.7, \quad y_2(0) = 19.2, \quad y_3(0) = 26.5 \tag{58}$$

As initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15.4, \quad \hat{b}(0) = 17.8 \tag{59}$$

Figures 7-9 depict the synchronization of the Rucklidge chaotic systems (40) and (41). Figure 10 depicts the time-history of the complete synchronization errors e_1, e_2, e_3.

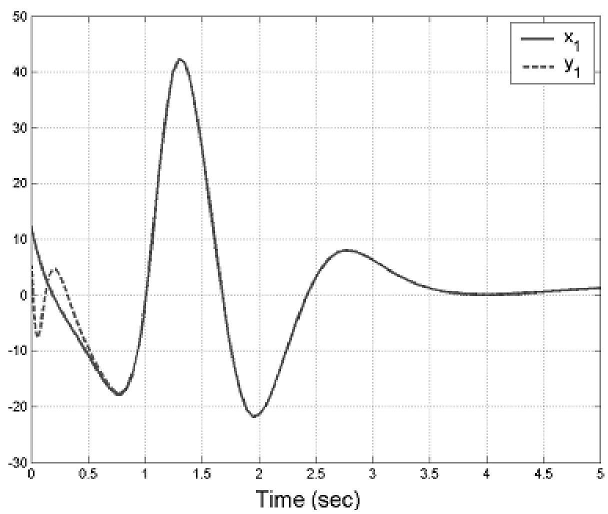

Figure 7: Synchronization of the states x_1 and y_1

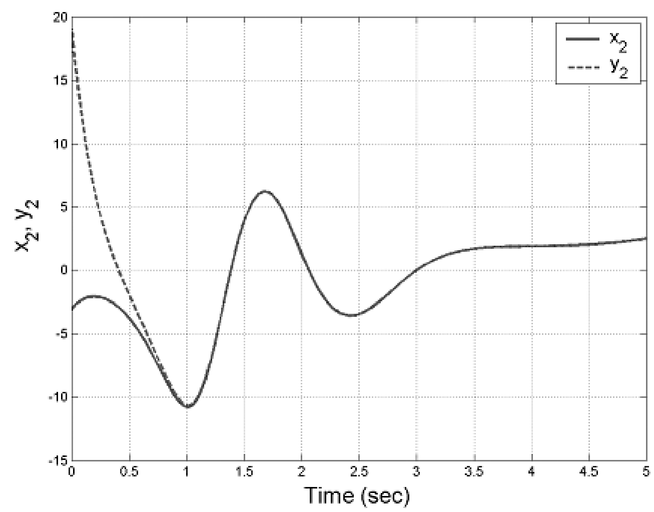

Figure 8: Synchronization of the states x_2 and y_2

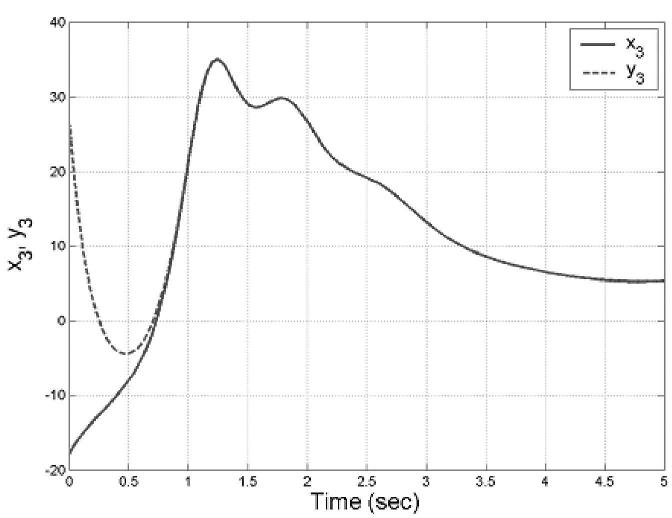

Figure 9: Synchronization of the states x_3 and y_3

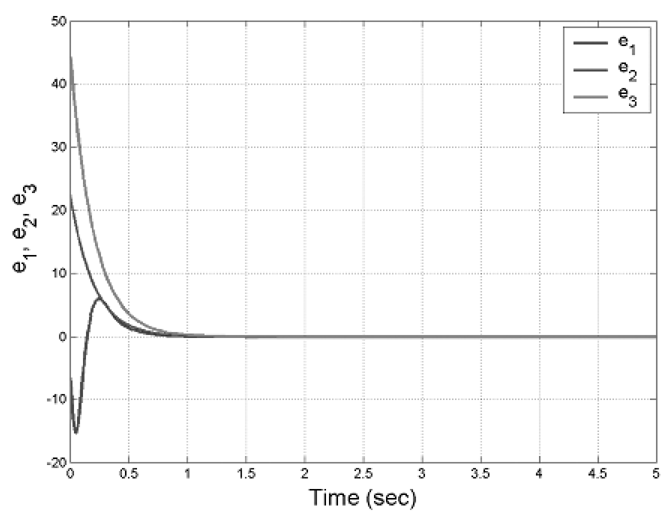

Figure 10: Time-history of the synchronization errors e_1, e_2, e_3

6. CIRCUIT SIMULATION AND LABVIEW IMPLEMENTATION

6.1. Labview Implementation of Adaptive Control Design for The Global Stabilization of The Rucklidge Chaotic System

The adaptive control method to derive an adaptive feedback control law for globally and exponentially stabilizing the Rucklidge chaotic system with unknown parameters was discussed in section 4. Figure 11 shows the Rucklidge system implemented in LabVIEW using the Control Design and Simulation Loop. Figure 12 the designed controller for stabilizing the chaotic system is implemented in LabVIEW using the

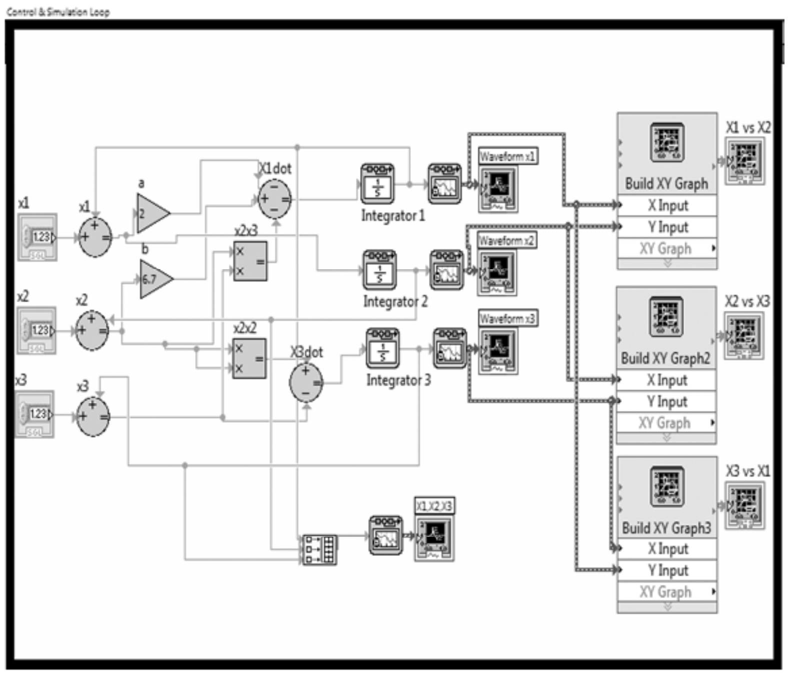

Figure 11: Block Diagram of the Rucklidge System

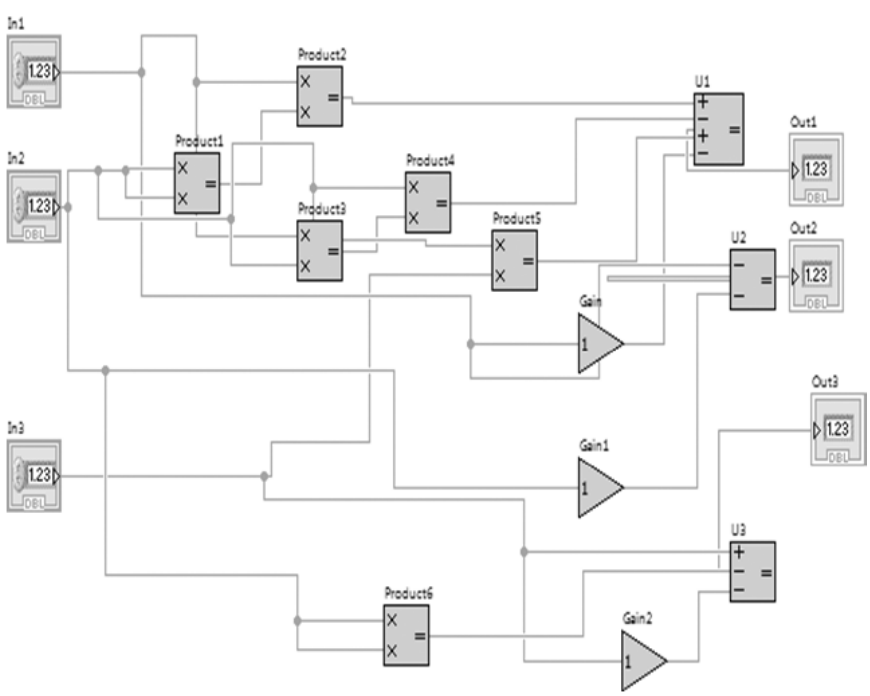

Figure 12: Block Diagram of the Controller as Subsystem

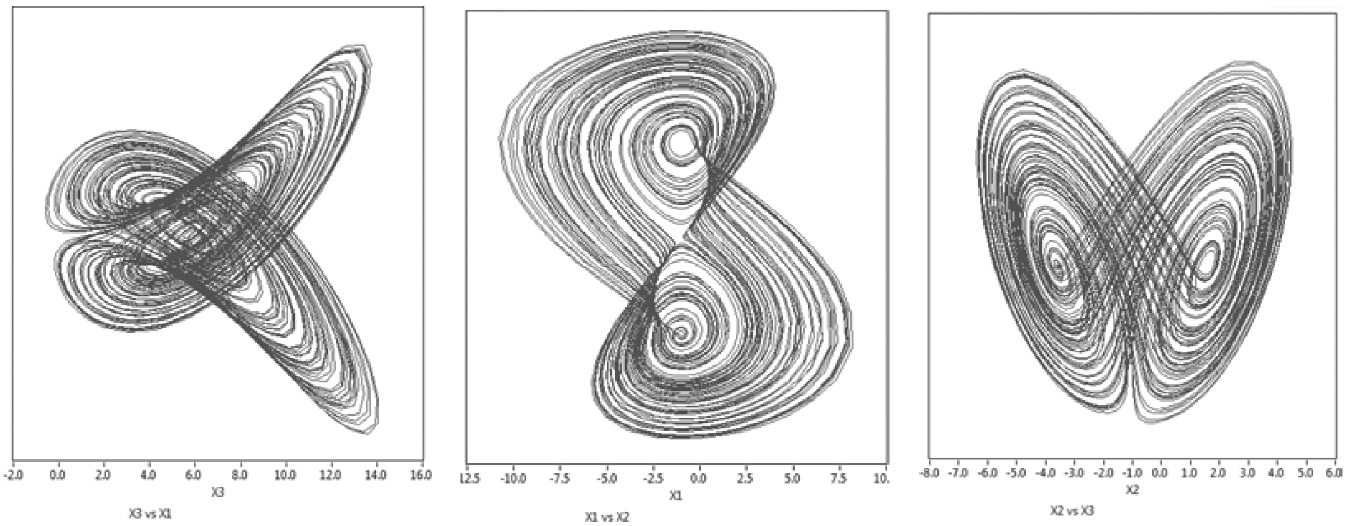

Figure 13: 2D Phase Portraits (x_3, x_1), (x_1, x_2), (x_2, x_3) of the Rucklidge System

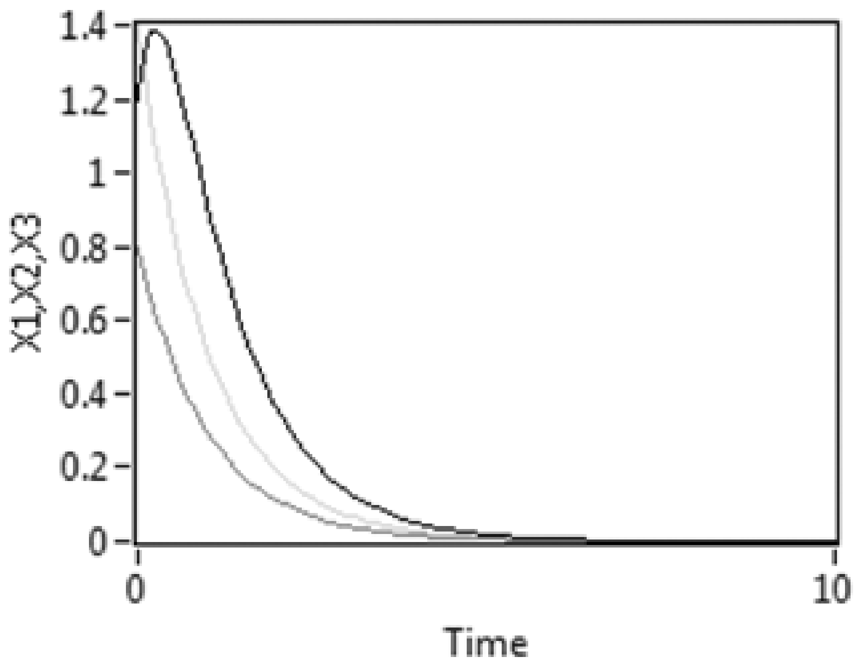

Figure 14: Time-History of the Stabilized States of the Rucklidge System

control design and simulation loop. Figure 13 shows the 2D Phase portraits (x_3, x_1), (x_1, x_2), and (x_2, x_3) of the Rucklidge system. Figure 14 shows the stabilized states of the Rucklidge system.

7. LABVIEW IMPLEMENTATION OF ADAPTIVE SYNCHRONIZATION OF THE IDENTICAL RUCKLIDGE CHAOTIC SYSTEMS

In this section, the adaptive control method discussed in Section 5 is implemented using LabVIEW. Figure 15 shows the slave subsystem. Figure 16 shows the designed adaptive controller U. Figure 17 shows the time history of the synchronization errors.

8. CONCLUSIONS

In this paper, a detailed description of the Rucklidge's nonlinear double convection system and the parameter values for which the Rucklidge system exhibits chaotic behavior is discussed. Next, an adaptive feedback

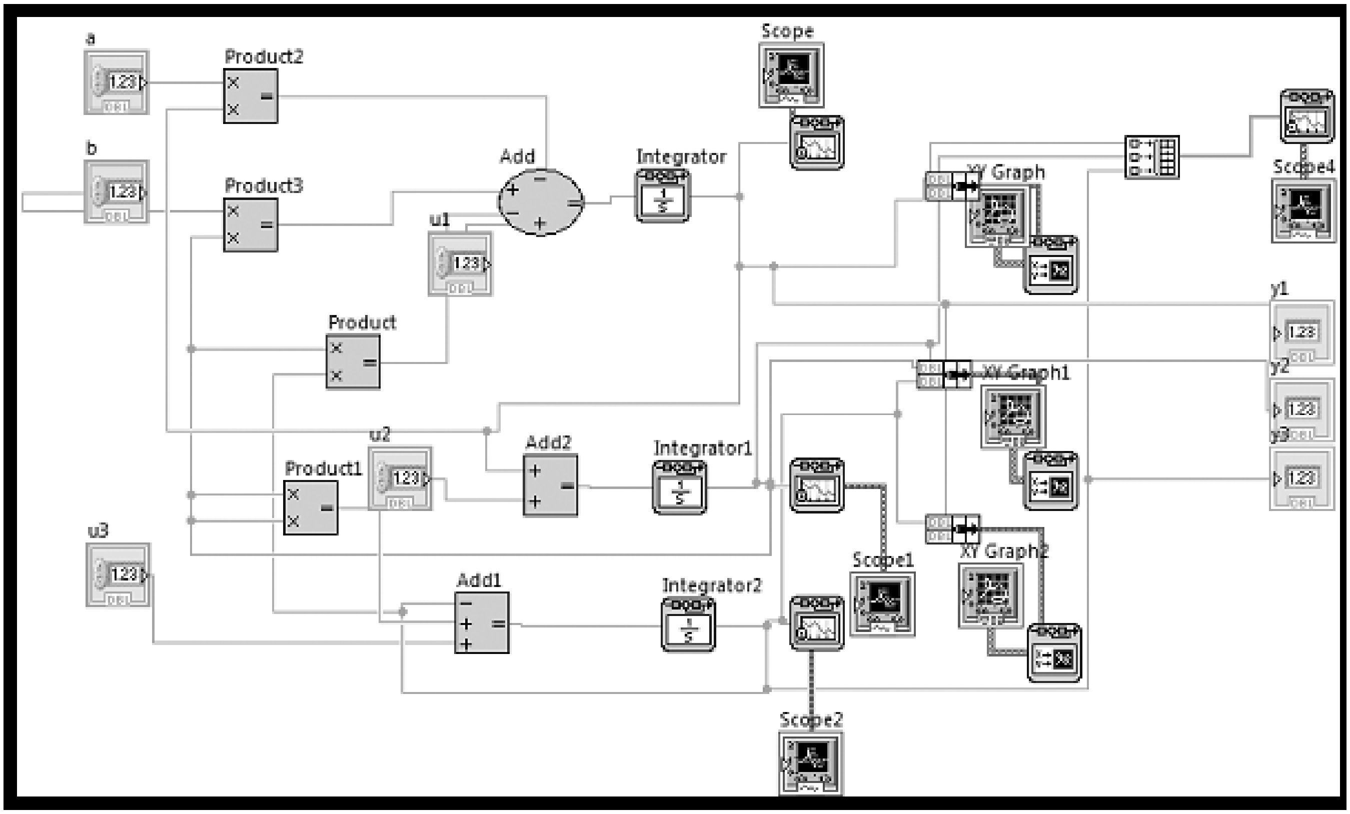

Figure 15: Block diagram of the Slave Rucklidge system.

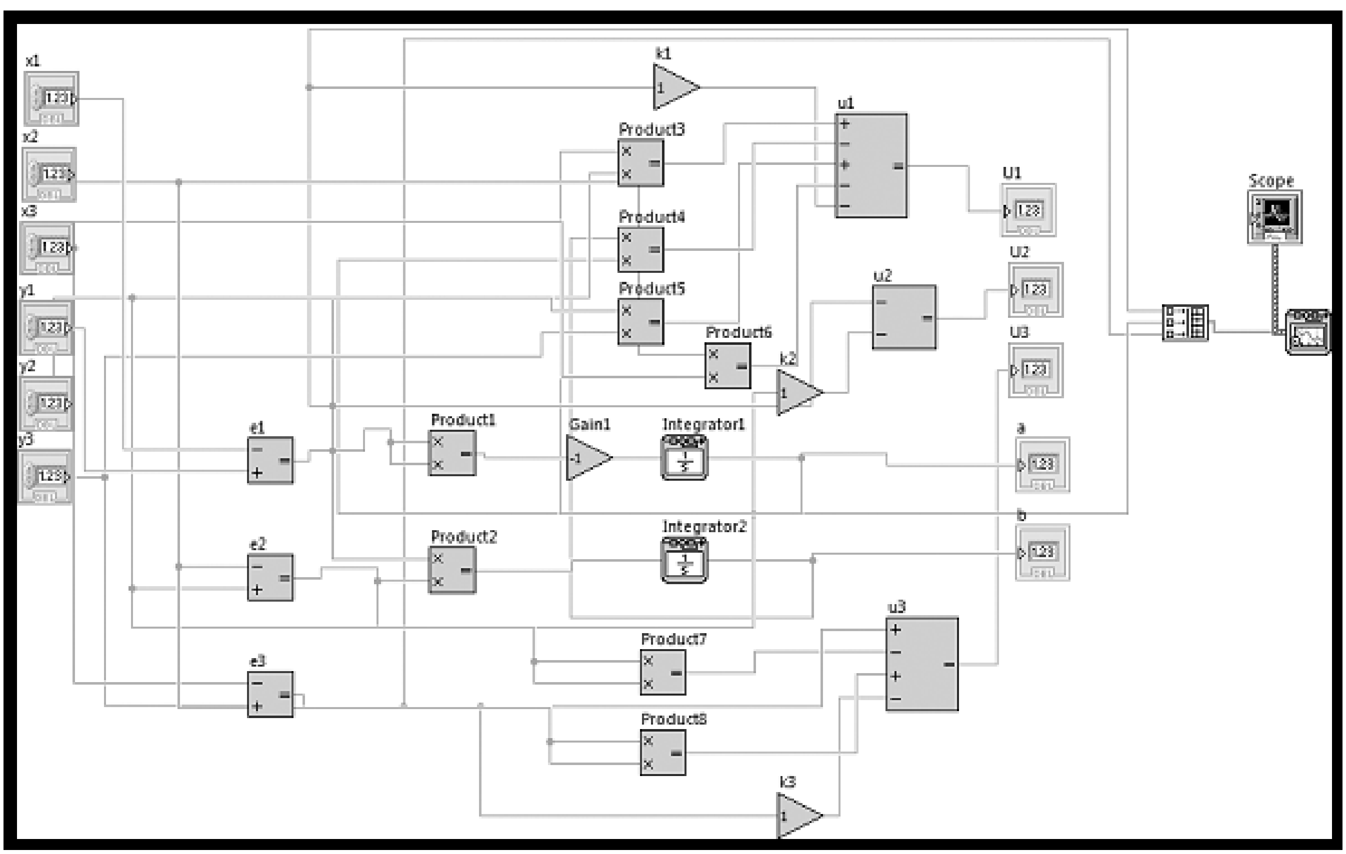

Figure 16: Block diagram of the Synchronization Controller.

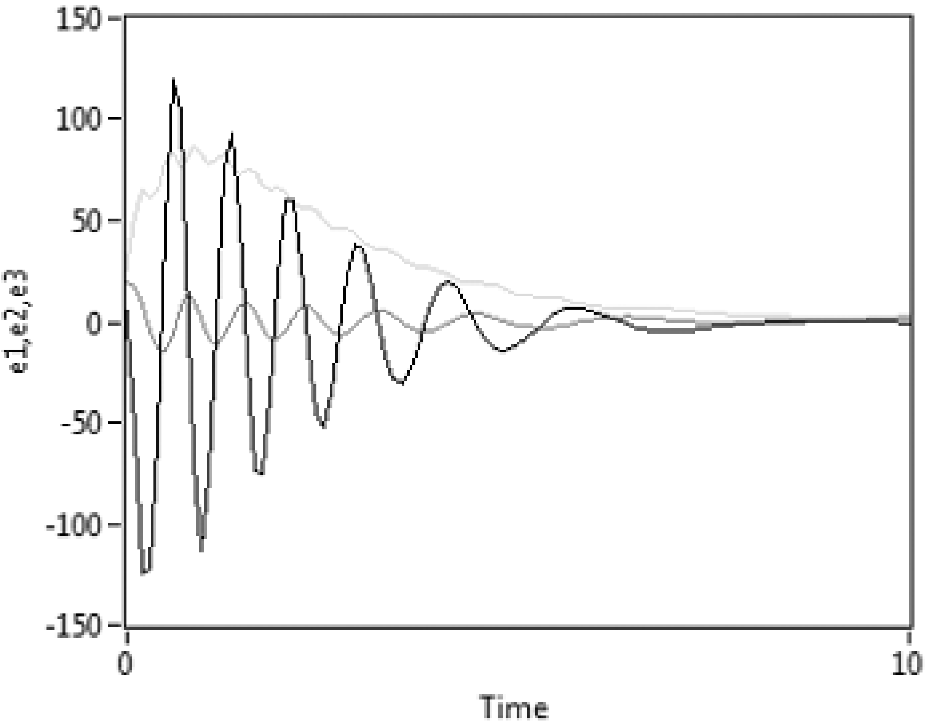

Figure 17: Time history of the Synchronization errors.

controller is designed for the global chaos control of the Rucklidge chaotic system with unknown parameters. Furthermore, an adaptive feedback controller is designed for the global chaos synchronization of the identical Rucklidge chaotic system with unknown parameters. Finally, a circuit design of the novel 3-D chaotic system is implemented in LABVIEW to validate the theoretical chaotic model.

References

[1] A.T. Azar and S. Vaidyanathan, *Chaos Modeling and Control Systems Design*, Springer, Berlin, 2015.

[2] E.N. Lorenz, "Deterministic nonperiodic flow", *Journal of the Atmospheric Sciences*, **20**, 130-141, 1963.

[3] O.E. Rössler, "An equation for continuous chaos", *Physics Letters A*, **57**, 397-398, 1976.

[4] A. Arneodo, P. Coullet and C. Tresser, "Possible new strange attractors with spiral structure," *Communications in Mathematical Physics*, **79**, 573-579, 1981.

[5] J.C. Sprott, "Some simple chaotic flows," *Physical Review E*, **50**, 647-650, 1994.

[6] G. Chen and T. Ueta, "Yet another chaotic attractor," *International Journal of Bifurcation and Chaos*, **9**, 1465-1466, 1999.

[7] J. Lü and G. Chen, "A new chaotic attractor coined," *International Journal of Bifurcation and Chaos*, **12**, 659-661, 2002.

[8] C.X. Liu, T. Liu, L. Liu and K. Liu, "A new chaotic attractor," Chaos, Solitons and Fractals, **22**, 1031-1038, 2004.

[9] G. Cai and Z. Tan, "Chaos synchronization of a new chaotic system via nonlinear control," *Journal of Uncertain Systems*, **1**, 235-240, 2007.

[10] G. Tigan and D. Opris, "Analysis of a 3D chaotic system," *Chaos, Solitons and Fractals*, **36**, 1315-1319, 2008.

[11] D. Li, "A three-scroll chaotic attractor," *Physics Letters A*, **372**, 387-393, 2008.

[12] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematical and Computer Modelling*, **55**, 1904-1915, 2012.

[13] V. Sundarapandian, "Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers," *Journal of Engineering Science and Technology Review*, **6**, 45-52, 2013.

[14] S. Vaidyanathan and K. Madhavan, "Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system," *International Journal of Control Theory and Applications*, **6**, 121-137, 2013.

[15] S. Vaidyanathan, "A new six-term 3-D chaotic system with an exponential nonlinearity," *Far East Journal of Mathematical Sciences*, **79**, 135-143, 2013.

[16] S. Vaidyanathan, "Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters," *Journal of Enginering Science and Technology Review*, **6**, 53-65, 2013.

[17] S. Vaidyanathan, "A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities," *Far East Journal of Mathematical Sciences*, **84**, 219-226, 2014.

[18] S. Vaidyanathan, "Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities," *International Journal of Modelling, Identification and Control*, **22**, 41-53, 2014.

[19] S. Vaidyanathan, C. Volos, V.-T. Pham, K. Madhavan and B.A. Idowu, "Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities," *Archives of Control Sciences*, **24**, 375-403, 2014.

[20] S. Vaidyanathan, "Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities," *European Physical Journal: Special Topics*, **223**, 1519-1529, 2014.

[21] S. Vaidyanathan, "Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control," *International Journal of Modelling, Identification and Control*, **22**, 207-217, 2014.

[22] S. Vaidyanathan, "Qualitative analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with a quartic nonlinearity," *International Journal of Control Theory and Applications*, **7**, 1-20, 2014.

[23] S. Vaidyanathan, C.K. Volos and V.-T. Pham, "Global chaos control of a novel nine-term chaotic system via sliding mode control," *Studies in Comptuational Intelligence*, **576**, 571-590, 2015.

[24] S. Vaidyanathan and A.T. Azar, "Analysis, control and synchronization of a nine-term 3-D novel chaotic system," *Studies in Computational Intelligence*, **581**, 19-38, 2015.

[25] S. Vaidyanathan, "Analysis, properties and control of an eight-term 3-D chaotic system with an exponential nonlinearity," *International Journal of Modelling, Identification and Control*, **23**, 164-172, 2015.

[26] S. Vaidyanathan, "A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters," *Journal of Engineering Science and Technology Review*, **8**, 106-115, 2015.

[27] S. Vaidyanathan, K. Rajagopal, C.K. Volos, I.M. Kyprianidis and I.N. Stouboulos, "Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW," *Journal of Engineering Science and Technology Review*, **8**, 130-141, 2015.

[28] S. Sampath, S. Vaidyanathan, C.K. Volos and V.-T. Pham, "An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation," *Journal of Engineering Science and Technology Review*, **8**, 1-6, 2015.

[29] S. Vaidyanathan and S. Pakiriswamy, "A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control," *Journal of Engineering Science and Technology Review*, **8**, 52-60, 2015.

[30] S. Vaidyanathan, C.K. Volos, I.M. Kyprianidis, I.N. Stouboulos and V.-T. Pham, "Analysis, adaptive control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation," *Journal of Engineering Science and Technology Review*, **8**, 24-36, 2015.

[31] S. Vaidyanathan, C.K. Volos and V.-T. Pham, "Analysis, adaptive control and adaptive synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation," *Journal of Engineering Science and Technology Review*, **8**, 174-184, 2015.

[32] S. Vaidyanathan and C. Volos, "Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system," *Archives of Control Sciences*, **25**, 333-353, 2015.

[33] S. Vaidyanathan, "Analysis, control and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities," *Kyungpook Mathematical Journal*, **55**, 563-586, 2015.

[34] V. Sundarapandian and I. Pehlivan, "Analysis, control, synchronization and circuit design of a novel chaotic system," *Mathematical and Computer Modelling*, **55**, 1904-1915, 2012.

[35] I. Pehlivan, I.M. Moroz and S. Vaidyanathan, "Analysis, synchronization and circuit design of a novel butterfly attractor," *Journal of Sound and Vibration*, **333**, 5077-5096, 2014.

[36] V.-T. Pham, C.K. Volos and S. Vaidyanathan, "Multi-scroll chaotic oscillator based on a first-order delay differential equation," *Studies in Computational Intelligence*, **581**, 59-72, 2015.

[37] V.-T. Pham, S. Vaidyanathan, C.K. Volos and S. Jafari, "Hidden attractors in a chaotic system with an exponential nonlinear term," *European Physical Journal: Special Topics*, **224**, 1507-1517, 2015.

[38] S. Jafari and J.C. Sprott, "Simple chaotic flows with a line equilibrium," *Chaos, Solitons and Fractals*, **57**, 79-84, 2013.

[39] S. Vaidyanathan, "A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control," *International Journal of Control Theory and Applications*, **6**, 97-109, 2013.

[40] S. Vaidyanathan, "Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities," *International Journal of Control Theory and Applications*, **7**, 35-47, 2014.

[41] S. Vaidyanathan and A.T. Azar, "Analysis and control of a 4-D novel hyperchaotic system," *Studies in Computational Intelligence*, **581**, 3-17, 2015.

[42] S. Vaidyanathan, Ch. K. Volos and V.T. Pham, "Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation," *Archives of Control Sciences*, **24**, 409-446, 2014.

[43] M. Lakshmanan and K. Murali, *Chaos in Nonlinear Oscillators: Controlling and Synchronization*, World Scientific: Singapore, 1996.

[44] S.K. Han, C. Kerrer and Y. Kuramoto, "Dephasing and bursting in coupled neural oscillators," *Physical Review Letters*, **75**, 3190-3193, 1995.

[45] S. Vaidyanathan, "Adaptive synchronization of chemical chaotic reactors," *International Journal of ChemTech Research*, **8** (2), 612-621, 2015.

[46] S. Vaidyanathan, "Adaptive control of a chemical chaotic reactor," *International Journal of PharmTech Research*, **8** (3), 377-382, 2015.

[47] S. Vaidyanathan, "Dynamics and control of Brusselator chemical reaction," *International Journal of ChemTech Research*, **8** (6), 740-749, 2015.

[48] S. Vaidyanathan, "Anti-synchronization of Brusselator chemical reaction systems via adaptive control," *International Journal of ChemTech Research*, **8** (6), 759-768, 2015.

[49] S. Vaidyanathan, "Dynamics and control of Tokamak system with symmetric and magnetically confined plasma," *International Journal of ChemTech Research*, **8** (6), 795-803, 2015.

[50] S. Vaidyanathan, "Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control," *International Journal of ChemTech Research*, **8** (6), 818-827, 2015.

[51] S. Vaidyanathan, "A novel chemical chaotic reactor system and its adaptive control," *International Journal of ChemTech Research*, **8** (7), 146-158, 2015.

[52] S. Vaidyanathan, "Adaptive synchronization of novel 3-D chemical chaotic reactor systems," *International Journal of ChemTech Research*, **8** (7), 159-171, 2015.

[53] S. Vaidyanathan, "Global chaos synchronization of chemical chaotic reactors via novel sliding mode control method," *International Journal of ChemTech Research*, **8** (7), 209-221, 2015.

[54] S. Vaidyanathan, "Sliding mode control of Rucklidge chaotic system for nonlinear double convection," *International Journal of ChemTech Research*, **8** (8), 25-35, 2015.

[55] S. Vaidyanathan, "Global chaos synchronization of Rucklidge chaotic systems for double convection via sliding mode control," *International Journal of ChemTech Research*, **8** (8), 61-72, 2015.

[56] S. Vaidyanathan, "Anti-synchronization of chemical chaotic reactors via adaptive control method," *International Journal of ChemTech Research*, **8** (8), 73-85, 2015.

[57] S. Vaidyanathan, "Adaptive synchronization of Rikitake two-disk dynamo chaotic systems," *International Journal of ChemTech Research*, **8** (8), 100-111, 2015.

[58] S. Vaidyanathan, "Adaptive control of Rikitake two-disk dynamo system," *International Journal of ChemTech Research*, **8** (8), 121-133, 2015.

[59] S. Vaidyanathan, "Adaptive backstepping control of enzymes-substrates system with ferroelectric behaviour in brain waves," *International Journal of PharmTech Research*, **8** (2), 256-261, 2015.

[60] S. Vaidyanathan, "Adaptive biological control of generalized Lotka-Volterra three-species biological system," *International Journal of PharmTech Research*, **8** (4), 622-631, 2015.

[61] S. Vaidyanathan, "3-cells Cellular Neural Network (CNN) attractor and its adaptive biological control," *International Journal of PharmTech Research*, **8** (4), 632-640, 2015.

[62] S. Vaidyanathan, "Adaptive synchronization of generalized Lotka-Volterra three-species biological systems," *International Journal of PharmTech Research*, **8** (5), 928-937, 2015.

[63] S. Vaidyanathan, "Synchronization of 3-cells Cellular Neural Network (CNN) attractors via adaptive control method," *International Journal of PharmTech Research*, **8** (5), 946-955, 2015.

[64] S. Vaidyanathan, "Chaos in neurons and adaptive control of Birkhoff-Shaw strange chaotic attractor," *International Journal of PharmTech Research*, **8** (5), 956-963, 2015.

[65] S. Vaidyanathan, "Adaptive chaos synchronization of enzymes-substrates system with ferroelectric behaviour in brain waves," *International Journal of PharmTech Research*, **8** (5), 964-973, 2015.

[66] S. Vaidyanathan, "Lotka-Volterra population biology models with negative feedback and their ecological monitoring," *International Journal of PharmTech Research*, **8** (5), 974-981, 2015.

[67] S. Vaidyanathan, "Chaos in neurons and synchronization of Birkhoff-Shaw strange chaotic attractors via adaptive control," *International Journal of PharmTech Research*, **8** (6), 1-11, 2015.

[68] S. Vaidyanathan, "Lotka-Volterra two species competitive biology models and their ecological monitoring," *International Journal of PharmTech Research*, **8** (6), 32-44, 2015.

[69] S. Vaidyanathan, "Coleman-Gomatam logarithmic competitive biology models and their ecological monitoring," *International Journal of PharmTech Research*, **8** (6), 94-105, 2015.

[70] S. Vaidyanathan, "Output regulation of the forced Van der Pol chaotic oscillator via adaptive control method," *International Journal of PharmTech Research*, **8** (6), 106-116, 2015.

[71] S. Vaidyanathan, "Adaptive control of the FitzHugh-Nagumo chaotic neuron model," *International Journal of PharmTech Research*, **8** (6), 117-127, 2015.

[72] S. Vaidyanathan, "Global chaos synchronization of the forced Van der Pol chaotic oscillators via adaptive control method," *International Journal of PharmTech Research*, **8** (6), 156-166, 2015.

[73] S. Vaidyanathan, "Adaptive synchronization of the identical FitzHugh-Nagumo chaotic neuron models," *International Journal of PharmTech Research*, **8** (6), 167-177, 2015.

[74] S. Vaidyanathan, "Global chaos synchronization of the Lotka-Volterra biological systems with four competitive species via active control," *International Journal of PharmTech Research*, **8** (6), 206-217, 2015.

[75] S. Vaidyanathan, "Anti-synchronization of 3-cells cellular neural network attractors via adaptive control method," *International Journal of PharmTech Research*, **8** (7), 26-38, 2015.

[76] S. Vaidyanathan, "Active control design for the anti-synchronization of Lotka-Volterra biological systems with four competitive species," *International Journal of PharmTech Research*, **8** (7), 58-70, 2015.

[77] S. Vaidyanathan, "Anti-synchronization of the FitzHugh-Nagumo chaotic neuron models via adaptive control method," *International Journal of PharmTech Research*, **8** (7), 71-83, 2015.

[78] S. Vaidyanathan, "Sliding controller design for the global chaos synchronization of enzymes-substrates systems," *International Journal of PharmTech Research*, **8** (7), 89-99, 2015.

[79] S. Vaidyanathan, "Sliding controller design for the global chaos synchronization of forced Van der Pol chaotic oscillators," *International Journal of PharmTech Research*, **8** (7), 100-111, 2015.

[80] S. Vaidyanathan, "Lotka-Volterra two-species mutualistic biology models and their ecological monitoring," *International Journal of PharmTech Research*, **8** (7), 199-212, 2015.

[81] B. Blasius, A. Huppert and L. Stone, "Complex dynamics and phase synchronization in spatially extended ecological system," *Nature*, **399**, 354-359, 1999.

[82] I. Suárez, "Mastering chaos in ecology", *Ecological Modelling*, **117**, 305-314, 1999.

[83] K. Aihira, T. Takabe and M. Toyoda, "Chaotic neural networks", *Physics Letters A*, **144**, 333-340, 1990.

[84] I. Tsuda, "Dynamic link of memory – chaotic memory map in nonequilibrium neural networks", *Neural Networks*, **5**, 313-326, 1992.

[85] S. Lankalapalli and A. Ghosal, "Chaos in robot control equations," *Interntional Journal of Bifurcation and Chaos*, **7**, 707-720, 1997.

[86] Y. Nakamura and A. Sekiguchi, "The chaotic mobile robot," *IEEE Transactions on Robotics and Automation*, **17**, 898-904, 2001.

[87] V.-T. Pham, C. K. Volos, S. Vaidyanathan and V. Y. Vu, "A memristor-based hyperchaotic system with hidden attractors: dynamics, synchronization and circuital emulating," *Journal of Engineering Science and Technology Review*, **8**, 205-214, 2015.

[88] C. K. Volos, I. M. Kyprianidis, I. N. Stouboulos, E. Tlelo-Cuautle and S. Vaidyanathan, "Memristor: A new concept in synchronization of coupled neuromorphic circuits," *Journal of Engineering Science and Technology Review*, **8,** 157-173, 2015.

[89] V.-T. Pham, C. Volos, S. Jafari, X. Wang and S. Vaidyanathan, "Hidden hyperchaotic attractor in a novel simple memristive neural network," *Optoelectronics and Advanced Materials, Rapid Communications*, **8**, 1157-1163, 2014.

[90] J.J. Buckley and Y. Hayashi, “Applications of fuzzy chaos to fuzzy simulation,” *Fuzzy Sets and Systems*, **99**, 151-157, 1998.

[91] C.F. Hsu, “Adaptive fuzzy wavelet neural controller design for chaos synchronization,” *Expert Systems with Applications*, **38**, 10475-10483, 2011.

[92] E. Ott, C. Grebogi and J.A. Yorke, “Controlling chaos,” *Physical Review Letters*, **64**, 1196-1199, 1990.

[93] J. Wang, T. Zhang and Y. Che, “Chaos control and synchronization of two neurons exposed to ELF external electric field,” *Chaos, Solitons and Fractals*, **34**, 839-850, 2007.

[94] V. Sundarapandian, “Output regulation of the Van der Pol oscillator,” *Journal of the Institution of Engineers (India): Electrical Engineering Division*, **88**, 20-14, 2007.

[95] V. Sundarapandian, “Output regulation of the Lorenz attractor,” *Far East Journal of Mathematical Sciences*, **42**, 289-299, 2010.

[96] S. Vaidyanathan, “Output regulation of the unified chaotic system,” *Communications in Computer and Information Science*, **198**, 1-9, 2011.

[97] S. Vaidyanathan, “Output regulation of Arneodo-Coullet chaotic system,” *Communications in Computer and Information Science*, **133**, 98-107, 2011.

[98] S. Vaidyanathan, “Output regulation of the Liu chaotic system,” *Applied Mechanics and Materials*, **110**, 3982-3989, 2012.

[99] V. Sundarapandian, “Adaptive control and synchronization of uncertain Liu-Chen-Liu system,” *International Journal of Computer Information Systems*, **3**, 1-6, 2011.

[100] V. Sundarapandian, “Adaptive control and synchronization of the Shaw chaotic system,” *International Journal in Foundations of Computer Science and Technology*, **1**, 1-11, 2011.

[101] S. Vaidyanathan, “Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system,” *International Journal of Control Theory and Applications*, **5**, 15-20, 2012.

[102] S. Vaidyanathan, “Global chaos control of hyperchaotic Liu system via sliding control method,” *International Journal of Control Theory and Applications*, **5**, 117-123, 2012.

[103] S. Vaidyanathan, “Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control,” *International Journal of Web and Grid Services*, **22**, 170-177, 2014.

[104] M. Feki, “An adaptive chaos synchronization scheme applied to secure communication,” *Chaos, Solitons and Fractals*, **18**, 141-148, 2003.

[105] L. Kocarev and U. Parlitz, “General approach for chaos synchronization with applications to communications,” *Physical Review Letters*, **74**, 5028-5030, 1995.

[106] K. Murali and M. Lakshmanan, “Secure communication using a compound signal using sampled-data feedback,” *Applied Math. Mech.*, **11**, 1309-1315, 2003.

[107] J. Yang and F. Zhu, “Synchronization for chaotic systems and chaos-based secure communications via both reduced-order and step-by-step sliding mode observers,” *Communications in Nonlinear Science and Numerical Simulation*, **18**, 926-937, 2013.

[108] L. Kocarev, “Chaos-based cryptography: a brief overview,” *IEEE Circuits and Systems*, **1**, 6-21, 2001.

[109] H. Gao, Y. Zhang, S. Liang and D. Li, “A new chaotic algorithm for image encryption,” *Chaos, Solitons and Fractals*, **29**, 393-399, 2006.

[110] Y. Wang, K.W. Wang, X. Liao and G. Chen, “A new chaos-based fast image encryption,” *Applied Soft Computing*, **11**, 514-522, 2011.

[111] X. Zhang, Z. Zhao and J. Wang, “Chaotic image encryption based on circular substitution box and key stream buffer,” *Signal Processing: Image Communication*, **29**, 902-913, 2014.

[112] L.M. Pecora and T.I. Carroll, “Synchronization in chaotic systems,” Phys. Rev. Lett., 64, 821-824, 1990.

[113] L.M. Pecora and T.L. Carroll, “Synchronizing in chaotic circuits,” IEEE Trans. Circ. Sys., 38, 453-456, 1991.

[114] L. Huang, R. Feng and M. Wang, “Synchronization of chaotic systems via nonlinear control,” *Physics Letters A*, **320**, 271-275, 2004.

[115] V. Sundarapandian and R. Karthikeyan, “Global chaos synchronization of hyperchaotic Liu and hyperchaotic Lorenz systems by active nonlinear control”, *International Journal of Control Theory and Applications*, **3**, 79-91, 2010.

[116] S. Vaidyanathan and S. Rasappan, “New results on the global chaos synchronization for Liu-Chen-Liu and Lü chaotic systems,” *Communications in Computer and Information Science*, **102**, 20-27, 2010.

[117] S. Vaidyanathan and K. Rajagopal, "Anti-synchronization of Li and T chaotic systems by active nonlinear control," *Communications in Computer and Information Science,* **198**, 175-184, 2011.

[118] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control," *Communications in Computer and Information Science*, **198**, 10-17, 2011.

[119] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control," *Communications in Computer and Information Science*, **204**, 84-93, 2011.

[120] P. Sarasu and V. Sundarapandian, "Active controller design for generalized projective synchronization of four-scroll chaotic systems," *International Journal of Systems Signal Control and Engineering Application*, **4**, 26-33, 2011.

[121] S. Vaidyanathan, "Hybrid chaos synchronization of Liu and Lü systems by active nonlinear control," *Communications in Computer and Information Science*, **204**, 1-10, 2011.

[122] P. Sarasu and V. Sundarapandian, "The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control," *International Journal of Soft Computing*, **6**, 216-223, 2011.

[123] S. Vaidyanathan and S. Rasappan, "Hybrid synchronization of hyperchaotic Qi and Lü systems by nonlinear control," *Communications in Computer and Information Science*, **131**, 585-593, 2011.

[124] S. Vaidyanathan and S. Pakiriswamy, "The design of active feedback controllers for the generalized projective synchronization of hyperchaotic Qi and hyperchaotic Lorenz systems," *Communications in Computer and Information Science*, **245**, 231-238, 2011.

[125] S. Vaidyanathan and K. Rajagopal, "Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 55-61, 2011.

[126] V. Sundarapandian and R. Karthikeyan, "Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control," *Journal of Engineering and Applied Sciences*, **7**, 254-264, 2012.

[127] S. Pakiriswamy and S. Vaidyanathan, "Generalized projective synchronization of three-scroll chaotic systems via active control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **85**, 146-155, 2012.

[128] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of double-scroll chaotic systems using active feedback control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 111-118, 2012.

[129] S. Pakiriswamy and S. Vaidyanathan, "Generalized projective synchronization of hyperchaotic Lü and hyperchaotic Cai systems via active control," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 53-62, 2012.

[130] S. Vaidyanathan, "Complete chaos synchronization of six-term Sundarapandian chaotic systems with exponential nonlinearity via active and adaptive control," *Proceedings of the 2013 International Conference on Green Computing, Communication and Conservation of Energy*, **ICGCE 2013**, 608-613, 2013.

[131] R. Karthikeyan and V. Sundarapandian, "Hybrid chaos synchronization of four-scroll systems via active control," *Journal of Electrical Engineering*, **65**, 97-103, 2014.

[132] S. Vaidyanathan, A. T. Azar, K. Rajagopal and P. Alexander, "Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control," *International Journal of Modelling, Identification and Control*, 23 (3), 267-277, 2015.

[133] B. Samuel, "Adaptive synchronization between two different chaotic dynamical systems," *Adaptive Commun. Nonlinear Sci. Num. Simul.*, **12**, 976-985, 2007.

[134] J.H. Park, S.M. Lee and O.M. Kwon, "Adaptive synchronization of Genesio-Tesi system via a novel feedback control," *Physics Letters A*, **371**, 263-270, 2007.

[135] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of System Signal Control and Engineering Applications*, **4**, 18-25, 2011.

[136] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control," *Communications in Computer and Information Science*, **205**, 193-202, 2011.

[137] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control," *European Journal of Scientific Research*, **64**, 94-106, 2011.

[138] V. Sundarapandian and R. Karthikeyan, "Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control," *International Journal of Systems Signal Control and Engineering Application*, **4**, 18-25, 2011.

[139] P. Sarasu and V. Sundarapandian, "Generalized projective synchronization of three-scroll chaotic systems via adaptive control," *European Journal of Scientific Research*, **72**, 504-522, 2012.

[140] V. Sundarapandian and R. Karthikeyan, "Adaptive anti-synchronization of uncertain Tigan and Li systems," *Journal of Engineering and Applied Sciences*, **7**, 45-52, 2012.

[141] S. Vaidyanathan and K. Rajagopal, "Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control," *International Journal of Soft Computing*, **7**, 28-37, 2012.

[142] P. Sarasu and V. Sundarapandian, "Generalized projective synchronization of two-scroll systems via adaptive control," *International Journal of Soft Computing*, **7**, 146-156, 2012.

[143] P. Sarasu and V. Sundarapandian, "Adaptive controller design for the generalized projective synchronization of 4-scroll systems," *International Journal of Systems Signal Control and Engineering Application*, **5**, 21-30, 2012.

[144] S. Vaidyanathan, "Adaptive controller and synchronizer design for the Qi-Chen chaotic system," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **85**, 124-133, 2012.

[145] S. Vaidyanathan, "Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control," *International Journal of Control Theory and Applications*, **5**, 41-59, 2012.

[146] V. Sundarapandian, "Adaptive control and synchronization design for the Lu-Xiao chaotic system," *Springer-Verlag Lecture Notes in Electrical Engineering*, **131**, 319-327, 2013.

[147] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of six-term Sundarapandian chaotic systems by adaptive control," *International Journal of Control Theory and Applications*, **6**, 153-163, 2013.

[148] S. Vaidyanathan and S. Pakiriswamy, "Generalized projective synchronization of Elhadj chaotic systems via adpative control," *Proceedings of the 2013 International Conference on Green Computing, Communication and Conservation of Energy*, **ICGCE 2013**, 614-618, 2013.

[149] S. Vaidyanathan, "Anaysis, control and synchronization of hyperchaotic Zhou system via adaptive control," *Advances in Intelligent Systems and Computing*, **177**, 1-10, 2013.

[150] X. Wu and J. Lü, "Parameter identification and backstepping control of uncertain Lü system," *Chaos, Solitons and Fractals*, **18**, 721-729, 2003.

[151] Y.G. Yu and S.C. Zhang, "Adaptive backstepping synchronization of uncertain chaotic systems," *Chaos, Solitons and Fractals*, **27**, 1369-1375, 2006.

[152] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of Chen-Lee systems via backstepping control," *IEEE-International Conference on Advances in Engineering, Science and Management*, **ICAESM-2012**, 73-77, 2012.

[153] R. Suresh and V. Sundarapandian, "Global chaos synchronization of WINDMI and Coullet chaotic systems by backstepping control," *Far East Journal of Mathematical Sciences*, **67**, 265-287, 2012.

[154] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback," *Archives of Control Sciences*, **22**, 343-365, 2012.

[155] S. Rasappan and S. Vaidyanathan, "Synchronization of hyperchaotic Liu via backstepping control with recursive feedback," *Communications in Computer and Information Science*, **305**, 212-221, 2012.

[156] S. Vaidyanathan, "Global chaos synchronization of Arneodo chaotic system via backstepping controller design", *ACM International Conference Proceeding Series*, **CCSEIT-12**, 1-6, 2012.

[157] R. Suresh and V. Sundarapandian, "Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping controller with recursive feedback," *Far East Journal of Mathematical Sciences*, **73**, 73-95, 2013.

[158] S. Rasappan and S. Vaidyanathan, "Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback," *Malaysian Journal of Mathematical Sciences*, **7**, 219-246, 2013.

[159] S. Rasappan and S. Vaidyanathan, "Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design," *Kyungpook Mathematical Journal*, **54**, 293-320, 2014.

[160] S. Vaidyanathan and S. Rasappan, "Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback," *Arabian Journal for Science and Engineering*, **39**, 3351-3364, 2014.

[161] S. Vaidyanathan, B.A. Idowu and A.T. Azar, "Backstepping controller design for the global chaos synchronization of Sprott's jerk systems," Studies in Computational Intelligence, **581**, 39-58, 2015.

[162] S. Vaidyanathan, "Global chaos synchronization of Lorenz-Stenflo and Qi chaotic systems by sliding mode control," *International Journal of Control Theory and Applications*, **4**, 161-172, 2011.

[163] S. Vaidyanathan and S. Sampath, "Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control," *Communications in Computer and Information Science*, **205**, 156-164, 2011.

[164] V. Sundarapandian and S. Sivaperumal, "Sliding controller design of hybrid synchronization of four-wing chaotic systems", *International Journal of Soft Computing*, **6**, 224-231, 2011.

[165] S. Vaidyanathan and S. Sampath, "Anti-synchronization of four-wing chaotic systems via sliding mode control", *International Journal of Automation and Computing*, **9**, 274-279, 2012.

[166] S. Vaidyanathan and S. Sampath, "Sliding mode controller design for the global chaos synchronization of Coullet systems," *Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering*, **84**, 103-110, 2012.

[167] S. Vaidyanathan, "Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control," *International Journal of Modelling, Identification and Control*, **22**, 170-177, 2014.

[168] C.H. Lien, L. Zhang, S. Vaidyanathan and H.R. Karimi, "Switched dynamics with its applications," *Abstract and Applied Analysis*, **2014**, art. no. 528532, 2014.

[169] S. Vaidyanathan and A.T. Azar, "Anti-synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan-Madhavan chaotic systems," *Studies in Computational Intelligence*, **576**, 527-545, 2015.

[170] S. Vaidyanathan and A.T. Azar, "Hybrid synchronization of identical chaotic systems using sliding mode control and an application to Vaidyanathan chaotic systems," *Studies in Computational Intelligence*, **576**, 549-569, 2015.

[171] A.M. Rucklidge, "Chaos in models of double convection," *Journal of Fluid Mechanics*, **237**, 209-229, 1992.

[172] H.K. Khalil, *Nonlinear Systems*, 3rd edition, Prentice Hall, New Jersey, USA.