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Risk Assessment Model and Resource Allocation Model (RAM^2): Model & Methodology

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ABSTRACT

We illustrate an application of multiple operations research and decision making tools to a risk assessment scenario involving threats and an energy grid network. We illustrate our proposed processes and methodologies that include network analysis, key node analysis the hybrid multi-attribute decision method: Analytical Hierarchy Process (AHP) and Technique of Order Preference by Similarity to Ideal Solution (TOPSIS), and optimization. We incorporation into a risk assessment tool, and then incorporate the results into a resource allocation optimization program. This report is a preliminary report and shows an example of the proposed methodology.

Keywords: *risk models, TOPSIS, AHP weights, key node network analysis, optimization models, integer programming, binary resource allocation models.*

Disclaimer: *The views in this research are the views of the author and are not the expressed views of the Naval Postgraduate School, the United States Navy, the Department of Defense, or the Department of Homeland Security.*

INTRODUCTION

Risk analysis and risk assessment are very important especially for government and military organizations. A possible risk assessment flow diagram is shown in Figure 1 from the Homeland Security website (accessed September 9, 2016). In this paper we address terrorism or a cyberattack on energy facilities as well as other major risk areas of concern for Homeland Security. Additionally, in order to keep this paper unclassified we assume the network grid is for vulnerabilities of a network and we assume the threats listed and used are credible.

Ersdale *et al.* (2008) and Aven (2016) discusses metrics that could be used in assessment and analysis. Figure 2 represent Aven's approach to a combination of fact- based and value-based analysis and data.

Let's assume that we have been asked to consider a situation to evaluate risks as well as develop a resource allocation plan to either provide security or clean up.

To illustrate an application of multiple operations research and decision making tools to a risk assessment scenario involving energy. We illustrate possible processes & methods including network analysis, key node analysis using the following hybrid method, Analytical Hierarchy Process (AHP), Technique of Order Preference by Similarity to Ideal Solution (TOPSIS), incorporation into a risk assessment tool, and then an resource allocation optimization program. This report is a preliminary report and shows an example of this methodology.

Network Analysis

According to Newman (2010) there are 4 metrics that contribute to identifying the key nodes of a network. These four are *Total Centrality, Closeness Centrality, Betweenness,* and *Eigenvector Centrality.* Although the model can handle all SN metrics, we restrict our analysis here to these four. We provide typical social network definitions of these four metrics.

Betweenness centrality: across all nodes pairs that have shortest paths containing v, the percentage that pass through v. Individuals or organizations are potentially influential or bear influence or serve as gatekeepers.

Total degree centrality: normalized sum of its row and column degrees. Individuals or organizations that are "in the know" as linked to many others so have more connections.

Eigenvector centrality: calculates the principle eigenvector of the network. Leaders of cliques that are connected to other cliques are important.

Closeness centrality: average closeness of a node to the other nodes in a network (also called out-closeness). Loosely speaking, it is the inverse of the average distance in a network from a node to all other nodes.

The network under consideration in our example is the US Energy Grid, Figure 3. In this grid there were 4941 nodes and 6594 links (Power Grid obtained from NPS CORE lab on January 2016).

PROPOSED METHODOLOGY

Figure 4 provides the basis for our proposed methodology. We note the initially there might not be a network to analyze and that this step might be skipped without loss of generality.

We describe these steps more in detail and illustrate with our energy grid example.

Methodology: Energy Network Phase

This step does not have to be energy but can be any system that can be described via a network.

Steps in this RAM Energy process

Step 1. Identify network of interest

Step 2. Obtain network metrics

Step 3. Use MADM to rank nodes according to weighted scheme for four key metrics. We used ORA, a social network software from our CORE Lab.

Step 4. Add key node as well as any additional metric's data to our RAM data base

Step 5. Run RAM model to get ranking (Benefit values from TOPSIS).

Step 6. Use these ranking as cost coefficients in an optimization-resource allocation model.

Step 7. Interpret model results

Step 8. Perform Sensitivity Analysis and re-interpret result

We use the energy grid previously shown and run the grid through ORA (Source). We provide a summarized result of the top nodes in Table 1.

We have our experts prioritize the four metrics according to importance. These are chosen as Betweenness, Closeness centrality, Total centrality, and Eigenvector centrality. We built a template to build the decision criteria weights based upon the Saaty method (1980) described for AHP.

We must ensure that this pairwise matrix is consistent according to Saaty's scheme to compute the Consistency Ratio, *CR, (1980)*. The value of *CR* must be less than or equal to 0.1 to be considered consistent. Saaty's (1980) computed the random index, *RI*, for random matrices for up to 10 criteria shown in Table 3.

Next, we approximate the largest eigenvalue, *l*, using the power method [16]. We compute the consistency index, *CI*, using the formula:

$$CI = \frac{(\lambda - n)}{(n-1)}$$

Then we compute the *CR* using:

$$CR = \frac{CI}{RI}$$

If $CR \le 0.10$, then our pairwise comparison matrix is consistent and we may continue the AHP process. If not, we must go back to our pairwise comparison and fix the inconsistencies until the $CR \le 0.10$. In general, the consistency ensures that if $A > B, B > C$, that $A > C$ for all *A, B*, and *C* all of which can be criteria or alternatives related by pairwise comparisons. We use an Excel template to input our values and obtain both the CR vale and weights, as shown in Figure 5.

The consistency ratio, *CR*, equals 0.012463167 which is less than the required 0.10. Thus, we accept the matrix below and continue to find the decision weights. We obtain the pairwise matrix and then the weights.

Pairwise matrix

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 1/3 & 1 & 3 & 5 \\ 1/5 & 1/3 & 1 & 3 \\ 1/7 & 1/5 & 1/3 & 1 \end{bmatrix}$$

The decision weights are found using methods described in previous research involving networks by Fox (2014), Fox & Everton (2013, 2014) and Fox, Ormand, & Williams *et al.* (2014): Betweenness = 0.593488372, Closeness centrality = 0.225348837, Total centrality = 0.112325581, and Eigenvector centrality = 0.068837209.

From some previous multi-attribute research work (Fox, 2016), we decided to try a method based upon ranks using the simple of additive weights (SAW) method. We compute the weighted average of the ranks using a modified simple of additive weights (SAW) method. We took the ranking 1-25 of the nodes from the ORA report. We then computed the weighted average where in this case, the lower average is better.

Simple of Additive Weights (SAW) Method

This is a very straight forward and easily constructed process. Fisburn has referred to this also as the weighted sum method (Fishburn, 1967). SAW is the simplest, and still one of the widest used of the MADM methods because it is very easy to use. Depending on the type of the relational data used, we might either want the larger sum (raw data) or the smaller sum (if using ranks). It is also important to use decision maker weights in the process.

Here, each criterion (attribute) is given a weight, and the sum of all weights must be equal to 1. If equally weighted criteria then we merely need to sum the alternative values. Each alternative is assessed with regard to every criterion (attribute). The overall or composite performance score of an alternative is given simply by Equation 1 with m criteria.

$$P_i = (\Sigma_{j=1}^{m} w_j\, m_{ij}) / m \tag{1}$$

It was previously though that all the units in the criteria must be identical units of measure such as dollars, pounds, seconds, etc). A normalization process can make the values unit less. So, we recommend normalizing the data as shown in equation 2:

$$P_i = \left(\Sigma_{j=1}^{m} w_j\, m_{ij\,Normalized}\right) / m \tag{2}$$

where ($m_{ijNormalized}$) represents the normalized value of m_{ij}, and P_i is the overall or composite score of the alternative A_i. The alternative with the highest value of P_i is considered the best alternative.

The reason that we choose SAW is its strengths. The strengths are (1) the ease of use and (2) the normalized data allow for comparison across many differing criteria. Limitations include larger is always better or smaller is always better. There is not the flexibility in this method to state which criterion should be larger or smaller to achieve better performance. This makes gathering useful data of the same relational value scheme (larger or smaller) essential. Our data for the network is not affected by the limitations so we can use SAW without reservation.

Sensitivity analysis should be applied to the weighting scheme employed to determine how sensitive the model is to the weights. Weighting can be arbitrary for a decision maker or in order to obtain weights you might choose to use a scheme to perform pairwise comparison as we show in AHP that we discuss later. Whenever subjectivity enters into the process for finding weights, then sensitivity analysis is recommended. Please see later sections for a suggested scheme for dealing with sensitivity analysis for individual criteria weights.

From ORA, a social network analysis software platform, we obtain the values shown in Table 4. We multiplied the values in Table 5 by the weights and averaged the row values to obtain Table 5.

We then ordered these smaller to larger because lower ranks are better (i.e. ranked 1st is better than ranked 10th). These are displayed in Table 6. We note that if larger and smaller can both be best depending on the data then TOPSIS should be used in lieau of SAW.

From this analysis, we conclude that energy Nodes: 1243, 2543, 4164, 1308, and 2528 are key nodes with vulnerabilities that cause the greatest failures across the systems. More data is needed on these nodes for the next model using AHP and TOPSIS. We used sensitivity analysis (Alinezhad et al., 2011) by modifying the decision weights and seeing the impact on our final node ranks. These are shown in Table 7 and Figure 6.

As we said, we prefer equation (3) (Alinezhad *et al.*, 2011) for adjusting weights which falls under the incremental analysis:

$$W_j' = \frac{1 - w_p'}{1 - w_p} W_j \tag{3}$$

where w_j' is the new weight and w_p is the original weight of the criterion to be adjusted and w_p' is the value after the criterion was adjusted. We found this to be an easy method to adjust weights to reenter back into our model.

Other risks need to be included. Table 8 shows the additional risk data and threats that we included as well as their data available. We were able to build a data set that included intelligence data gathered on threat scenarios using unclassified resources. The criteria of data gathered are: reliability of threat, number of intelligence tips, location, psychological profile of damage, cost of successful threat in dollars, and casualty estimation.

Next, we describe the second part of our methodology.

Steps in the RAM II process

Step 1. Identify data

Step 2. Prioritize criteria

Step 3. Use AHP and pairwise comparisons to obtain initial criteria weights

Step 4. Enter real/estimated data into TOPSIS in the same order as the prioritized criteria.

Step 5. Run TOPSIS RAM model to get ranking (Benefit values from TOPSIS).

Step 6. Use these ranking as cost coefficients in an optimization-resource allocation model.

Step 7. Interpret model results

Step 8. Perform Sensitivity Analysis and re-interpret results.

First, we need to prioritize the decision criteria and enter those into our template. The prioritized list is: reliability, destructive psychological impact, deaths, location, cost, and threats as the number of related tips as shown in Figure 7.

Pairwise matrix

Reliability	1	3	4	5	6	7
Destructive psychological	0.3333333					
impact	3	1	2	3	4	5
Deaths	0.25	0.5	1	2	3	4
		0.3333333				
Location	0.2	3	0.5	1	2	2
Cost	0.1666666		0.3333333			
	7	0.25	3	0.5	1	2
Threats	0.1428571					
	4	0.2	0.25	0.5	0.5	1

The weights are found as: Reliability (0.4771), Destructive psychological impact (0.1933), Deaths (0.12967), Location (0.0.8776), Cost (0.06458), and Threat Tips (0.05053). Our *CR* is 0.00438117 which is less than 0.10 indicating the matrix is consistent. Our weights as shown above and used with data in table 6. The TOPSIS methodology is described fully in previous research (Hwang *et al.* 1981; Fox 2014, 2015, 2015, 2015).

The TOPSIS process is carried out using the following steps:

Step 1 Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criterion given as x_{ij}, giving us a matrix $(\boldsymbol{X}_{ij})_{m\,x\,n}$.

$$
\begin{array}{c c}
 & \begin{array}{cccccc} x_1 & x_2 & x_3 & \cdot & \cdot & \cdot \; x_n \end{array} \\
D = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix}
\end{array}
$$

Step 2 The matrix shown as $\boldsymbol{D}$ above then is normalized to form the matrix $\boldsymbol{R} = (\boldsymbol{R}_{ij})_{m\,x\,n}$ as shown using the normalization method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\Sigma x_{ij}^2}}$$

for $i = 1,2\ldots,m;\ j = 1,2,\ldots n$

Step 3 Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 3 a. Use either the decision maker's weights for the attributes $x_1, x_2, ..x_n$ or compute the weights through the use of Saaty's (1980) AHP decision maker weights method to obtain the weights as the eigenvector to the attributes versus attribute pairwise comparison matrix.

$$\sum_{j=1}^{n} w_j = 1$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

Step 3b. Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, ***T***.

$$T = (t_{ij})_{m \times n} = (w_j\, r_{ij})_{m \times n},\ i = 1,2,\ldots,m$$

Step 4 Determine the worst alternative (A_w) and the best alternative (A_b) : Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit), then the best alternatives are the largest values, and if the values imply smaller is better (such as cost), then the best alternative is the smallest value.

$$A_w = \{\langle \max(t_{ij} \mid i = 1,2,\ldots,m \mid j \in J_-\rangle, \langle \min(t_{ij} \mid i = 1,2,\ldots,m) \mid j \in J_+\rangle\} \equiv \{t_{wj} \mid j = 1,2,\ldots,n\},$$

$$A_{wb} = \{\langle \min(t_{ij} \mid i = 1,2,\ldots,m \mid j \in J_-\rangle, \langle \max(t_{ij} \mid i = 1,2,\ldots,m) \mid j \in J_+\rangle\} \equiv \{t_{bj} \mid j = 1,2,\ldots,n\},$$

where,

$J_+ = \{j = 1, 2, \ldots n|j)$ associated with the criteria having a positive impact, and

$J_- = \{j = 1, 2, \ldots n|j)$ associated with the criteria having a negative impact.

We suggest that if possible make all entry values in terms of positive impacts.

Step 5 Calculate the L2-distance between the target alternative i and the worst condition A_w

$$d_{iw} = \sqrt{\Sigma_{j=1}^{n} (t_{ij} - t_{wj})^2},\ i = 1,2,\ldots,m$$

and then calculate the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\Sigma_{j=1}^{n}(t_{ij} - t_{bj})^2}\,,\ , i = 1,2,\ldots m$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 6 Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{iw}}{(d_{iw} + d_{ib})}, 0 \le s_{iw} \le 1,\ i = 1,2,\ldots,m$$

$S_{iw} = 1$ if and only if the alternative solution has the worst condition; and

$S_{iw} = 0$ if and only if the alternative solution has the best condition.

Step 7 Rank the alternatives according to their value from S_{iw} $(i = 1,2,\ldots,m)$.

Normalization

Two methods of normalization that have been used to deal with incongruous criteria dimensions are linear normalization and vector normalization. Normalization can be calculated as in *Step 2* of the TOPSIS process above. Vector normalization was incorporated with the original development of the TOPSIS method (Yoon, 1987), and is calculated using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\Sigma x_{ij}^2}} \text{ for } i = 1,2\ldots,m;\ j = 1,2,\ldots n$$

In using vector normalization, the non-linear distances between single dimension scores and ratios should produce smoother trade-offs (Hwang et al., 1981).

Let's suggest two options for the weights in Step 3. First, the decision maker might actually have a weighting scheme that they want the analyst to use. If not, we suggest using Saaty's 9-point pairwise method developed for the Analytical Hierarchy Process (AHP) (Saaty, 1980) to obtain the criteria weights as described in the previous section.

TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution.

TOPSIS is a method of many steps that compares a set of alternatives by identifying weights for each criterion, normalizing scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion.

The decision weights are subject to sensitivity analysis to determine how they affect the final ranking. Sensitivity analysis is essential to good analysis. Additionally, Alinehad (2011) suggests sensitivity analysis for TOPSIS for changing an attribute weight. We will again use equation (3) in our sensitivity analysis.

TOPSIS is conducted using our EXCEL template on our alternatives and the resulting TOPSIS values and ranks are shown in Table 9:

No model is complete without sensitivity analysis. We utilized sensitivity methods to modify the decision criteria weights. Clearly, we see from Figure 8 that changes in the criteria weights affect the rankings.

The TOPSIS values found in the model are used as benefit coefficient in the recourse allocation model. This is a typical resource allocation model that can be formulated either as a linear or integer program.

RESOURCE ALLOCATION MODEL

What do we mean by resource allocation model in this context? We are referring to resource available to mitigate, investigate, and prepare to "repair" the damages caused by the threats. The United States as well as many other countries receive or perceive threats daily. The threats might be intelligence discovered or remotely discovered. The number of these threats is too great for one or many organizations to investigate or prepare for. The first part of these models ranks the threats based upon weighted criteria in order to rank these threats in some meaningful way. Next, with the resources available to agencies which of these threats should now be investigated or prepared for recovery?

It is this answer to this previous question that we build a resource allocation model. Basically, this resource allocation optimization model (either integer, mixed-integer, or linear) matches the resources to the prevailing threats.

In general terms a resource allocation model is a common class of optimization models found in many optimization and modeling books (Winston, 2003, Giordano *et al.* 2013). Typical model might include:

Parameters

n_j: Number of activities $j = 1,2,\ldots n$.
m_i: Number of resources, $i = 1,2\ldots m$
B_j: Benefit of activity j.
b_i: Amount of resource i available.
a_{ij}: amount of resource i used for activity j.

Variables

xj: amount of activity j selected
Model

Maximize Benefit $B = \Sigma_{j=1}^{n} b_j x_j$

Subject to:

$$\sum_{j=1}^{n} a_{ij}\, x_j \le b_i,\ i = 1,2\ldots,m$$

$x_j \geq 0$ for all j and binary integers

In our resources, we do not want to maximize profit. We want to maximize the benefit of reducing the threats by applying resources to thwart the threat. There are packages that allow for quick solutions to this type of optimization model. These include, are not limited to, LINDO, LINGO, EXCEL, and MAPLE. We will illustrate a solution with our example.

First, we define our twelve decision variables.

Dirty Bomb Threat	x1
Anthrax-Bio Terror Threat	x2
DC-Road & Bridge network threat	x3
NY subway threat	x4
DC Metro Threat	x5
Major bank robbery	x6
FAA Threat	x7
Energy Node 1243	x8
Energy Node 2543	x9
Energy Node 4164	x10
Energy Node 1308	x11
Energy Node 2528	x12

We then extract the initial TOPSIS values for our threat alternatives as the benefit values.

Dirty Bomb Threat	0.387927404
Anthrax-Bio Terror Threat	0.477959146
DC-Road & Bridge network threat	0.452059844
NY subway threat	0.552324312
DC Metro Threat	0.51025919
Major bank robbery	0.586251914
FAA Threat	0.565680203
Energy Node 1243	0.732135261
Energy Node 2543	0.721220283
Energy Node 4164	0.681322031
Energy Node 1308	0.675568667
Energy Node 2528	0.641296638

These values form the basis of our objective function. Initially, we used only three constraints: people available, time available, and equipment available to illustrate the model. We realize that there are many more constraints that might added such as budget, teams available (investigative, rescue, etc), or even units of National Guard for callout in response just to name a few.

We assume technology coefficients, a_{ij}, for these three resources as:

	x1	*x2*	*x3*	*x4*	*x5*	*x6*	*x7*	*x8*	*x9*	*x10*	*x11*	*x12*
people	52	25	100	100	100	10	10	75	75	75	75	75
time hours	72	98	188	188	188	48	48	200	200	200	200	200
equipment	12	12	24	24	24	5	0	15	15	15	15	15

We further assume in this example that our decision variables are binary integers: supported is defined as a one (1) while not supported is defined as a zero (0). We run our optimization model using the Solver in Excel. We find that *x1, x2, x6, x7, x8, x9*, and *x10 = 1* and *x3, x4, x5, x11*, and *x12 = 0*.

Thus, due to our imposed resource constraints we can only handle *x1, x2, x6, x7, x8, x9*, and *x10* from our list. We also not that we have available resources left over. There are 228 people (550-322 = 228), 134 (1000-866 = 134) units of time, and 1 (75-74 = 1) equipment units not used. This suggests to the analyst that perhaps they need to find ways to get more equipment in order to advance the coverage.

RESULTS AND CONCLUSION

We have shown and illustrated a methodology to assess threats and risks as well as to assign resources to protect and defend such threats. We realize this is an unclassified draft model and more work in a larger version might be required. However, we believe this stands as an initial model to examine threats and resources available or needed to respond to these threats. In summary we utilized multiple operations research procedures in order to obtain a useful solution to a complicated problem.

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Appendix

Table 1: Summarized ORA output for the energy nodes

Ranks Nodes	*TDC*	*CC*	*BETW*	*EC*
4164	100	16	1	100
2543	100	12	2	100
1243	100	6	3	100
4219	100	13	4	100
2528	100	9	5	100
1267	100	17	6	100
1308	100	1	7	100
1244	100	100	8	100
426	100	20	9	100
2606	100	5	10	100
2594	100	2	11	100
2605	100	3	12	100
69	100	100	13	100
108	100	100	14	100
1167	100	100	15	100
4120	100	24	16	100
2235	100	100	17	100
70	100	100	18	100
2223	100	100	19	100
393	100	100	20	100

Table 2: Saaty's (1980) 9-Point Scale

Intensity of Importance in Pair-wise Comparisons	*Definition*
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements *i* and *j* if *i* is 3 compared to *j*, then j is 1/3 compared to *i*.
Rationale	Force consistency; measure values available

Table 3: Random Index (RI) I for n = 1 to 10

n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
RI	*0*	*0*	*0.52*	*0.89*	*1.1*	*1.24*	*1.35*	*1.4*	*1.45*	*1.49*

Table 4: ORA's output on the top 20 nodes

Nodes	*TDC*	*CC*	*BETW*	*EC*
4164	100	16	1	100
2543	100	12	2	100
1243	100	6	3	100
4219	100	13	4	100
2528	100	9	5	100
1267	100	17	6	100
1308	100	1	7	100
1244	100	100	8	100
426	100	20	9	100
2606	100	5	10	100
2594	100	2	11	100
2605	100	3	12	100
69	100	100	13	100
108	100	100	14	100
1167	100	100	15	100
4120	100	24	16	100
2235	100	100	17	100
70	100	100	18	100
2223	100	100	19	100
393	100	100	20	100

Table 5: Weighted values for nodes

Nodes	*TDC*	*CC*	*Betw*	*EC*	*Average*
4164	11.23255814	3.605581	0.593488	6.883721	5.578837
2543	11.23255814	2.704186	1.186977	6.883721	5.50186
1243	11.23255814	1.352093	1.780465	6.883721	5.312209
4219	11.23255814	2.929535	2.373953	6.883721	5.854942
2528	11.23255814	2.02814	2.967442	6.883721	5.777965
1267	11.23255814	3.83093	3.56093	6.883721	6.377035
1308	11.23255814	0.225349	4.154419	6.883721	5.624012
1244	11.23255814	22.53488	4.747907	6.883721	11.34977
426	11.23255814	4.506977	5.341395	6.883721	6.991163
2606	11.23255814	1.126744	5.934884	6.883721	6.294477
2594	11.23255814	0.450698	6.528372	6.883721	6.273837
2605	11.23255814	0.676047	7.12186	6.883721	6.478547
69	11.23255814	22.53488	7.715349	6.883721	12.09163
108	11.23255814	22.53488	8.308837	6.883721	12.24
1167	11.23255814	22.53488	8.902326	6.883721	12.38837
4120	11.23255814	5.408372	9.495814	6.883721	8.255116
2235	11.23255814	22.53488	10.0893	6.883721	12.68512
70	11.23255814	22.53488	10.68279	6.883721	12.83349
2223	11.23255814	22.53488	11.27628	6.883721	12.98186
393	11.23255814	22.53488	11.86977	6.883721	13.13023

Table 6: Ranked weighted averages for key nodes

1243	5.31221
2543	5.50186
4164	5.57884
1308	5.62401
2528	5.77797
4219	5.85494
2594	6.27384
2606	6.29448
1267	6.37703
2605	6.47855
426	6.99116
4120	8.25512
1244	11.3498
69	12.0916
108	12.24
1167	12.3884
2235	12.6851
70	12.8335
2223	12.9819
393	13.1302

Table 7: Ranks from model and with sensitivity analysis

	ranks 1	*ranks 2*
Node 1243	1	2
Node 2543	2	5
Node 4164	3	8
Node 1308	4	1
Node 2528	5	4
Node 4219	6	7
Node 2594	7	3
Node 2606	8	6

Table 8: Threat data matrix for our example from ranking threats (Fox, 2016)

Threat Alternatives\ Criterion	*Reliability of threat assessment*	*Approximate associated deaths (000)*	*Cost to fix damages in (Millions)*	*Location density in in millions*	*Destructive psychological Influence*	*Number of intelligence related tips*
Dirty Bomb Threat	0.40	10	150	4.5	9	3
Anthrax-Bio Terror Threat	0.45	.8	10	3.2	7.5	12
DC-Road & Bridge network threat	0.35	0.005	300	.85	6	8
NY subway threat	0.73	12	200	6.3	7	5
DC Metro Threat	0.69	11	200	2.5	7	5
Major bank robbery	0.81	0.0002	10	.57	2	16
FAA Threat	0.70	0.001	5	.15	4.5	15
Node 1243	0.85	0.00001	50	4	8	10
Node 2543	0.77	0.00001	65	5	8	8
Node 4164	0.75	0.00001	43	3	8	5
Node 1308	0.72	0.00001	38	4	8	4
Node 2528	0.71	0.00001	25	2	8	4

Table 9: TOPSIS ranks for threats

Topsis Value	*Final Rank*	
0.388	12	Dirty Bomb Threat
0.478	10	Anthrax-Bio Terror Threat
0.452	11	Road & Bridge network threat
0.552	8	NY subway threat
0.510	9	DC Metro Threat
0.586	6	Major bank robbery
0.566	7	FAA Threat
0.732	1	Node 1243
0.721	2	Node 2543
0.681	3	Node 4164
0.676	4	Node 1308
0.641	5	Node 2528

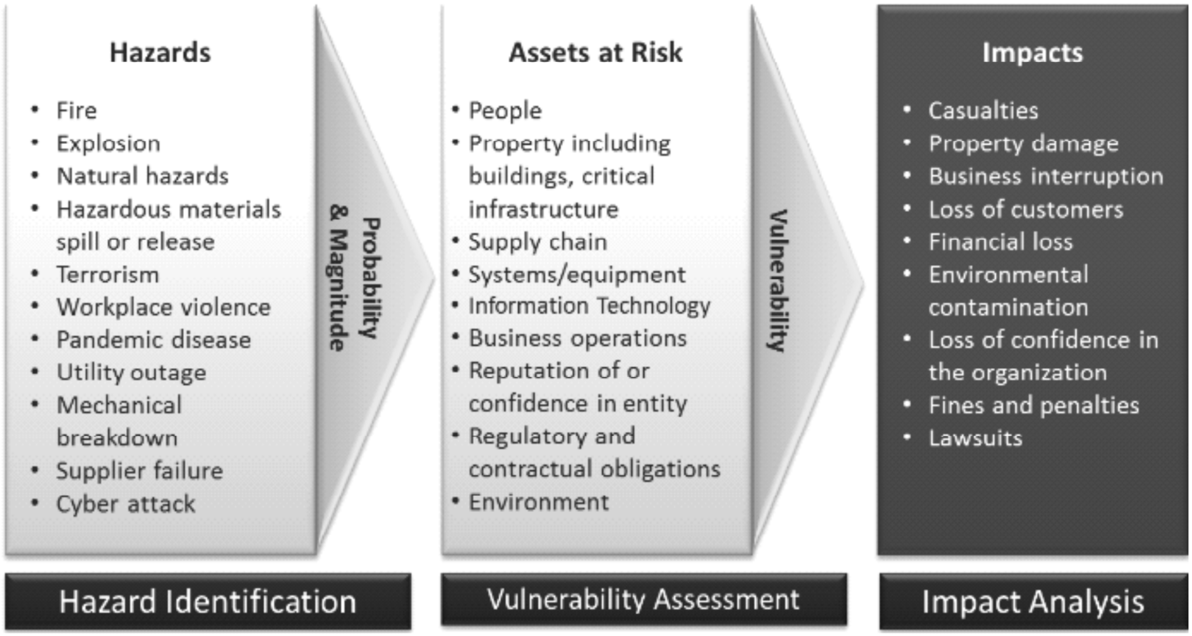


Figure 1: Risk assessment process diagram (Source: *https://www.ready.gov/risk-assessment* Accessed September 9, 2016)

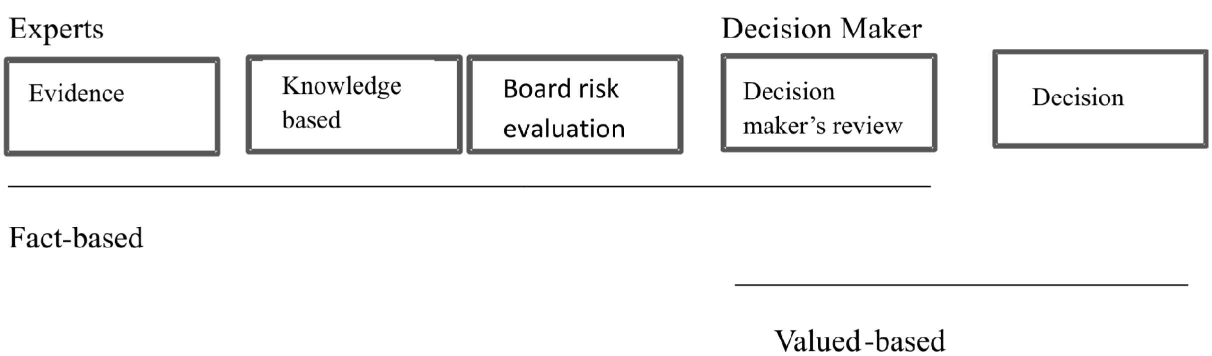


Figure 2: Model for linking various stages of risk assessment and decision making (Ersdale *et al.* 2008 & Aven 2016)

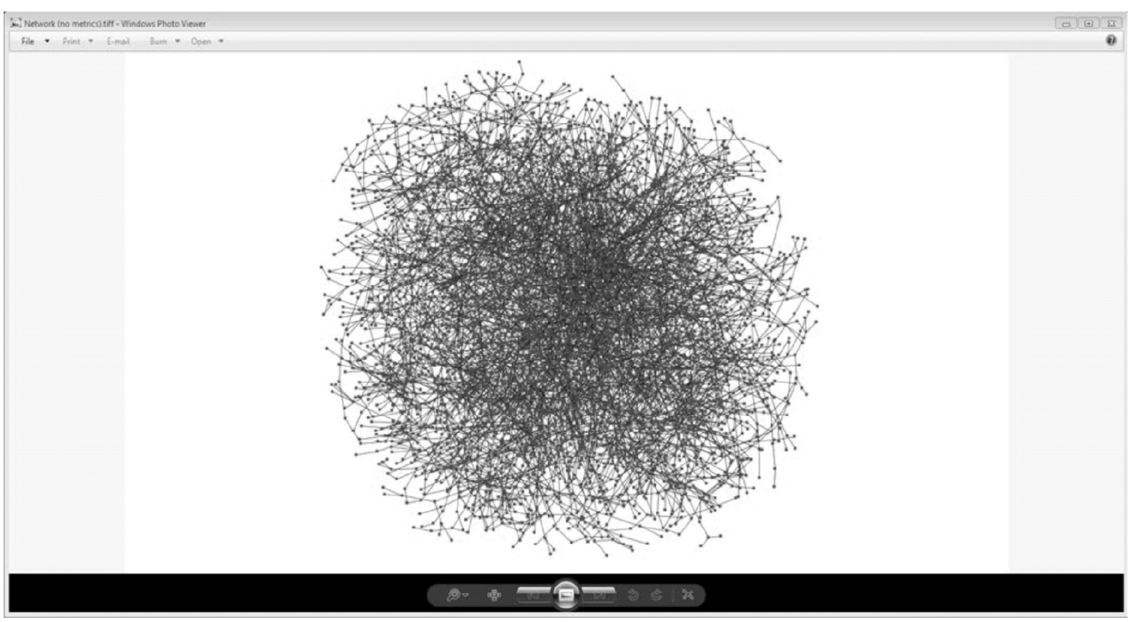
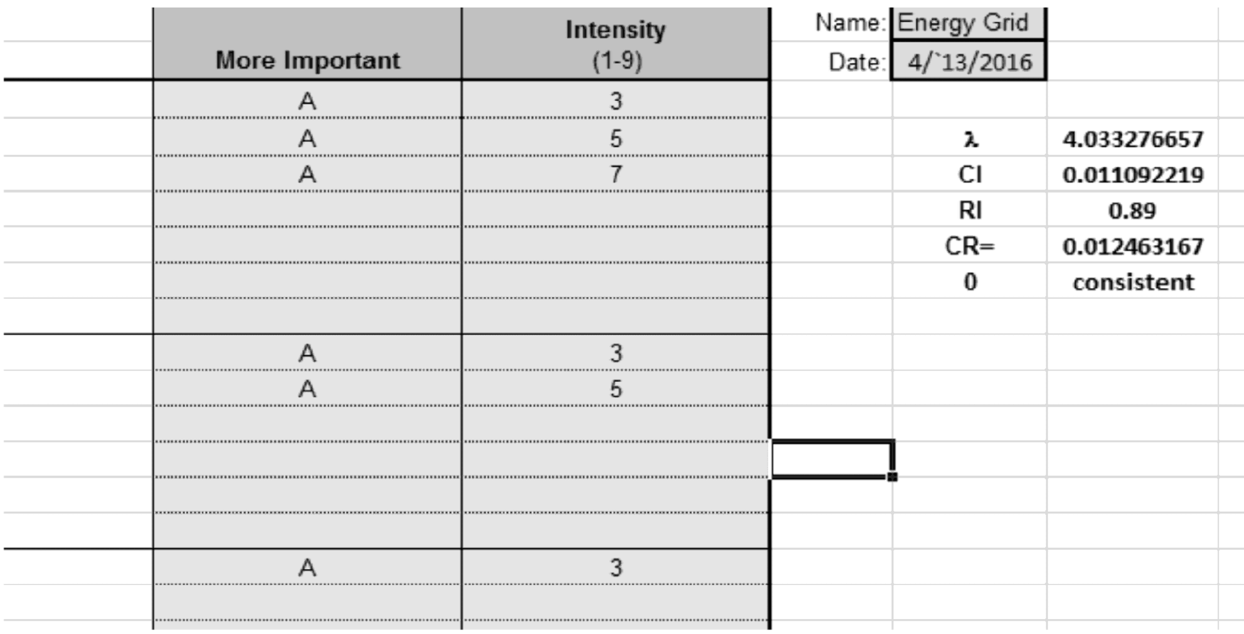


Figure 3: US Energy Grid from Department of Defense Analysis CORE Lab (2016)

Step 1. If applicable, perform a Key Node Network Analysis as described by Couch et al. (2015) and Fox et al. (2013, 2014, 2014, & 2105)
Step 2. Perform multi -attribute decision making analysis with an AHP -TOPSIS Hybrid approach previously described by Fox (2014, 2016), Fox et al. (2015) and include sensitivity analysis described by Alinezhad et al (2011).
Step 3. Using the output of the TOPSIS program as benefit coefficient in an optimization process perform a resource allocation optimization analysis to maxim ize overall benefit discussed by Winston (2003).

Figure 4: Proposed methodology for RAM Part I



More Important	Intensity (1-9)		
		Name:	Energy Grid
		Date:	4/`13/2016
A	3		
A	5	λ	4.033276657
A	7	CI	0.011092219
		RI	0.89
		CR=	0.012463167
		0	consistent
A	3		
A	5		
A	3		

Figure 5: Excel template screenshot

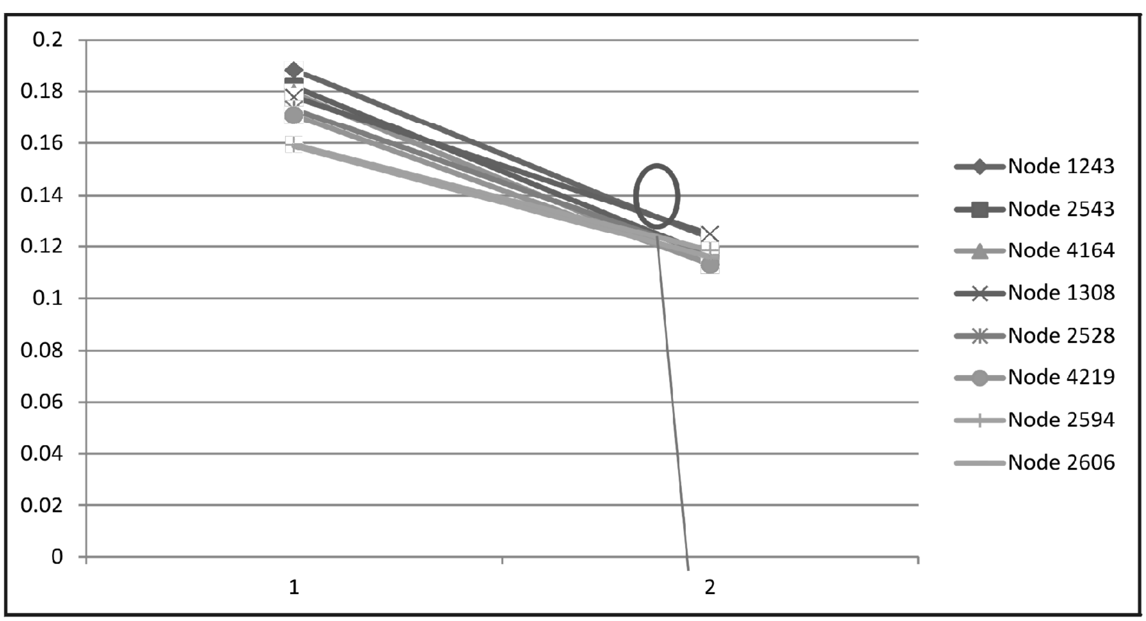
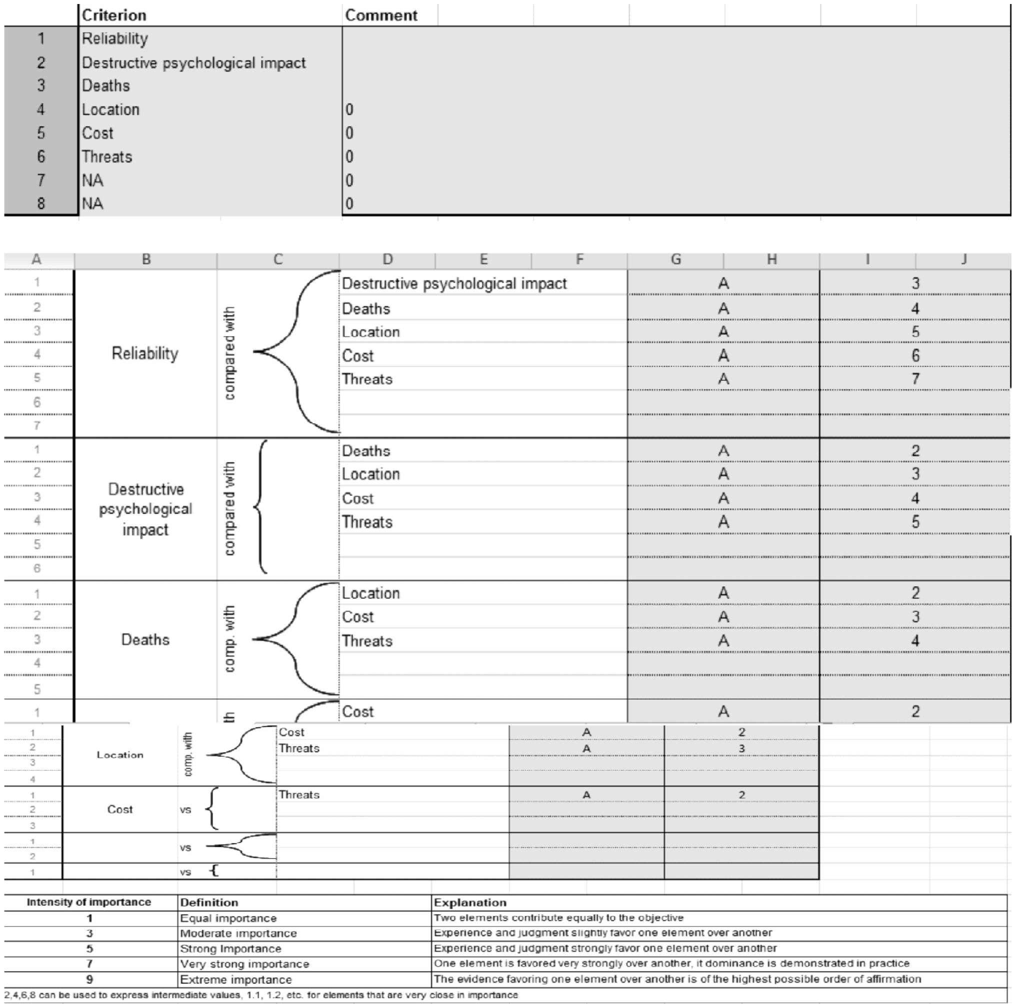


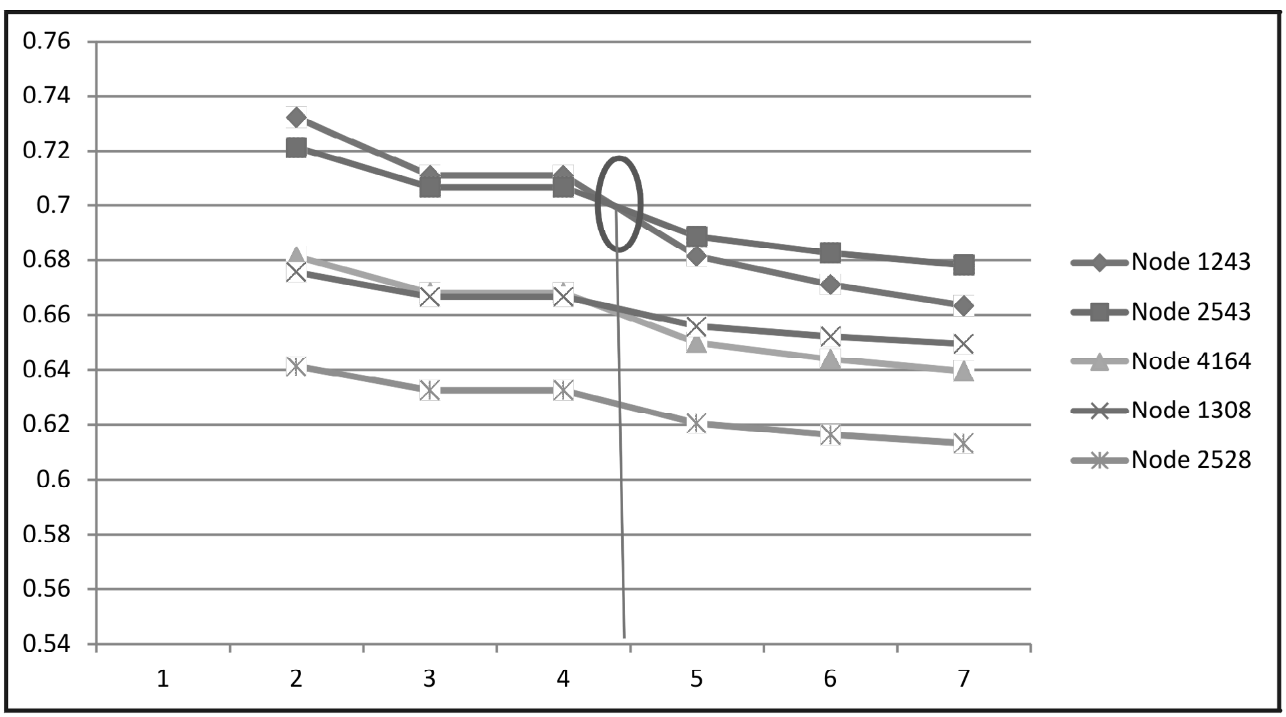
Figure 6: Sensitivity analysis graphical output for energy nodes



	Criterion	Comment
1	Reliability	
2	Destructive psychological impact	
3	Deaths	
4	Location	0
5	Cost	0
6	Threats	0
7	NA	0
8	NA	0

Intensity of importance	Definition	Explanation
1	Equal importance	Two elements contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one element over another
5	Strong Importance	Experience and judgment strongly favor one element over another
7	Very strong importance	One element is favored very strongly over another, it dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of affirmation
2,4,6,8 can be used to express intermediate values, 1.1, 1.2, etc. for elements that are very close in importance		

Figure 7: Excel template for criteria weights



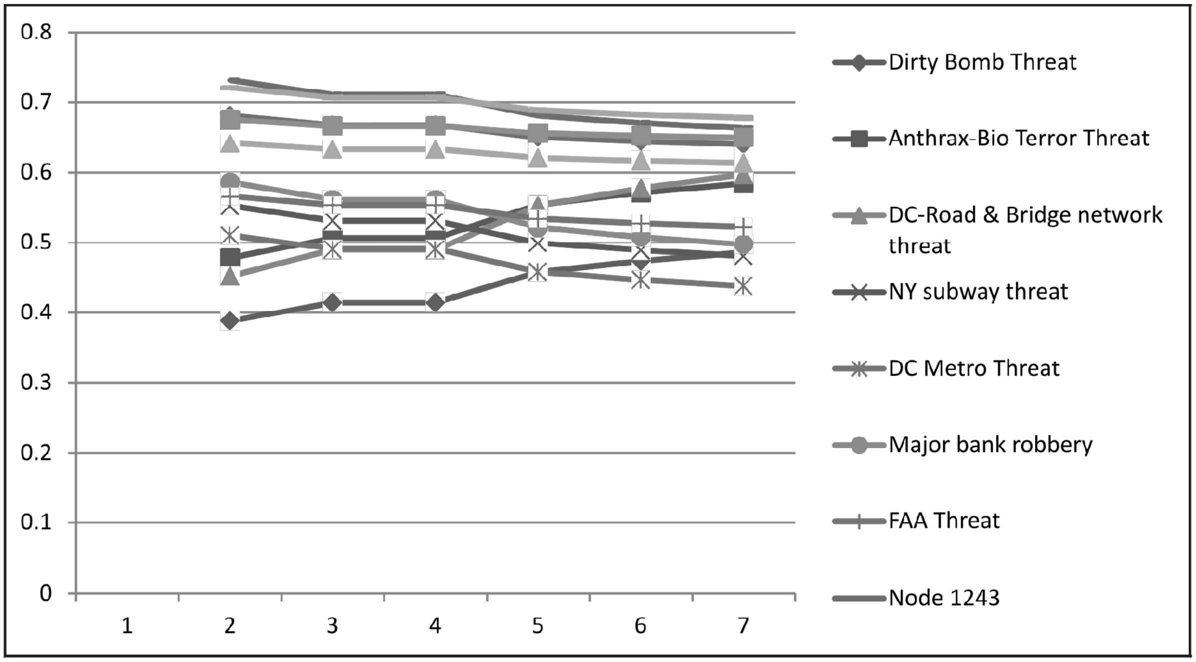


Figure 8: Sensitivity Analysis of the Top 5 Threats and all the remaining threats