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INTUITIONISTIC RANDOM APPROXIMATION OF THE RECIPROCAL-CUBIC FUNCTIONAL EQUATION USING DIFFERENT APPROACHES

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Abstract: In this paper,we will prove stability of reciprocal-cubic functional equation

$$f((c+1)x-cy)-f((c+1)x+cy)=\frac{2cf(x)f(y)[c^2f(x)+3(c+1)^2f^{1/3}(x)f^{2/3}(y)]}{[(c+1)^2f^{2/3}(y)-c^2)f^{2/3}(y)]^3}$$

(where c is a positive integer) using fixed point and direct method in Intuitionistic Random normed space.

Keywords- Hyers-Ulam-Rassias stability, Reciprocal-Cubic functional equation,Intuitionistic Random normed spaces.

Mathematical subject classification- 39B72,39B82,39B52,34K36,46S50,47S50.

1. INTRODUCTION

Hyers[12] was first to speak in response to query of Ulam[4] about stability of group homomorphism under Banach spaces,which was further generalised by Aoki[5] for additive mapping and Th. M. Rassias[17] for linear mapping.Afterwards, these results were generalised by JM Rassias [15], Gavruta[10] under different consideration.The analysis of stability of various types of functional equations have been considered by number of mathematicians and there are lot of results available in the literature.

In 2010,K.Ravi and B.V.S. Kunar[19] introduced and proved the generalized Hyers-Ulam stability of the reciprocal functional equation

$$f(x+y)=\frac{f(x)f(y)}{f(x)+f(y)} \tag{1}$$

in $\mathbf{R}^+$.It is easily seen that the reciprocal function $f(x)=\frac{c_0}{x}$ is a solution (1). In 2014, kim and second author [7] introduced and proved the generalized Hyers-Ulam stability of the quadratic reciprocal functional equation

$$f(2x+y)+f(2x-y)=\frac{2f(x)f(y)(4f(x)+f(y))}{(4f(y)-f(x))^2} \tag{2}$$

Recently, K.Ravi et al. [20] studied the generalized Hyers-Ulam stability of the cubic reciprocal functional equations

$$f(2x+y)+f(2x-y)=\frac{4f(x)f(y)(4f(y)+3f(x)^{2/3}f(y)^{2/3})}{(4f(y)^{2/3}-f(x)^{2/3})^3} \quad (3)$$

in non-Archimedean fields.Some more results about the stability of various types of functional equation can be studied from[[11],[16],[18],[21]].

In this paper, we introduce a new generalised reciprocal-cubic functional equation and investigate the generalized Hyers-Ulam stability of this reciprocal-cubic functional equation in the framework of intuitionistic Random Normed spaces.

$$f((c+1)x-cy)-f((c+1)x+cy)=\frac{2cf(x)f(y)[c^2f(x)+3(c+1)^2f^{1/3}(x)f^{2/3}(y)]}{[(c+1)^2f^{2/3}(y)-c^2)f^{2/3}(y)]^3} \quad (4)$$

2. Preliminaries

Chang et al.[8] introduced the concept of intuitionistic random normed spaces. In this section we define the notion of intuitionistic random normed spaces as in [[1],[2],[3],[22],[23]].

2.1 Definitions

• A measure distribution function is a function m:R→[0,1] which is non-decreasing and left continuous on R with $inf_{t\in R}m(t)=0$ and $sup_{t\in R}m(t)=1.$ Let us denote the family of all measure distribution functions by D and a special element of D by H defined as

$$H(t)=\begin{cases}1 & t\leq 0\\ 0 & t>0,\end{cases}$$

If X is a nonempty set, then m:X→D is called a probabilistic measure on X and m(x) is represented as m_x.

• A non-measure distribution function is a function n:R→[0,1] which is non-decreasing and right continuous on R with $inf_{t\in R}m(t)=0$ and $sup_{t\in R}m(t)=1.$. Let us denote the family of all non-measure distribution functions by B and a special element of B by G defined as

$$H(t)=\begin{cases}1 & t\leq 0\\ 0 & t>0,\end{cases}$$

If X is a nonempty set, then n:X→D is called a probabilistic non-measure on X and n(x) is represented as n_x.

2.2 Lemma ([6],[9])

Consider the set $L^{\bullet}$ and operation $\leq_{L^{\bullet}}$ defined by:

$$L^{\bullet} = \{(a_1, a_2): (a_1, a_2) \in [0,1]^2 \ and \ a_1 + a_2 \leq 1\},$$

$(a_1, a_2) \leq_{L^{\bullet}} (b_1, b_2) \Leftrightarrow a_1 \leq b_1, a_2 \geq b_2, \ for\ all\ (a_1, a_2), (b_1, b_2) \in L^{\bullet}$. Then $(L^{\bullet}, \leq_{L^{\bullet}})$ is a complete lattice.Also we denote the units by $0_{L^{\bullet}} = (0, 1)$ and $1_{L^{\bullet}} = (1, 0)$.

2.3 Definitions [6]

• intuitionistic fuzzy set $(A_{\zeta,\eta})$: An intuitionistic fuzzy set $A_{\zeta,\eta}$ in a universal set U is an object $A_{\zeta,\eta} = \{(\zeta_A(u), \eta_A(u)): u \in U\}$ with $\zeta_A(u) + \eta_A(u)) \leq 1$,where $\zeta_A(u) \in [0,1]$ are called membership degree and non-membership degree, respectively, of u in $A_{\zeta,\eta}$.

• Triangular norm (ⅹ = T): A triangular norm ⅹ = T on [0, 1] is an associative,commutative and increasing mapping $T:[0,1]^2 \rightarrow [0,1]$ with T(1,t)=1 ⅹ t=t for all $t \in [0,1]$.

• Triangular conorm (♦ = S): A triangular conorm ♦ = S is an associative,commutative and increasing mapping S:[0,1]^2→[0,1] with S(0,t)=0♦t=t for all $t \in [0,1]$.

Using the lattice ($L^{\bullet}$, $\leq_{L^{\bullet}}$), these definitions can be straightforwardly extended.

2.4 Definition [6]

A triangular norm (t-norm) on $L^{\bullet}$ is a mapping $T:(L^{\bullet})^2 \rightarrow L^{\bullet}$ satisfying the following conditions:

1. boundary condition:- $(T(a, 1_{L^{\bullet}}) = a)$, for all $a \in L^{\bullet}$;
2. commutativity:- (T (a, b) = T (b, a)), for all $(a, b) \in (L^{\bullet})^2$;
3. associativity:-(T (a, T (b, c)) = T (T (a, b) , c)), for all $(a, b, c) \in (L^{\bullet})^3$;
4. monotonicity:- $(a \leq_{L^{\bullet}} a'$ and $b \leq_{L^{\bullet}} b' \Rightarrow T(a, b) \leq_{L^{\bullet}} T(a', b'))$, for all $(a, a', b, b') \in (L^{\bullet})^4$.

If $(L^{\bullet}, \leq(L^{\bullet}), T)$ is an Abelian topological monoid with unit $1_L^{\bullet}$, then T is said to be a continuous t-norm.

2.5 Definition [6]

A continuous t-norms T on $L^{\bullet}$ is said to be continuous t-representable if there exist a continuous t-norm ⅹ and a continuous t-conorm ♦ on [0, 1] such that, for all $a=(a_1,a_2)$, $b=(b_1,b_2) \in L^{\bullet}$, $T(a,b)=(a_1$ ⅹ b_1, a_2♦$b_2)$. For example,

$$M(x, y) = (min\{x_1, y_1\}, max\{x_2, y_2\})$$

for all $x=(x_1,x_2), y=(y_1,y_2) \in L^{\bullet}$ are continuous t-representable.Also next, we define a sequence T^n recursively by $T^n(x^{(1)},\ldots,x^{(n+1)}) = T(T^{n-1}(x^{(1)},\ldots,x^{(n)}),x^{(n+1)})$, for all $n \geq 2, x^{(i)} \in L^{\bullet}$.

2.6 Definition [24]

A negator on $L^{\bullet}$ is any decreasing mapping $N:L^{\bullet}\to L^{\bullet}$ satisfying $N(0_{L^{\bullet}}) = 1_{L^{\bullet}}$ and $N(1_{L^{\bullet}}) = 0_{L^{\bullet}}$ If $N(N(x))=x$ for all $x \in L^{\bullet}$, then N is called an involutive negator. A negator on [0, 1] is a decreasing mapping $N:[0,1]\to[0,1]$ satisfying $N(0)=1$ and $N(1)=0$. N_s denotes the standard negator on [0, 1] defined by $N_s(x)=1-x$, for all $x \in [0,1]$.

2.7 Definition [24]

Let μ and ν be measure and non-measure distribution functions from $X\times(0,+\infty)$ to [0,1] such that $\mu_x(t)+\nu_x(t)\leq 1$ for all $x \in X$ and all $t>0$. The triple $(X,P_{\mu,\nu},T)$ is said to be an intuitionistic random normed space (IRN-space) if X is a real vector space, T is a continuous t-representable and $P_{\mu,\nu}$ is a mapping $X\times(0,\infty)\to L_{\bullet}$ satisfying the following conditions, for all

$x,y \in X$ and $t,r > 0$,

- $P_{\mu,\nu}(x,0) = 0_{L^{\bullet}}$;
- $P_{\mu,\nu}(x,t) = 1_{L^{\bullet}}$ if x = 0;
- $P_{\mu,\nu}(\alpha x,t) = P_{\mu,\nu}(x,\frac{t}{|\alpha|})$ for all $\alpha \neq 0$;
- $P_{\mu,\nu}(x+y,t+r) \geq_{L^{\bullet}} T(P_{\mu,\nu}(x,t),P_{\mu,\nu}(y,r))$.

In this case, $P_{\mu,\nu}$ is called an intuitionistic random norm. Here, $P_{\mu,\nu}(x,t)=(\mu_x(t),n_x(t))$ .For example[24] Let (X,||.||) be a normed space. Let $T(a,b)=(a_1,b_1,\min(a_2+b_2,1))$ for all $a=(a_1,a_2)$, $b=(b_1,b_2) \in L^{\wedge\bullet}$ and μ,ν be measure and non-measure distribution functions defined by

$$P_{\mu,\nu}(x,t) = (\mu_x(t),\nu_x(t)) = (\frac{t}{t+||x||},\frac{||x||}{t+||x||})$$

for all $t \in R^+$. Then $(X,P_{\mu,\nu},T)$ is an IRN-sapce.

2.8 Definitions[24]

• A sequence $\{x_n\}$ in an IRN-space $(X,P_{\mu,\nu},T)$ is called a Cauchy sequence if, for any ε > 0 and t > 0, there exists $n_0 \in N$, $\forall$ $n,m\geq n_0$,such that

$$P_{\mu,\nu}(x_n-x_m,t)\geq_L (N_s(\varepsilon),\varepsilon).$$

• The sequence {xn} is said to be convergent to a point x∈X if $P_{\mu,\nu}(x_n-x,t)\to 1_{L^\bullet}$) as n→∞ for every t>0.

• An IRN-space (X,Pμ,,T) is said to be complete if every Cauchy sequence in X is convergent to a point x∈X.

Now let us define a Difference operator Δ:X→Y as following

$$\Delta(x,y)=\frac{2cf(x)f(y)[c^2f(x)+3(c+1)^2f^{1/3}(x)f^{2/3}(y)]}{[(c+1)^2f^{2/3}(y)-c^2)f^{2/3}(y)]^3}-f((c+1)x-cy)+f((c+1)x+cy)$$

3 STABILITY OF FUNCTIONAL EQUATION(4):DIRECT METHOD

3.1 Theorem

Let X be a real linear space and $(Y,Z_{\alpha,\beta},T)$ be a complete intuitionistic random normed space.Let f:X→Y be a mapping with f(0)=0, with the condition

$$Z_{\alpha,\beta}(\Delta(x,y),t)\geq_{L^\bullet} Z'_{\mu,\nu}(x,y,t) \tag{5}$$

where $\mu_{x,y},\nu_{x,y}:X^2\to D^+$ and $(\mu_{x,y}(t),\nu_{x,y}(t))$ is denoted by $Z'_{\mu,\nu}(x,y,t)$ for all $x,y\in X$ and $t>0$.If

$$T_{i=1}^{\infty}Z'_{\mu,\nu}(\frac{x}{(2c+1)^{i+n}},\frac{x}{(2c+1)^{i+n}},(2c+1)^{2(i-1)+3n}t)=1_{L^\bullet} \tag{6}$$

(and)

$$lim_{n\to\infty}Z'_{\mu,\nu}(\frac{x}{(2c+1)^{n}},\frac{y}{(2c+1)^{n}},(2c+1)^{3n}t)=1_{L^\bullet} \tag{7}$$

for all x∈X and t > 0,then there exists a unique reciprocal-cubic mapping C_R:X→Y such that for all x,y∈X and t > 0, we have

$$Z_{\alpha,\beta}(f(x)-C_R(x),t)\geq_{L^\bullet} T_{i=1}^{\infty}Z'_{\mu,\nu}(\frac{x}{(2c+1)^{i}},\frac{x}{(2c+1)^{i}},(2c+1)^{2(i-1)}t) \tag{8}$$

Proof: Replacing (x, y) by $(\frac{x}{(2c+1)},\frac{x}{(2c+1)})$ in (5), we get

$$Z_{\alpha,\beta}(f(x)-\frac{1}{(2c+1)^3}f(\frac{x}{(2c+1)}),t)\geq_{L^\bullet} Z'_{\mu,\nu}(\frac{x}{(2c+1)},\frac{x}{(2c+1)},t) \tag{9}$$

for all x∈X and all t > 0.Again replacing x by $x/(2c+1)^n$ in (9)and using property of IRN-space, we get

$$Z_{\alpha,\beta}(\frac{1}{(2c+1)^{3n}}f(\frac{x}{(2c+1)^{n}})-\frac{1}{(2c+1)^{3(n+1)}}f(\frac{x}{(2c+1)^{n+1}}),\frac{t}{(2c+1)^{3n}})$$
$$\geq_{L^\bullet} Z'_{\mu,\nu}(\frac{x}{(2c+1)^{n+1}},\frac{x}{(2c+1)^{n+1}},t) \tag{10}$$

Again using property of IRN-space, we get

$$Z_{\alpha,\beta}\left(\frac{1}{(2c+1)^{3n}}f\left(\frac{x}{(2c+1)^{n}}\right)-\frac{1}{(2c+1)^{3(n+1)}}f\left(\frac{x}{(2c+1)^{n+1}}\right),\frac{t}{(2c+1)^{n}}\right) \qquad (11)$$

$$\geq_{L^*} \; Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{n+1}},\frac{x}{(2c+1)^{n+1}},(2c+1)^{2n}t\right)$$

for all n∈N and all t > 0. As (2c+1)>1/(2c+1)+1/(2c+1)^2+...+1/(2c+1)^k, by the triangle inequality it follows

$$Z_{\alpha,\beta}\left(f(x)-\frac{1}{(2c+1)^{3r}}f\left(\frac{x}{(2c+1)^{r}}\right),t\right)\geq_{L^*} \qquad (12)$$

$$T_{n=0}^{r-1}\{Z_{\alpha,\beta}\left(\frac{1}{(2c+1)^{3n}}f\left(\frac{x}{(2c+1)^{n}}\right)-\frac{1}{(2c+1)^{3(n+1)}}f\left(\frac{x}{(2c+1)^{n+1}},\sum_{n=0}^{r-1}\frac{t}{(2c+1)^{n}}\right)\right)\}$$

$$\geq_{L^*} \; T_{i=1}^{r}\{Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{i}},\frac{x}{(2c+1)^{i}},(2c+1)^{2(i-1)}t\right)\}$$

for all x∈X and all t > 0.Now replacing x by x/(2c+1)^k in (12), we get

$$Z_{\alpha,\beta}\left(\frac{1}{(2c+1)^{3k}}f\left(\frac{x}{(2c+1)^{k}}\right)-\frac{1}{(2c+1)^{3(k+r)}}f\left(\frac{x}{(2c+1)^{(k+r)}}\right),\frac{t}{(2c+1)^{3k}}\right)$$

$$\geq_{L^*} \; T_{n=1}^{r}\{Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{(i+k)}},\frac{x}{(2c+1)^{(i+k)}},(2c+1)^{2(i-1)}t\right)\} \qquad (13)$$

$$Z_{\alpha,\beta}\left(\frac{1}{(2c+1)^{3k}}f\left(\frac{x}{(2c+1)^{k}}\right)-\frac{1}{(2c+1)^{3(k+r)}}f\left(\frac{x}{(2c+1)^{(k+r)}}\right),t\right)$$

$$\geq_{L^*} \; T_{n=1}^{r}\{Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{(i+k)}},\frac{x}{(2c+1)^{(i+k)}},(2c+1)^{2(i-1)+3k}t\right)\}$$

for all x∈X and all t,i,k > 0. Taking the limits in above equation and using (6),we can say that the sequence $\frac{1}{(2c+1)^{3n}}f\left(\frac{x}{(2c+1)^{n}}\right)$ is a Cauchy sequence. Therefore, we can define $C_R(x)=lim_{n\to\infty}\frac{1}{(2c+1)^{3n}}f\left(\frac{x}{(2c+1)^{n}}\right)$ for all x∈X. Now replace (x,y) by $\left(\frac{x}{(2c+1)^{n}},\frac{y}{(2c+1)^{n}}\right)$ in (5),we get

$$Z_{\alpha,\beta}\left(\frac{1}{(2c+1)^{3n}}\Delta\left(\frac{x}{(2c+1)^{n}},\frac{y}{(2c+1)^{n}}\right),t\right)\geq_{L^*} Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{n}},\frac{y}{(2c+1)^{n}},(2c+1)^{3n}t\right)$$

for all x,y∈X and t > 0. Letting n→∞ in the above inequality and considering the definition of C_R (x), we get that C_R satisfies (4) for all x,y∈X.

Uniqueness:Let us suppose that there exists another reciprocal-cubic function C_S which satisfies (8).Hence for all x,y∈X and t > 0,we get

$$Z_{\alpha,\beta}(C_R(x) - C_S(x),2t) \geq_{L^\bullet} Z_{\alpha,\beta}(C_R(\tfrac{x}{(2c+1)^n}) - C_S(\tfrac{x}{(2c+1)^n}),2(2c+1)^{3n}t)$$

$$\geq_{L^\bullet} T(Z_{\alpha,\beta}(C_R(\tfrac{x}{(2c+1)^n}) - f(\tfrac{x}{(2c+1)^n}), (2c+1)^{3n}t), Z_{\alpha,\beta}(f(\tfrac{x}{(2c+1)^n}) -$$

$$C_S(\tfrac{x}{(2c+1)^n}), (2c+1)^{3n}t))$$

$$\geq_{L^\bullet} T(T_{i=1}^{\infty}(Z'_{\mu,\nu}(\tfrac{x}{(2c+1)^{i+n}}, \tfrac{x}{(2c+1)^{i+n}}, (2c+1)^{2(i-1)+3n}t)),$$

$$T_{i=1}^{\infty}(Z'_{\mu,\nu}(\tfrac{x}{(2c+1)^{i+n}}, \tfrac{x}{(2c+1)^{i+n}}, (2c+1)^{2(i-1)+3n}t)))$$

$$= T(1_{L^\bullet}, 1_{L^\bullet}) = 1_{L^\bullet}$$

This proves the uniqueness of C_R. This completes the proof.

3.2 Corollary

Let p be any real number and θ is non-negative real number.If f:X→Y satisfies

$$Z_{\alpha,\beta}(\Delta(x,y),t) \geq_{L^\bullet} \begin{cases} Z'_{\mu,\nu}(\theta,t); \\ Z'_{\mu,\nu}(\theta(||x||^p + ||y||^p),t); \\ Z'_{\mu,\nu}(\theta||x||^p||y||^p,t); \\ Z'_{\mu,\nu}(\theta(||x||^p||y||^p + ||x||^{2p} + ||y||^{2p}),t); \end{cases},$$

for all x,y∈X and t > 0,then there exists a unique reciprocal-cubic mapping C_R:X→Y satisfying (4) and the inequality for all x∈X and t > 0

$$Z_{\alpha,\beta}(f(x) - C_R(x),t) \geq_{L^\bullet} \begin{cases} Z'_{\mu,\nu}(|\frac{(2c+1)^6}{(2c+1)^6-1}|\theta,t); \\ Z'_{\mu,\nu}(\frac{2\theta||x||^p}{|(2c+1)^p-(2c+1)^{(-6)}}|,t) & p \neq -6; \\ Z'_{\mu,\nu}(\frac{\theta||x||^{2p}}{|(2c+1)^{2p}-(2c+1)^{(-6)}|},t) & p \neq -3; \\ Z'_{\mu,\nu}(\frac{3\theta||x||^{2p}}{|(2c+1)^{2p}-(2c+1)^{(-6)}|},t) & p \neq -3; \end{cases},$$

Proof: Choosing appropriate $Z'_{\mu,\nu}$(x,y,t) in above theorem we can get the results.

4 STABILITY OF FUNCTIONAL EQUATION(4):FIXED POINT METHOD

4.1 Theorem[13]

Let (X,d) be a complete generalized metric space and J:X→X a strictly contractive mapping with Lipschitz constant L < 1. Then, for all x∈X, either $d(J^n x, J^{n+1} x)=\infty$ for all nonnegative integers n or there exists a positive integer n_0 such that

1. $d(J^n x, J^{n+1} x)<\infty$ for all $n\geq n_0$;
2. the sequence $\{J^n x\}$ converges to a fixed point y^* of J;
3. y^* is the unique fixed point of J in the set $Y=\{y\in X: d(J^n{}_0\,) x,y)<\infty\}$;
4. $d(y,y^*)\leq(1/(1-L))d(y,Jy)$ for all y∈Y.

4.2 Theorem

Let X be a real linear space and $(Y,Z_{\alpha,\beta},T)$ be a complete intuitionistic random normed space.Let f:X→Y be a mapping with f(0)=0 and satisfies equation (5).If

$$T_{i=1}^{\infty} Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{i+n}}, \frac{x}{(2c+1)^{i+n}}, (2c+1)^{2(i-1)+3n}t\right) = 1_{L^*} \tag{14}$$

and

$$lim_{n\to\infty} Z'_{\mu,\nu}\left(\frac{x}{(2c+1)^{n}}, \frac{y}{(2c+1)^{n}}, (2c+1)^{3n}t\right) = 1_{L^*} \tag{15}$$

for all x∈X and t > 0.If there exists L such that the function

$$x \to \zeta(x) = \left(\frac{x}{(2c+1)}, \frac{x}{(2c+1)}\right) \tag{16}$$

with the property

$$Z'_{\mu,\nu}(L\delta_i^3 \zeta(\delta_i x), t) = Z'_{\mu,\nu}(\zeta(x), t) \tag{17}$$

for all x∈X and t > 0,then a there exist a unique reciprocal-cubic mapping C_R:X→Y such that for all x,y∈X and t > 0, we have

$$Z_{\alpha,\beta}(f(x) - C_R(x), t) \geq_{L^*} Z'_{\mu,\nu}\left(\frac{L^{1-i}}{1-L}\zeta(x), t\right) \tag{18}$$

Proof: Consider the set

$$S=\{g:X\to Y; g(0)=0\} \tag{19}$$

and a constant δ_i such that

$$\delta_i = \begin{cases} (2c+1) & for \quad i = 0, \\ (2c+1)^{-1} & for \quad i = 1 \end{cases},$$

$$d(g,h)=\inf\{A\in(0,\infty): Z_{\alpha,\beta}(g(x)-h(x),t)\geq_{L^*} Z'_{\mu,\nu}(A\zeta(x),t); x \in X, t>0\}$$

As in the proof of[[14],Lemma 2.1],we can show that (S,d) is a generalised complete metric space.Define η:S→S by $\eta g(x)=(\delta_i)^3 g(\delta^i x)$ for all x∈X.For g,h ∈ Ω we have d(g,h)≤A. Next,

$$Z_{\alpha,\beta}(g(x)-h(x),t) \geq_{L^\bullet} Z'_{\mu,\nu}(A\zeta(x),t)$$

$$\Rightarrow Z_{\alpha,\beta}(\delta_i^3 g(\delta_i x)-\delta_i^3 h(\delta_i x),t) \geq_{L^\bullet} Z'_{\mu,\nu}(A\zeta(\delta_i x),t/(\delta_i)^3) \quad (20)$$
$$\Rightarrow Z_{\alpha,\beta}(\eta g(x)-\eta h(x),t) \geq_{L^\bullet} Z'_{\mu,\nu}(AL\zeta(x),t)$$
$$\Rightarrow d(\eta g(x),\eta f(x)) \leq AL \leq Ld(g,h)$$

for all g,h∈S.Therefore, η is strictly contractive mapping on S with lipschitz constant L.Replacing (x,y) by (x,x) in (5),we get

$$Z_{\alpha,\beta}(f((2c+1)x)-\frac{f(x)}{(2c+1)^3},t)) \geq_{L^\bullet} Z'_{\mu,\nu}(x,x,t) \quad (21)$$

for all x∈X, t > 0.Using property of IRN-spaces in above equation ,we get

$$Z_{\alpha,\beta}((2c+1)^3 f((2c+1)x)-f(x),t)) \geq_{L^\bullet} Z'_{\mu,\nu}(x,x,t/(2c+1)^3) \quad (22)$$

for all x∈X, t > 0.Now when i = 0, it follows from above equation and (17) that

$$Z_{\alpha,\beta}((2c+1)^3 f((2c+1)x)-f(x),t)) \geq_{L^\bullet} Z'_{\mu,\nu}(L\zeta(x),t)$$
$$\Rightarrow d(\eta f,f) \leq L = L^{1-i}. \quad (23)$$

for all x∈X, t>0.Now from (23) and (25) we have,

$$d(f,\eta f) \leq L^{1-i} < \infty.$$

then from theorem[4.1],we can say that there exists a fixed point C^R of η in S such that

$$lim_{n\to\infty} Z_{\alpha,\beta}(\delta_i^{3n} f(\delta_i^n x)-C_R(x),t) \to 1_{L^\bullet} \quad (26)$$

for all x∈X, t > 0. Replacing (x,y) by ($\delta_i x,\delta_i x$) in (5),we get

$$Z_{\alpha,\beta}(\delta_i^{3n}\Delta(\delta_i x,\delta_i y),t) \geq_{L^\bullet} Z'_{\mu,\nu}(\delta_i x,\delta_i y,\delta_i^{-3n} t) \quad (27)$$

for all x∈X, t>0.Uniqueness of the function C_R (x):X→Y satisfying

$$Z_{\alpha,\beta}(f(x)-C_R(x),t) \geq_{L^\bullet} Z'_{\mu,\nu}(A\zeta(x),t) \quad (28)$$

for all x∈X, t > 0,is clear by theorem[4.1],since C_R is a unique fixed point of η in

$$\nabla = \{f \in S : d(f,S) < \infty\}.$$

We can easily prove using procedure used in Theorem [3.1] ,that C_R (x):X→Y satisfies functional equation (4).Finally using theorem[4.1] ,we get

$$d(f,C_R) \leq \frac{1}{1-L} d(f,Sf)$$

$$\Rightarrow d(f,C_R) \leq \frac{L^{1-i}}{1-L} \quad (29)$$

$$\Rightarrow Z_{\alpha,\beta}(f(x) - C_R(x),t) \geq_{L^*} Z'_{\mu,\nu}(\frac{L^{1-i}}{1-L}\zeta(x),t)$$

for all x∈X, t > 0.This completes the proof.

4.3 Remark

Same result as in corollary[3.2] can be easily reproduced in this section with suitable choice of $Z'_{\mu,\nu}$ (x,y,t) and L.

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