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Realisation of High Performance Controllers for Nonlinear Drive System

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ABSTRACT

Implementation of effective control schemes for Nonlinear Systems is currently being studied comprehensively. In this paper, robust control schemes for trajectory tracking of Nonlinear Permanent Magnet Stepper Motor system has been proposed. Direct Quadrature (DQ) transformation technique has been employed to convert the system dynamics to DQ frame. Initially, a conventional PID controller along with its control law is discussed. Also, a Self-Tuning Adaptive Minimum Variance controller is tested and found to be better than PID controller. Finally, an optimal control strategy using Feedback Linearization is formulated; where in, the desired tracking of current vectors (direct and quadrature) are obtained. The performances of all the aforementioned control algorithms are analysed in detail, with better results observed in Feedback Linearization controller.

Keywords: Permanent Magnet Stepper Motor (PMSM), Controllers, PID, Minimum Variance Control, Feedback linearization.

1. INTRODUCTION

There have been considerable researches occurring in field of elementary precision control of Drives and Motors. In particular, Permanent Magnet Stepper Motor is one of the widely exploited branches. The control of permanent magnet stepper motor using Marc Nonlinear state feedback was proposed by [1]. The position control of permanent magnet stepper motor and its simple field weakening methods are discussed in [2]. The model based control law for a high performance nonlinear feedback control of PMSM is discussed in [3-4]. The mathematical model and methodology of the linearization of PMSM and its positional control are discussed. It is non-adaptive but still superior to open loop controller. The parameters vary with time and the machines are highly nonlinear. So, nonlinear adaptive control technique as discussed in [5] has been applied to PMSM.

The static PID control law [6], which is used as a reference PID law for testing the PMSM tracking operation is analyzed. The minimum variance controller for the optimum tracking of PMSM has also been considered to emphasize their advantages over static PID controller. [7-11]. As discussed in [13-15], there has been a considerable development in nonlinear control theory in the last 15 years with such ideas as feedback linearization, input/output linearization, and passivity theory. The least squares identification procedure [16-17] is also employed for the tracking of output positional vector. This paper is divided into VI sections. Section I consists of Mathematical Model of Permanent Magnet Stepper Motor (PMSM) and its DQ transformation. Section II delineates about PID Control. Section III deals with Minimum Variance Control. Section IV describes about the Feedback Linearization Control and its Reference Position Generation. Section V comprises of Simulation results and the required Motor parameters. Finally, Section VI furnishes the conclusion and the cited references.

Permanent Magnet Stepper Motor (PMSM) works on the principle of electromagnetism. It is incorporated with a rotor contrived of permanent magnet and a stator made of electromagnet. When the winding of the

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stator is energized with the supply, it makes the rotor to get magnetized permanently and moves with revolving magnetic field produced by the stator. The speed of the PMSM is in turn directly proportional to the number of poles of the PMSM. Thus by the variation of voltage between the windings, the rotor can be made to rotate.

It could be seen that the system of equations governing the PMSM is nonlinear and this may result in several uncertainties of the dynamic system thereby causing problems in effective controlling of the motor. If the modelling of these uncertainties is ignored, then this might damage the performance of the motor resulting in inaccuracies and unacceptable errors in applications involving the positioning of the motor. Therefore transformation of these equations from the existing coordinate to a nonlinear coordinate known as Direct-Quadrature transformation is done.

SECTION I

MATHEMATICAL MODEL OF PMSM

The mathematical dynamics of the PMSM is given by the following set of differential equations [12]:

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \frac{1}{J}\left(-Ki_a \cos\left(\frac{\pi}{2} - N_r\theta\right) + Ki_b \cos N_r\theta - B\omega - \tau_f\right)$$

$$i_a = \frac{1}{L}\left(v_a - Ri_a + K\omega\cos\left(\frac{\pi}{2} - N_r\theta\right)\right)$$

$$i_b = \frac{1}{L}\left(v_b - Ri_b + K\omega\cos\left(\frac{\pi}{2} - N_r\theta\right)\right) \quad (1)$$

Here the variables v_a, v_b are voltages (V) and i_a, i_b are current (A) in phases A and B respectively. Also R is the resistance (Ω) of the phase winding, ω is rotor's speed (rad/sec), θ is the rotor's position(rad), K is the motor torque constant (N.m/A), N_r is the number of rotor teeth, L is the inductance of the phase winding (H), J is the moment of inertia of the motor (kg.m^2), B is a viscous friction coefficient (N.m.s/rad)and τ_f is the friction torque (N.m). The DQ transformation of phase voltage and current is given by the following set of equations:

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \cos\left(\frac{\pi}{2} - N_r\theta\right) \\ -\cos\left(\frac{\pi}{2} - N_r\theta\right) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \cos\left(\frac{\pi}{2} - N_r\theta\right) \\ -\cos\left(\frac{\pi}{2} - N_r\theta\right) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \quad (2)$$

In (2), i_d (direct current), i_q (quadrature current) & v_d (direct voltage), v_q (quadrature voltage) are DQ transforms of the stator voltage and current. Applying this DQ transformation to the system yields:

$$i_d = \frac{1}{L}\left(v_d - Ri_d + N_r L\omega i_q\right)$$

$$i_d = \frac{1}{L}\left(v_q - Ri_q + N_r L\omega i_d - K\omega\right)$$

$$\dot{\omega} = \frac{1}{J}\left(Ki_q - B\omega - \tau_f\right)$$

$$\dot{\theta} = \omega \tag{3}$$

Here we take the voltage inequality as follows:

$$v_a^2 + v_b^2 = v_d^2 + v_q^2 \leq V_{\max}$$

where V_{max} is the maximum voltage that could be applied to the system. V_{max} can have a value less than or equal to 40 Volts which is a standard value.

SECTION II

PID CONTROL

The most common control algorithms involve the use of PID (Proportional Integral Derivative) controllers. The PID controller, with some minor variations is used in many practical feedback loops. It comprises of three parameters: the proportional, integral and derivative. PID calculation necessitates the use of these three parameters. The reception to current error is determined by proportional value; the reaction to sum of recent errors is determined by integral value; and the derivative value determines the action based on the rate at which error has been changing. The sum of these three actions is used in different process control schemes. The equation of a PID controller is given as,

$$U(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + Kd\frac{de}{dt}$$

Where, K_p is the proportional gain, K_i is the integral gain and K_d is the derivative gain. We use this PID controller for the tracking and control of Permanent Magnet Stepper Motor (PMSM). The results are simulated using MATLAB. The following control equations (4) are used,

$$U_d = -pL\omega i_d - k_4\left(i_q - i_{qr}\right) - k_5 \int_0^t \left[i_d(\tau) - i_{dr}(\tau)\right] d\tau$$

$$U_q = K_m\omega - k_4\left(i_q - i_{qr}\right) - k_5 \int_0^t \left[i_d(\tau) - i_{dr}(\tau)\right] d\tau \tag{4}$$

Where U_d is direct control variable, U_q is quadrature control variable. In this paper, we assume that the direct reference current is almost zero because it does not provide any torque to the motor system.

$$i_{dr} = 0$$

$$i_{qr} = \left(-J / K_m\right)\left\{k_1\left(\theta - \theta_r\right) + k_2 \int_0^t \left[\theta(\tau) - \theta_r(\tau)\right] d\tau + k_3\left(\omega - \omega_r\right)\right\} \tag{5}$$

In (5) i_{dr} is desired direct current and i_{qr} is the desired quadrature current. All values are in SI units. The performance of the PID controller is shown in the simulation results. This being the conventional one gives poor tracking performances compared to the next two.

SECTION III

MINIMUM VARIANCE CONTROL

Minimum variance controller is just one form of self-tuning controllers that falls under the adaptive control strategies. We often want fast response time at the initial stages of a process, but later on put more prominence on reducing steady state error. This section discusses the use of minimum variance controller for the tracking of Permanent Magnet Stepper Motor (PMSM). Consider the following model (6):

$$A\left(q^{-1}\right)*y\left(k\right)=F\left(q^{-1}\right)q^{-d}U\left(k\right)+G\left(q^{-1}\right)e\left(k\right) \quad (6)$$

The objective function is measured by the equation (9) given below:

$$I=E\left\{\left[y\left(k+d\right)-\dot{y}\left(k+d\right)\right]^2\right\} \quad (7)$$

The above equation yields input $U(k)$ after simplifying, which minimizes the mean square error between the predicted output and the desired value $\dot{y}\left(k+d\right)$. The case where $\dot{y}\left(k+d\right)=0$ is considered, the above equations (7) becomes (8),

$$I=E\left\{\left[y^2\left(k+d\right)^2\right]^2\right\} \quad (8)$$

The process variable of the system at time $(k + d)$ is given by:

$$y\left(k+d\right)=T\left(q^{-1}\right)/G\left(q^{-1}\right)Y\left(k\right)+B\left(q^{-1}\right)/G\left(q^{-1}\right)D\left(q^{-1}\right)+N$$

Where, $N = B(q^{-1})\, e(k + d)$ and B and T are unique polynomials defined as

$$B(q^{-1}) = 1 + b_1q^{-1} + b_2q^{-2} + \ldots\, b_{d-1}\, q^{-(d-1)}$$
$$T(q^{-1}) = t_0 + t_1q^{-1} + t_2q^{-2} + \ldots\, t_{d-1}\, q^{-(d-1)}$$

Also $B(q^{-1})\, \mathrm{E}(q^{-1}) + q^{-d}\, T\,(q^{-1}) = G(q^{-1})$

Thus, equation (9) becomes,

$$I = E\{[T/G)\, y(k) + (FB/G)\, u(k) + Be(k + d)]^2\} \quad (9)$$

The whole objective here is to simplify the above equation to its minimal value. Control variable $u(k)$ depends upon the previous values $u(k-1)$, $u(k-2)$ and on outputs $y(k)$, $y(k-1)$. Thus, if the following equation (10) is satisfied, minimum variance control of the above form is obtained.

$$U(K) = -(T(q^{-1})/F(q^{-1}) + B(q^{-1}))y(k) \quad (10)$$

SECTION IV

FEEDBACK LINEARIZATION CONTROL

This controller is proposed to eliminate the rigorous recursive least square identification procedure that is carried out in Minimum Variance Control strategy. Generally for control of nonlinear systems, the generic method adopted is transformation of the system into a linear one and finally applying an apropos linear control technique. This might not suffice for processes having continuously changing process variables. So in PMSM where the position vector and speed vectors are constantly accelerating and decelerating, Feedback Linearization is used which alleviates the aforementioned problem. Both linearization as well as the control action are taken care of by the Feedback Linearization controller. In effect, this actually reduces both complexity as well as the time required for our desired tracking. This strategy transforms the Nonlinear Permanent Magnet Stepper Motor system into a linear system by converting the input voltage vector into a different coordinate plane which is called the Direct-Quadrature (DQ) frame. Thus this DQ transformation helps us to easily carry out the Feedback Linearization control action. Now, the voltage vectors in Direct and Quadrature frame as given as:

$$v_d = v_{dr} + N_r L\omega_r\left(I_{qr} - i_q\right) + Lu_d$$
$$v_q = v_{qr} + N_r L\omega_r\left(i_{dr} - i_d\right) + Lu_q \quad (11)$$

By substituting the above equations in equation (3) the nonlinear terms get cancelled out i.e. they are eliminated completely, thereby linearizing the entire system. The control signal U_d is a function error in current vector in direct frame. In order to compensate for the errors in position as well as speed vector, their errors are also incorporated in U_q dynamics. The control signals U_d and U_q are given as follows:

$$U_d = P1\left(i_{dr} - i_d\right)$$
$$U_q = P2\left(i_{qr} - i_q\right) + P3\left(\omega_r - \omega\right) + P4\left(\theta_r - \theta\right) + P5\xi \tag{12}$$

where $\xi = \int_0^t \left(\theta_r\left(t\right) - \theta\left(t\right)\right)\, dt$

The acceleration α_r is acquired by taking the derivative of ω_r. The desired current vectors are obtained by the following formula (13), (14):

$$i_{dr} = -\frac{N_r L K \omega_r^2}{R^2 + \left(N_r \omega_r L\right)^2} \tag{13}$$

$$i_{qr} = \frac{J}{K}\alpha_r + \frac{B}{K}\omega_r \tag{14}$$

Here $K\omega$ term represents the back-emf of the motor. This back-emf is quite undesirable because it limits the motor in reaching high speeds. So to eliminate it, the back-emf term has been included in direct reference current vector. Also the reactance of the motor represented by the term $N_r\omega_r L$ has been incorporated together with resistance R to determine the total impedance provided by the PMSM. Also the Moment of Inertia of the motor J should be known so that control action and change in position of the motor would be in sync with each other. Motors having larger inertia would change their position slowly as compared to motors which have lesser inertia. Depending upon the value of J, control action should be taken by the controller.

Finally the tracking error is found to be dependent on errors in speed, position as well as current vectors in both direct and quadrature frame. The feedback term u is a function of ε i.e.,

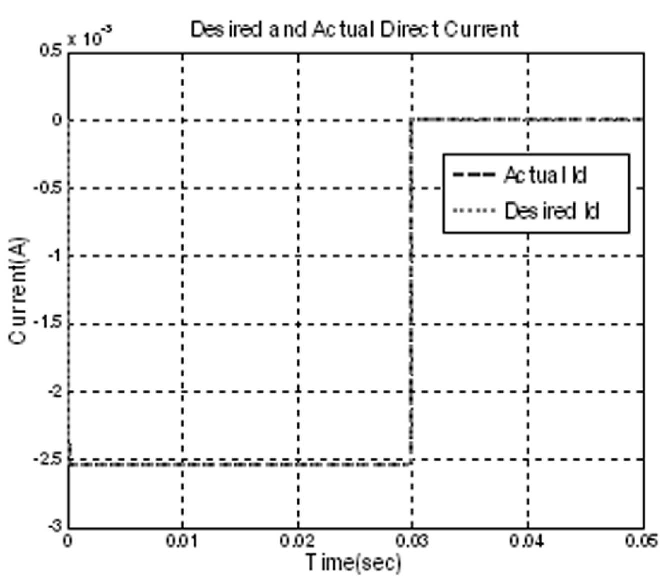

Figure 1: Desired and Actual Direct Current

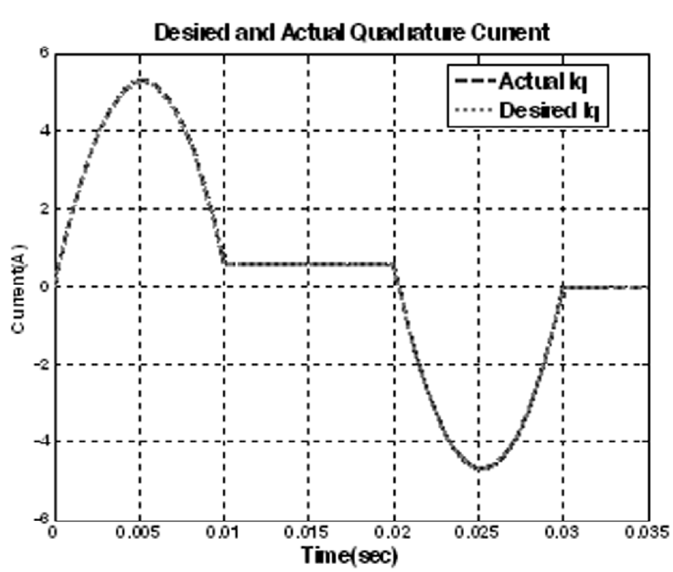

Figure 2: Desired and Actual Quadrature Current

$$u = -P\varepsilon \tag{15}$$

where P is the gain matrix which is obtained by pole placement technique using Ackermann's formula.

$$P = \begin{bmatrix} P1 & 0 & 0 & 0 & 0 \\ 0 & P2 & P3 & P4 & P5 \end{bmatrix}$$

Now with the current vectors obtained in (13) and (14), the equations are solved to get v_{dr} and v_{qr} in (16).

$$v_{dr} = Li_{dr} + Ri_{dr} - N_r L\omega_r i_{qr}$$

$$v_{qr} = Li_{qr} + Ri_{dr} - N_r L\omega_r i_{dr} + K\omega_r \tag{16}$$

REFERENCE SPEED AND POSITION GENERATION

The reference speed trajectory generated here is a trapezoidal curve. The trapezoidal curve consists of three sections with each portion occupying a fixed time length. Till time T1, it is a linear straight line of the form $y = mx$ with constant positive slope. From time T1 to T2, it is a constant line at 50 rad/sec with slope zero and from time T2 to T3, it is a linear straight of the form $y = mx + c$ with negative slope falling down to zero at time T3. The reference trajectory for position is obtained by integrating it. The reference trajectory used in this paper are given below:

$$y = \begin{matrix} 666.67x & 0 \le t < 0.075 \\ 50 & 0.075 \le t < 0.275 \\ -1000x + 325 & 0.275 \le t < 0.325 \end{matrix} \tag{17}$$

SECTION V

SIMULATION RESULTS

The simulation is carried out in MATLAB and the results are shown in the figure. In these results, it is seen that the Feedback Linearization Controller tracks the reference trajectory much closer than the other two. The simulation is carried out for 0.5 seconds and the output is recorded for the same. Further, the comparison between the controllers in the speed tracking trajectory and their corresponding errors in position as well as

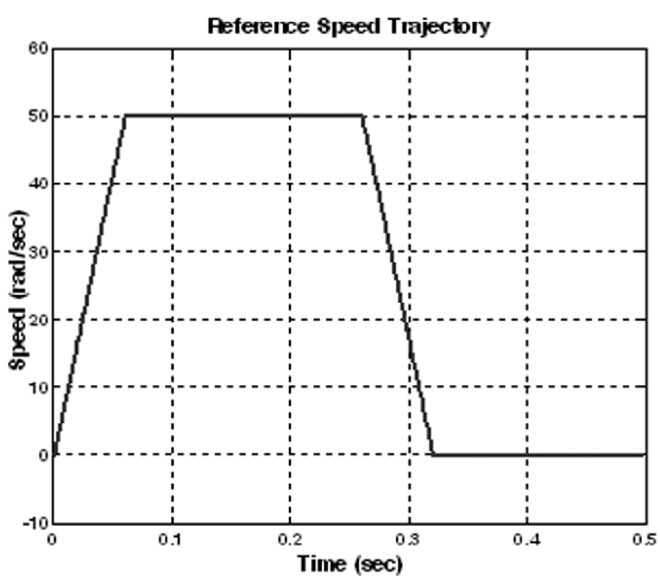

Figure 3: Reference Speed Trajectory

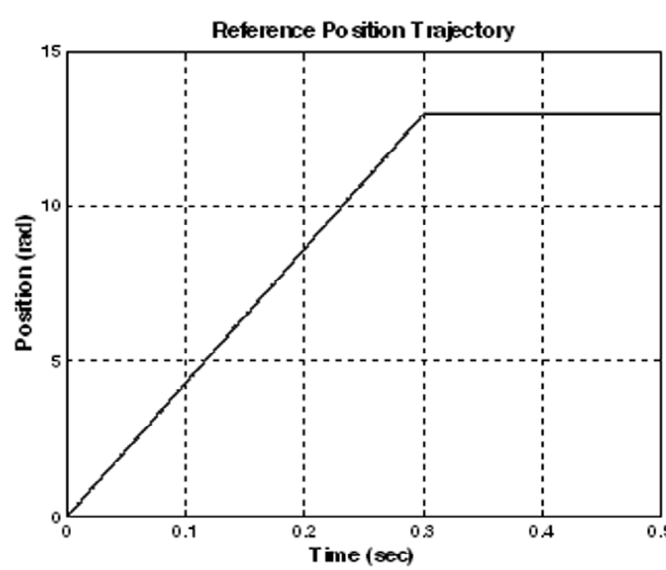

Figure 4: Reference Position Trajectory

speed is noted. Here, although the error converges to zero for all the three controllers, the tracking performance is the best for Feedback Linearization Controller. In addition to this, the actual Quadrature and direct current is found to move along with the desired values. For stability analysis, the Eigen values of the system matrix is found to be negative. Also, the root locus of all the three controllers show that the poles of the feedback linearization controller is comparatively towards the left half of the s-plane indicating a higher stability then the others. In the table given below which shows the comparison between the controllers, it is found that the performances indices have minimum values for feedback linearization controller. The minimum variance controller provides a better result when compared to PID controller. Yet the settling time and time constant remains almost the same for all the controllers. The performance criterions for evaluating the efficiency of a controller are Integral Absolute value of magnitude of error (IAE), Integral Square Error (ISE), and Integral Time multiplexed Absolute value of Error (ITAE). Their vales should be maintained as minimum as possible. These depend on the absolute value of error thereby preventing the error cancellation. The formulae for calculating them are as follows:

$$ISE = \int e^2 \, dt$$

$$IAE = \int |e| \, dt$$

$$ITAE = \int |e| t \, dt \tag{18}$$

Table for Comparison of the Three Controllers:

Performance Indices	*FBL*	*Minimum Variance*	*PID*
ISE	0.05	0.55	1.12
IAE	2.3	7.94	11.34
ITAE	0.92	2.38	3.4

MOTOR PARAMETERS

The following motor parameters are taken for simulation [19] are as follows:

Parameter	*Value*
J	4.5 × 10-5 kg.m2
N	50
Km	0.19 N.m/A
L	1.5 mH
R	0.55Ω
V	40 V
B	0.0008 N.m.s/rad

SECTION VI

CONCLUSION

Thus the performances of all the proposed controllers has been analysed based on various performance indices. Although the Minimum Variance Controller had slightly better results than PID Controller, it is evidently seen that Feedback Linearization Controller in a Permanent Magnet Stepper Motor (PMSM) has close tracking of trajectory position of the rotor and its current vectors with respect to reference values. The minimal amount of errors present was due to the inherent delays in the system, all of which could be undermined when put to use in an industrial setup. There is no amount of quantisation noise present in any

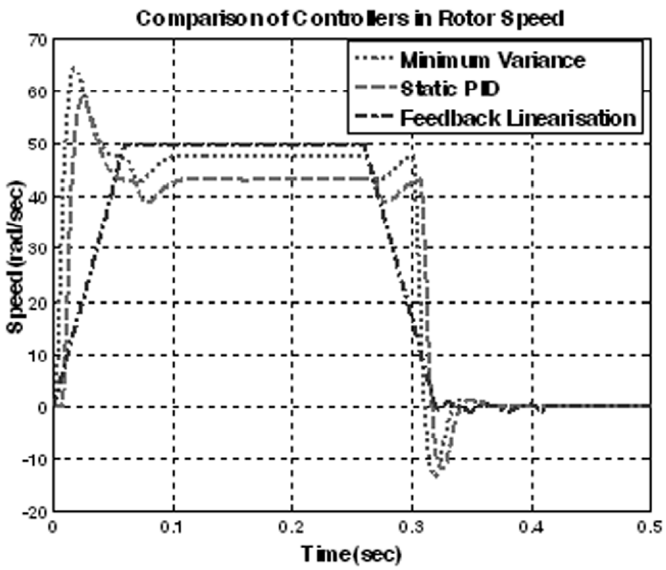

Figure 5: Comparison of Controllers in Rotor Speed

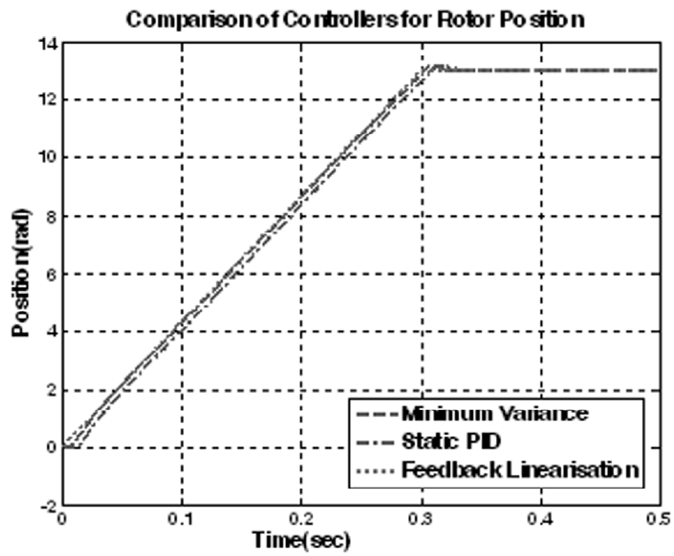

Figure 6: Comparison of Controllers in Rotor Position

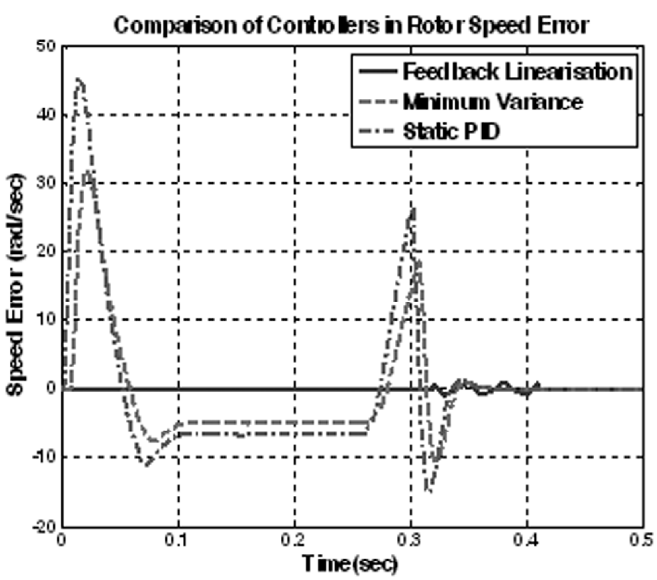

Figure 7: Comparison of Controllers in Rotor Speed Error

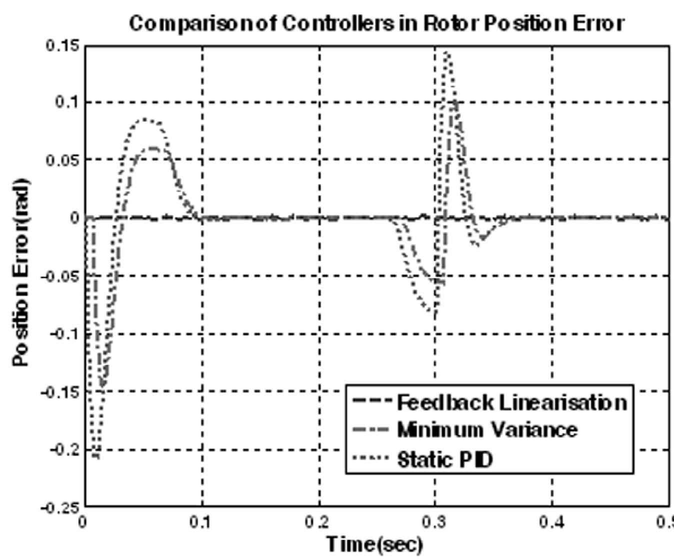

Figure 8: Comparison of Controllers in Rotor Position Error

of the tracking simulations of Feedback Linearization Controller thereby providing sufficient evidence that it outweighs the other controllers.

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