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CONTROL OF NONLINEAR PHENOMENA AT INCEPTION OF VOLTAGE COLLAPSE

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Abstract: In the present Paper, we shall investigate and analyze the concept of control of nonlinear Phenomena at inception of voltage Collapse . The presence of the dynamic bifurcations and the resulting implications for dynamic behavior, necessitate a re-examination of the role of saddle node bifurcations in the voltage collapse phenomenon more generally, catastrophic bifurcations are identified as important mechanisms for voltage collapse.

Keywords: Voltage collapse, Hopf. Bifurcation and nonlinear bifurcation Control

INTRODUCTION

We consider local control of voltage collapse at its inception. That is, we design controllers which can delay the occurrence of voltage collapse, as opposed to controllers for recovery from voltage collapse. The controllers we seek do not involve forced system operation in parameter ranges where voltage collapse does not occur, but are designed to work in the parameter ranges of difficulty. In order to control voltage collapse in power system models such as the one studied in the preceding section, one has to design control laws to deal with bifurcations, chaos and crises. We have seen that control laws which significantly reduce the amplitude of a bifurcated solution, or significantly enhance its stability over a nontrivial parameter range, are viable tools in the taming of chaos. Here similar techniques will be employed to control the bifurcations, chaos, and crises. In doing so we expect to increase the stability margin of the system in parameter space. In other words, voltage collapse will be 'postponed' so that stable operation of the system will be allowed beyond the point of impending collapse in the open loop system. In particular, the control laws are designed to increase two types of stability margin: a static stability margin and a dynamic stability margin. The Static stability margin is measured in parameter space from the point where the nominal equilibrium loses its stability. The dynamic stability margin is measured in parameter space from the point where voltage collapse takes place. As discussed in the previous section, these two stability margins are not necessarily the same. In the system models of under study. the static stability margin is measured from the Hopf Bifurcation point *Q*?. In the model of section 5.2.1, dynamic stability is measured either from *Q*? or the crisis point Q; depending on the route to collapse. In the model of Section 5.2.3, the dynamic stability is measured from the crisis point Qi.

In the models under study, the stable equilibrium point loses its stability through the subcritical Hopf bifurcation HBC!). The subcriticality of the hopf bifurcation has several negative effects on the system: the system may exhibit a jump from the stable equilibrium to the coexisting attractor under perturbation, and the boundary crisis is also a direct consequence of the subcriticality of the Hopf bifurcation. Moreover, the region of attraction of the stable equilibrium is bounded by the stable manifold of the unstable limit cycle, and so this region shrinks as criticality is approached. These factors motivate the design of feedback control laws directed at the Hopf bifurcation which reduce the negative effects and increase the stability margin of the system in parameter space. As shown in [3], such control action can also suppress the chaos and crises by 'squeezing' the period doubling cascades. Next we present a brief summary of the bifurcation the period doubling cascades. Next we present a brief summary of the bifurcation control approach in the context of Hopf [1] and period doubling [3] bifurcation control and the proceed to use these techniques in the voltage collapse control problem.

5.3.1 NONLINEAR BIFURCATION CONTROL

Consider a one-parameter family of nonlinear autonomous control systems

$$X = fm(x, u). \tag{1}$$

Where $x\ E\ IRn$ is the state vector, $\mu\ E\ IR$ is the system parameter, I; is a smooth map from $IRn\ x\ IR$ to IRn and u is a scalar input. Local bifurcation control deals with the design of smooth control laws $u = u(x)$ which stabilize a bifurcation occurring in the one-parameter family of systems. These control laws exist generically, even if the critical eigenvalues of the linearized system at the equilibrium of interest are uncontrollable. The feedback control designs of [1] transform a subcritical (unstable) Hopf bifurcation to a supercritical (stable) bifurcation.

For Hopf bifurcation. the design procedure recalled aims to ensure the asymptotic stability of the Hopf bifurcation point as well as or bital asymptotic stability of the periodic solutions emerging from the bifurcation point for a range of parameter values. It is well known that only the quadratic and cubic terms occurring in a nonlinear system undergoing a Hopf bifurcation influence the value of $(3_2$. Thus only the linear, quadratic, and cubic terms in an applied control u have potential for influencing $(3_2$. If the critical mode is controllable, a linear stabilizing feedback exists in such a case. On the other hand, if the critical mode is uncontrollable, the system may still be stabilizable by a quadratic feedback control law.

Now suppose the periodic solution emerging from the Hopf bifurcation point undergoes a cascade of period doubling bifurcation to chaos. As shown in [3], nonlinear feedback control laws can be designed which influence the degree of stability and amplitude of a given period-double orbit. If the amplitude of such an orbit can be constrained sufficiently, then the cascade of period doublings to chaos can be eliminated.

5.3.2 VOLTAGE COLLAPSE CONTROL I

In this subsection, we carry out the design of controllers for voltage collapse. Consider system subject to a frequency dependent control $u = u(w)$ by adding a control function u to the right side of give

$$\delta m = w \tag{2}$$

$$Mw = -dmw + Pm + Em\ VYm\ \sin)\delta - \delta m - \theta n) + E_m^2\ Ym\ Sin\ \theta m$$

$$K_q\ w\delta = -K_{qu2}\ V^2 - K_{qu}\ V + Q(\delta_m, \delta, V) - Q_0 - Q_1 \tag{3}$$

$$TK_{qw}\ K_{vv}\ V = -K_p\ wK_{qv2}\ V^2 + (K_{pw}\ K_{qv} - K_{qw}\ K_{vv})V \tag{4}$$

$$+ K_{qw}\ (P(\delta m, \delta_r\ V) - P^0 - p)$$

$$-K_{pw}\ (Q(\delta m, \delta, V) - Q0 - Q1) + u(w) \tag{5}$$

where $P(\delta m, \delta, V)$, $Q(\delta m, \delta, V)$

Note that the control is implemented by injecting a speed signal into the load node. The speed signal needs no washout since it does not affect the system equilibrium structure at steady state. Note also that such a controller does not affect the position of the saddle node bifurcation SNB(ID.

One control law design transforms a sub critical Hopf bifurcation into a super critical bifurcation and ensures a sufficient degree of stability of the bifurcated periodic solutions so that chaos and crises are eliminated. This control law allows stable operation very close to the point of impending collapse (saddle node bifurcation). Because the critical mode in this case in controllable, a purely cubic control is designed to handle all these tasks. Another control design in valves changing the critical parameter value at which the Hopf bifurcation occur through a linear feedback control. Because of the special structure of the system under study, this linear feedback law eliminates the Hopf bifurcations and the resulting chaos and crises. Thus, the linearly controlled system can operate at a stable equilibrium up to the saddle node bifurcation.

Nonlinear Bifurcation Control

To render the Hopf bifurcation HB(D supercritical, we employ a cubic feedback with measurement of w. The closed loop system is E_q, (2) – (5) and is of the form

$$u = K_n\ \omega^3 \tag{6}$$

Where $kn > o$ is the nonlinear (Cubic) Feedback gain.

Values of kn which give a supercritical HB are determined by computing the stability coefficient /3$_2$ of the closed loop system. Since transforming HB(D to a supercritical Hopf bifurcation is strictly a local result, computational analysis techniques must be used to assess the effects of the nonlinear control on the global dynamical behavior of the closed-loop system. As shown in [5], [2], larger values of the gain kn not only enhance the stability of the bifurcation but also result in a reduced amplitude of the stable limit cycle over a range of parameter values. Recalling the discussion in Subsection 5.3.1, if the amplitude of the periodic orbit can be constrained sufficiently, then the cascade of period doublings to chaos can be eliminated, Figure 5.1 shows a bifurcation diagram for the closed loop system with control gain *kn* = 0.5. In the closed-loop system, HB(D is rendered supercritical. Moreover, the period doubling bifurcations, including the two period doubling cascades and the resulting two strange attractors and their crises are all eliminated. The benefits of changing HB (D to a

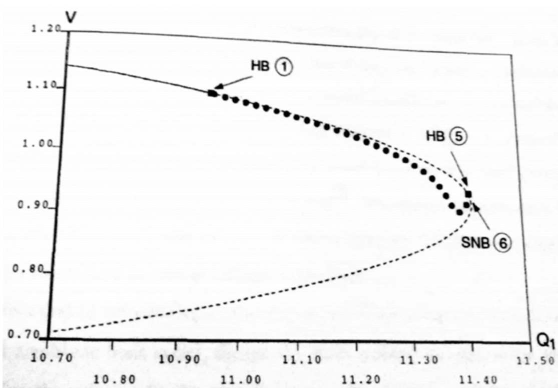

Figure 5.1 Bifurcation diagram of closed loop system with cubic control (*kn* = 0.5)

Supercritical bifurcation can be seen in Fig. 5.3 where the dynamic response of the system to increasing $Q_1$to a value beyond HB(D is shown. Transient trajectory (a) shows the increasing oscillations and ultimate voltage collapse without control. With nonlinear control and identical initial conditions, however, we see that voltage settles to a small amplitude oscillation in trajectory (b) rather than collapsing.

Evidently such a control has a very favorable effect on the voltage collapse dynamics. By transforming the Hopf bifurcation HB(D to a supercritical bifurcation, the multistability near HB(D is eliminated and hence the occurrence of jump behavior of the system operating point under perturbation is prevented. More significantly, the system can operate at a small amplitude limit cycle as Q_1 crosses the previous collapse point *Q*? and this transition can be done in a continuous fashion. Also because of the supercriticality of the Hopf bifurcation in the closed loop system, the region of attraction of the nominal stable equilibrium is increased. As Q_1 increases further, the nominal equilibrium regains stability at Q_1 = *Q(f* > through the supercritical

Hopf bifurcation HB@. The operation of the system takes yet another continuous transition from the small periodic orbit to the stable equilibrium as Q_1 Crosses $Q(f$ >. The system can operate at eh stable equilibrium until the saddle node bifurcation point SNB@ is encountered. Then a sharp drop in voltage collapse takes place.

Note that by introducing an alternative type of operating condition (a stable small amplitude limit cycle), though the static stability margin of the system remains the same as in the open loop case, the dynamic stability margin is increased up to the saddle node bifurcation point. Thus, in the closed loop system, the fatal voltage collapse then occurs at the saddle node bifurcation point (now agreeing with the scenario in [4]).

Linear Bifurcation Control

Since the critical mode is controllable, a linear stabilizing control exists for the stabilization of the Hopf bifurcation point. However, in the context of voltage collapse control, one has to consider the effect of such a control over a range of parameters. The effect may be difficult to determine since linear feedback will affect all the Eigen values and eigenvectors. In particular, a high gain linear feedback may well destabilize modes that are open-loop stable. Also it should not be surprising that in some situations a linear feedback which locally stabilizes an equilibrium may result in globally unbounded behavior[2]. For small feedback gains, however, one can expect that the bifurcation will reappear at a different parameter value. Fortunately, in this particular example, linear feedback with measurement of w in the form can impart desirable effects on the system. In (7), $K_1 > 0$ is the (scalar) linear feedback gain.

$$u = K_1\,\omega \qquad (7)$$

Since system (2)-(5) is a parametrized system, it is very difficult to study the effect of the linear control (7) over a range of parameter values by standard pole placement techniques directed at a particular equilibrium for a particular parameter value. However, if we consider the control gain, K_1 as a second parameter in the system in addition to the parameter Qi, the effect of linear control can be tracked with two-parameter continuations of the Hopf bifurcation points (HB(I) and HB@. Recall that the control design does not affect the position of NSB@.

Fig. 5.2 shows a two-parameter (depending on K_1 and Qi) curve of the Hopf bifurcation points (I) and @. It can be seen that as K_1 increases form 0, HB(I) and HB@ move closer to each other with HB(I) having a much faster pace. As k, increases further, HB(I) merges with BH@ leading to their disappearance. Figs. 5.2, 5.2 and 5.2 show that the Hopf window shrinks and ceases to exist. The benefits of eliminating the Hopf window are seen in trajectory (c) of fig. 5.3 where increasing Qi to a point which is beyond the location of the original HB(I) results in the system settling down to the original, high voltage equilibrium branch. The initial conditions of trajectory (c) coincide with those of (a) and (b).

With linear bifurcation control (7), both the static and dynamic stability margins can be increased. When the Hopf bifurcations cease to exist (approxi-

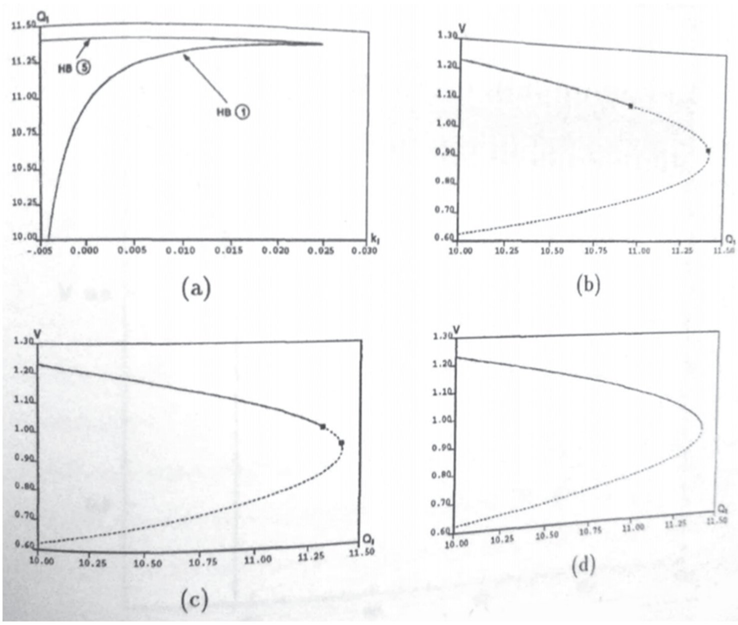

Figure 5.2 With linear bifurcation control: (a) two-parameter curves of the Hopf bifurcation points; bifurcation diagrams with (b) $K_z = 0$ (no control), (c) $KI = 0.01$, (d) $K_1 = 0.025$.

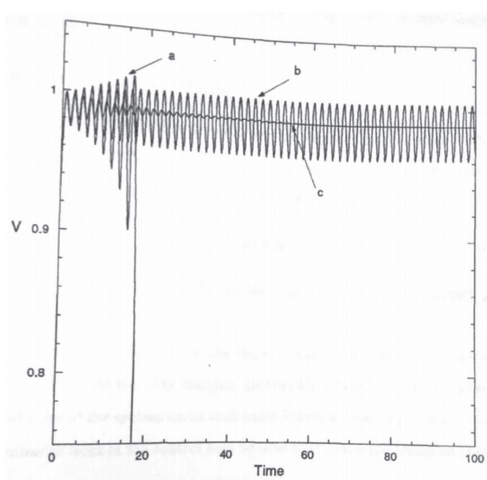

Figure 5.3 Dynamic responses of the system at $Q_1 = 11.35$: (a) without control; (b) with nonlinear control $kn = 0.5$; (c) with linear control $K_2 = 0.025$.

Mately for $K_1 > 0.0245$).the static and dynamic stability margins are maximized in the system can operate at a stable nominal equilibrium up to the saddle node bifurcation. Also, all the dynamic bifurcations. Including Hopf bifurcations, period doubling bifurcations, period doubling cascade to chaos and crises, are extinguished.

Composite Bifurcation Control

The two types of control law given above, namely the cubic control (6) and the linear control (7) can be combined to result in a composite control law. The closed loop system is Eq. (2)-(5) with u of the form where $kl > 0$ and $kn > 0$ are the (scalar) linear and nonlinear feedback gains, respectively.

$$u = K_1\,\omega + K_n\,\omega^3$$

With this composite control, the designer has the freedom to choose proper static and dynamic stability margins. Besides the flexibility in terms of achievable behavior of the system under such control (over a range of parameter values), the nonlinear term in the control may be used to improve the transient response.

5.3.3 VOLTAGE COLLAPSE CONTROL II

In the section, we consider local control of voltage collapse at its inception in the model of Section 5.2.3. The results parallel those of the subsection above, which were obtained for the model of section 5.2.1.

In the model of section 5.2.3, the stable equilibrium point loses its stability through the subcritical Hopf bifurcation (local), and the boundary crisis (global) of the strange attractor triggers the voltage collapse. The subcriticality of the Hopf bifurcation has several negative effects on the system as we discussed before. The same control strategy is employed as in the previous subsection. It can be easily seen in this example that this is an approach to influence the various aspects of the global bifurcations (e.g., blue sky catastrophes) through local control.

We carry out the design of controllers for voltage collapse control for the model subject to control u which is inserted additively at the right hand side.

The objectives of control are 1) to prevent the occurrence of the jump behavior, 2) to increase the region of attraction of the stable equilibrium point, and 3) to delay the collapse (in parameter space). One control law design transforms the subcritical Hopf bifurcation into a supercritical bifurcation and ensures a sufficient degree of stability of the bifurcated periodic solutions over a range of parameter values of interest. These control laws allow stable operation close to the point of saddle node bifurcation. Another control design involves changing the critical parameter value at which the Hopf bifurcation occurs through a linear feedback control. The voltage collapse can be delayed by such a linear control. Note that controllability of the critical mode facilitates the design.

Stabilizing the Hopf Bifurcation

To render the Hopf bifurcation HB(D supercritical, we employ a cubic feedback with measurement of w. The control *u* is of the form

$$u = -k_n \omega^3$$

Where = K ≪> 0 is the (:, *ca*, *la*, *r*) cubic feedback Q^a. in.

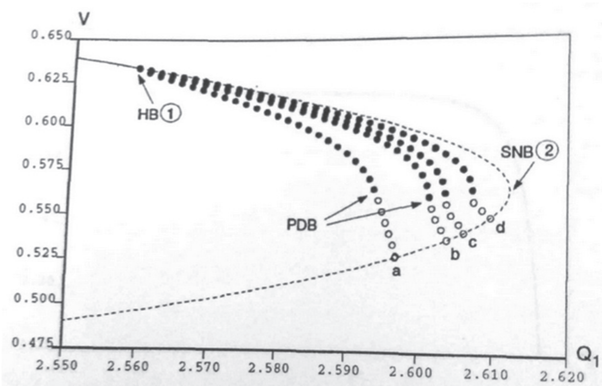

Figure 5.4: Superimposed bifurcation diagrams for cubic control with different gains: (a) –0.1; (b) –0.5; (c) –1.0; (d) –5.0

Fig.l 5.4 shows superimposed bifurcation diagrams for the closed loop system with various control gains *kn*. This along with simulation evidence indicates that larger values of the gain |*kn*| result in a reduced amplitude of the stable limit cycle. Note that the boundary crisis is delayed and the possible operation range of the system in parameter space is increased.

Delaying the Hopf Bifurcation

A linear control exists for delaying the Hopf bifurcation point:

$$u = -K_Z w. \tag{10}$$

Where –*K*| > 0 is the (scalar) linear feedback gain.

Such a control is found to be able to delay the subcritical Hopf bifurcation BHQ) to parameter values extremely close to Q<|>, where the saddle node bifur-cation SNB@ takes place. The relationship between the critical value Q? and the gain K| in the closed loop system is found by a two-parameter (Qi and K|) continuation of the Hopf bifurcation and is depicted in Fig. 5.5. Also another interesting consequence is that chaos and the boundary crisis can be eliminated by such a control.

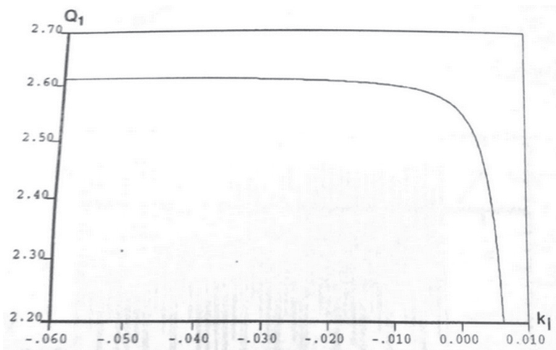

Figure 5.5 Relationship between critical Q_1 at which the Hopf bifurcation occurs and gain K| of linear control

Figure 5.6 illustrates the system responses for a value of Qi greater than the critical crisis value Qi. Trajectory (a) is with no control, (b) is with nonlinear control (9), and (c) is with linear control (10).

The two types of control law given above, namely the cubic control (9) and the linear control (10) can be combined to result in a composite control law which both delays and stabilizes the Hopf bifurcation.

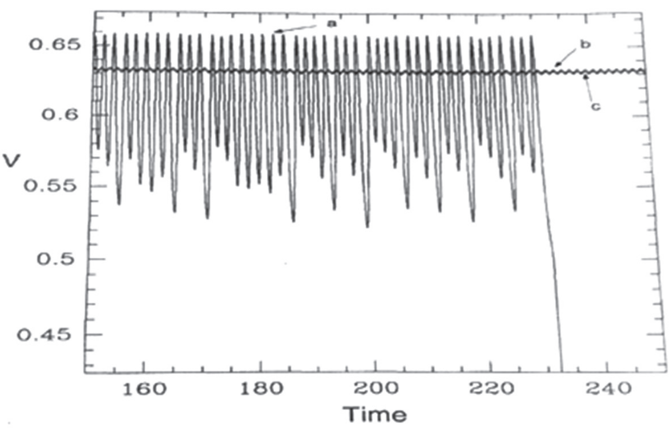

Figure 5.6: Sample trajectories of the system at Q_1 = 2.56.379: (a) without control; (b) with nonlinear control *kn* = –0.1; (c) with linear control K_1 = –0.0618.

5.4 CONCLUSION

Nonlinear phenomena, including bifurcations and chaos, occurring in power system models exhibiting voltage collapse have been studied in this paper. A new theory of voltage collapse is suggested. The theory emphasizes the role of catastrophic bifurcations in voltage collapse in electric power systems. The presences of the various nonlinear phenomena have also been determined to be crucial factors in the inception of voltage collapse in these models. Moreover, the problem of controlling voltage collapse in the presence of these nonlinear phenomena is addressed. The bifurcation control approach is employed to modify the bifurcations and to suppress chaos. The control law is shown to result in improved performance of the system for a greater range of parameter values. Although the relative importance of the effects of these nonlinear phenomena in general power systems under stressed conditions is still a topic for further research the bifurcation control approach appears to be a viable technique for control of these systems.

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