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ANALYSIS OF MAGNETIC EFFECT ON BLOOD FLOW IN STENOSED ARTERY FOR A TWO-LAYERED PULSATILE FLOW MODEL WITH BODY ACCELERATION AND SLIP AT WALL

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Abstract : This paper concerns with the pulsatile flow of blood through a mildly stenosed artery assuming blood as a two - layer fluid, where the erythrocytes remain suspended in the core region and the plasma constitutes the peripheral region. In the fluid flow model, it is assumed that the core region is Bingham plastic and the peripheral region is Newtonian fluid. This flow model includes the effects of uniform magnetic field , periodic body acceleration, velocity slip and pressure gradient on the blood flow through the constricted arteries. The analytical expressions for the velocity, flow rate, wall shear stress and the effective viscosity, are obtained employing the perturbation technique. Some of the aforesaid expressions have been depicted pictorially and then their behaviours (under the influence of various flow parameters) have been interpreted. It has been observed that with increase of magnetic field the velocity of core region and wall shear stress decreases whereas the velocity of peripheral region increases. Further, the velocity and the wall shear stress increase with increase in body acceleration and pressure gradient parameter. On the other hand, with increase in slip velocity, the velocity of blood increases but the wall shear stress decreases.

Mathematics Subject Classification : 76Z05, 74G10, 76W05.

Keywords : Bingham Plastic , Newtonian , Peripheral Plasma Layer , Slip velocity , Body Acceleration , Mild Stenosis , Blood , Magnetic Field , Pulsatile Flow .

INTRODUCTION:

Blood is a multi-component material that consists of gel-like formed elements in aqueous plasma viz. Red blood cells (RBC 98% by volume), white blood cells (WBC), platelets and variety of Lipoproteins. Plasma, which is an aqueous solution, contains numerous low molecular weight organic and inorganic materials in low concentration including clotting factors (fibrinogen, prothrombin etc.) and various ions. RBCS are in large numbers , which contain haemoglobin transport oxygen around the body. Henderson and Thurston have reported that though platelets are very small, they played an important rule on coagulation of blood both in the healing of wounds and in the formation of thrombi.

Blood flow in the human circulatory system is caused by the pumping action of the heart, which inturn produces a pressure gradient throughout the system [Fung (1981), Misra et al (2008)]. Human heart is a muscular pump which, by the contraction and expansion of heart muscles, produces a pressure difference in its systolic and diastolic conditions, popularly known as pressure pulse. Flow of blood due to this pressure pulse is know as pulsatile flow of blood [Chaturani and Samy (1985); Guyton and Hall (2006)]. Again, the human body may also be subjected to acceleration (or vibrations) which are quite common in normal life viz. while riding a vehicle or while landing, taking off and flying in an aircraft, sudden fast movement of the body, operating a jack hammer, during sports and gymnastic activities etc. In such circumstances, the human body gets unintentionally subjected to external acceleration, which is known as body acceleration. Due to this body acceleration, the flow of blood in the arteries is affected greatly [Sud and Sekhon (1985, 1987)].

Although the human body can generally adjust to such effects, yet prolonged exposure to such accelerative disturbances may lead to health problem, like headache, abdominal pain, loss of vision and increase pulse rate etc. Sud and Sekhon (1985) proposed a mathematical model of blood flow in a single artery subject to pulsatic pressure gradient as well as body acceleration. Nagarani and Sarojamma (2008) presented a theoretical model of pulsatile blood flow in a stenosed artery under the action of periodic body acceleration, considering blood as a Casson fluid.

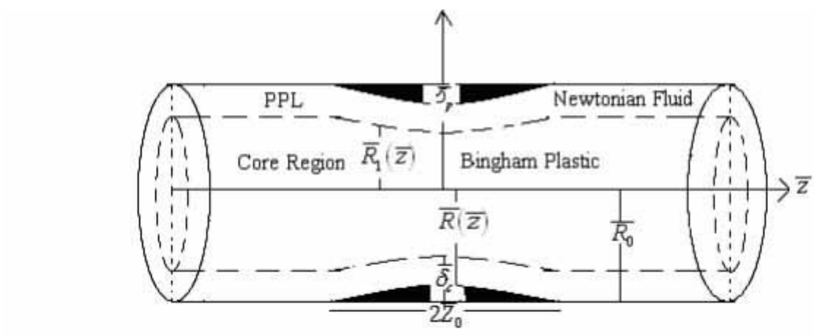
Many theoretical and experimental analysis were presented on the study of blood flow characteristics in presence of Stenosis [Mc Donald (1979), Mandal (2005), Bali and Awasthi (2007); Shankar and Ismail (2009); Biswas and Chakraborty (2009 a, 2009 b, 2010a), Sankar and Lee (2009, 2010)]. It has been observed from various reports that, though at high Shear rates blood exhibits Newtonian behaviour in large arteries like aorta [Taylor (1959)], blood being a suspension of corpuscles , at low Shear rates and while flowing through narrow vessels, blood behaves like a non- Newtonian fluid [Merrill et al (1965); Charm and Kurland (1974)]. Bugliarellow and Sevilla (1970) and Cokelet (1972) have experimentally proved that for blood flowing through small vessels, there exists cell poor plasma (Newtonian fluid) layer in the peripheral region and a core region of suspension containing almost all the erythrocytes.

It is to be noted that for certain flow models, many investigators pointed out that blood possesses a finite yield stress [Fung (1981), Kapur et al (1982)]. We know that Bingham plastic is an interesting and specialized material with yield stress whose consistency curve or flow behaviour seems to be a straight line curve [Fung (1981); Kapur et al (1982)]. This particular material deforms elastically, until the yield stress is reached, but once this stress is exceeded, it flows as a Newtonian fluid with Shear Stress being linearly related to the rate of Shear Strain [Schlichting (1968)]. Hence, in reality, blood behaves as Bingham Plastic in the core region of a constricted artery. Biswas and Chakraborty (2010 b) presented a model of two-layer pulsatile blood flow in a stenosed artery with body acceleration, where the peripheral layer was considered as Newtonian and the core region was assumed as Bingham plastic.

Here, in view of the above discussion, a blood flow model has been proposed for a stenosed artery in presence of a uniform magnetic field, where blood is assumed to be a two - layer fluid where the erythrocytes remain suspended in the core region and the plasma constitutes the peripheral region. The fluid flow model assumes that the core region behaves as Bingham plastic and the peripheral region as Newtonian fluid. This flow model includes the effects of a uniform magnetic field, periodic body acceleration, pressure gradient and velocity slip on the blood flow through constricted arteries.

MATHEMATICAL FORMULATION:

In this problem we consider an axially symmetric, laminar, pulsatile and fully developed flow of blood (assumed to be incompressible) through a circular tube with an axially non-symmetric mild stenosis in presence of a uniform magnetic field of low strength. So, the induced magnetic field is negligible. Next, it is assumed that the body fluid be represented by a two-layer fluid model with the core region (suspension of all erythrocytes) as Bingham plastic and the peripheral layer (constituted by plasma) as Newtonian fluid, where the wall of the tube is rigid. The length of the artery is assumed to be so great in comparison to its radius that the entrance and exit special wall effect may be neglected.



Schematic diagram of a stenosed artery

The geometry of stenosis in the peripheral region is given by [Nagarani and Sarojamma (2008); Biswas and Chakraborty (2010 b)]

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - \dfrac{\bar{\delta}_p}{2}\left(1+\cos\dfrac{\pi \bar{z}}{\bar{z}_0}\right), & \text{for } |\bar{z}| \le \bar{z}_0 \\ \bar{R}_0, & \text{for } |\bar{z}| > \bar{z}_0 \end{cases}, \tag{1}$$

and the geometry of the core region is given by

$$\bar{R}_1\left(\bar{z}\right)=\begin{cases}\beta\bar{R}_0-\dfrac{\bar{\delta}_c}{2}\left(1+\cos\dfrac{\pi\bar{z}}{\bar{z}_0}\right), & \text{for } \left|\bar{z}\right|\le\bar{z}_0\\ \beta\bar{R}_0, & \text{for } \left|\bar{z}\right|>\bar{z}_0\end{cases}\quad; \tag{2}$$

where , $\bar{R}(\bar{z})$ is the radius of the stenosed artery in the peripheral layer , $\bar{R}_1(\bar{z})$ is the radius of the artery in stenosed core region such that $\bar{R}_1(\bar{z})=\beta\bar{R}(\bar{z})$; where $\bar{R}_0$, $\beta\bar{R}_0$ are the radii of the normal artery and core region of the normal artery respectively ; $\bar{\delta}_p$ is the maximum height of the stenosis in the peripheral region , β is the ratio of central core radius to the normal artery radius, $\bar{\delta}_c$ is the maximum height of the stenosis in the core region such that $\bar{\delta}_c=\beta\bar{\delta}_p$ and $\bar{z}_0$ is the half length of the stenosis.

Further it is to be noted that the radial velocity is negligibly small for low Reynolds number flow in a tube with mild stenosis [Nagarani and Sarojamma(2008) ; Sankar and Lee (2009)].

The equation of motions governing the flow in core region and peripheral region are given by

$$\frac{\partial\bar{U}_B}{\partial\bar{t}}=-\frac{1}{\bar{\rho}_B}\frac{\partial\bar{p}}{\partial\bar{z}}-\frac{1}{\bar{\rho}_B\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\ \bar{\tau}_B)+\frac{1}{\bar{\rho}_B}\bar{F}(\bar{t})-\frac{\sigma\bar{U}_B\bar{B}^2}{\bar{\rho}_B}\ ,\quad 0\le\bar{r}\le\bar{R}_1(\bar{z})\ ; \tag{3}$$

and

$$\frac{\partial\bar{U}_N}{\partial\bar{t}}=-\frac{1}{\bar{\rho}_N}\frac{\partial\bar{p}}{\partial\bar{z}}-\frac{1}{\bar{\rho}_N\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{r}\ \bar{\tau}_N)+\frac{1}{\bar{\rho}_N}\bar{F}(\bar{t})-\frac{\sigma\bar{U}_N\bar{B}^2}{\bar{\rho}_N}\ ,\quad \bar{R}_1(\bar{z})\le\bar{r}\le\bar{R}(\bar{z})\ ; \tag{4}$$

where, $\bar{U}_B$ and $\bar{U}_N$ are the fluid velocities in the core region and peripheral region respectively, $\bar{\tau}_B$, $\bar{\tau}_N$ are the shear stresses for Bingham plastic fluid and Newtonian fluid respectively ; $\bar{p}$ is the pressure and $\bar{F}(\bar{t})$ is the body acceleration ; $\bar{\rho}_B$, $\bar{\rho}_N$ are the densities of Bingham plastic fluid and Newtonian fluid respectively, σ is the electrical conductivity and $\bar{B}$ is the applied magnetic field.

The constitutive equations for Bingham plastic fluid and Newtonian fluid are respectively given by

$$\bar{\tau}_B = \bar{\tau}_Y - \bar{\mu}_B \frac{\partial \bar{U}_B}{\partial \bar{r}}, \text{ if } \bar{\tau}_B \geq \bar{\tau}_Y, \bar{R}_p \leq \bar{r} \leq \bar{R}_1(\bar{z}) \tag{5}$$

$$\frac{\partial \bar{U}_B}{\partial \bar{r}} = 0, \qquad \text{if } \bar{\tau}_B \leq \bar{\tau}_Y, \ 0 \leq \bar{r} \leq \bar{R}_p$$

and

$$\bar{\tau}_N = -\bar{\mu}_N \frac{\partial \bar{U}_N}{\partial \bar{r}} \quad \text{if } \bar{R}_1(\bar{z}) \leq \bar{r} \leq \bar{R}(\bar{z}), \tag{6}$$

where, $\bar{R}_p$ is the radius of the plug flow region and $\bar{\tau}_Y$ is the Yield Stress [Sankar and Lee (2010)] .

The periodic body acceleration in axial direction is given by

$$\bar{F}(\bar{t}) = a_0 \cos(\bar{\omega}_b \bar{t} + \phi), \tag{7}$$

where, a_0 is the amplitude, $\bar{\omega}_b = 2\pi \bar{f}_b$, $\bar{f}_b$ is its frequency in Hz, ϕ is the angle of $\bar{F}(\bar{t})$ with respect to the heart action. The frequency of body acceleration $\bar{f}_b$ is taken to be small so that wave effect can be ignored.

The pressure gradient at any position $\bar{z}$ and time $\bar{t}$ may be represented as under:

$$-\frac{\partial \bar{p}}{\partial \bar{z}}(\bar{z}, \bar{t}) = A_0 + A_1 \cos(\bar{\omega}_p \bar{t}) \quad , \tag{8}$$

where, A_0 is the steady component of the pressure gradient, A_1 is the amplitude of fluctuating component of pressure gradient and $\bar{\omega}_p = 2\pi \bar{f}_p$,where $\bar{f}_p$ is the pulse frequency. Both A_0 and A_1 are functions of $\bar{z}$.

Let us introduce the following non-dimensional variables:

$$z = \frac{\bar{z}}{\bar{R}_0}, \quad R(z) = \frac{\bar{R}(\bar{z})}{\bar{R}_0}, \quad R_1(z) = \frac{\bar{R}_1(\bar{z})}{\bar{R}_0}, \quad r = \frac{\bar{r}}{\bar{R}_0}, \quad t = \bar{t}\,\bar{\omega}_p, \ \omega = \frac{\bar{\omega}_b}{\bar{\omega}_p}, \quad \delta_p = \frac{\bar{\delta}_p}{\bar{R}_0}, \quad \delta_c = \frac{\bar{\delta}_c}{\bar{R}_0},$$

$$U_B = \frac{\bar{U}_B}{A_0 \bar{R}_0^2 / 4\bar{\mu}_B}, \ U_N = \frac{\bar{U}_N}{A_0 \bar{R}_0^2 / 4\bar{\mu}_N}, \quad U_S = \frac{\bar{U}_S}{A_0 \bar{R}_0^2 / 4\bar{\mu}_N}, \ \tau_B = \frac{\bar{\tau}_B}{A_0 \bar{R}_0 / 2}, \ \tau_N = \frac{\bar{\tau}_N}{A_0 \bar{R}_0 / 2},$$

$$e = \frac{A_1}{A_0}, \ B = \frac{a_0}{A_0}, \ \theta = \frac{\bar{\tau}_Y}{A_0 \bar{R}_0 / 2}, \quad \alpha^2{}_B = \frac{\bar{R}_0^2\, \bar{\omega}_p\, \bar{\rho}_B}{\bar{\mu}_B}, \quad \alpha^2{}_N = \frac{\bar{R}_0^2\, \bar{\omega}_p\, \bar{\rho}_N}{\bar{\mu}_N}, \tag{9}$$

where, α_B and α_N are the pulsatile Reynolds number for Bingham plastic fluid and Newtonian fluid respectively.

Using the above non-dimensional variables, equations (1) and (2) become

$$R(z)=\begin{cases}1 - \dfrac{\delta_p}{2}\left(1+\cos\dfrac{\pi z}{z_0}\right), & \text{for } |z|\le z_0\\ 1\ , & \text{for } |z|>z_0\end{cases}\ , \tag{10}$$

and

$$R_1(z)=\begin{cases}\beta - \dfrac{\delta_c}{2}\left(1+\cos\dfrac{\pi z}{z_0}\right), & \text{for } |z|\le z_0\\ \beta\ , & \text{for } |z|>z_0\end{cases}\ . \tag{11}$$

The governing equations of motion in (3) and (4) , in non-dimensional form are

$$\alpha_B^2\frac{\partial U_B}{\partial t}=4f(t)-\frac{2}{r}\frac{\partial}{\partial r}(r\,\tau_B)-U_B M_B\ , \tag{12}$$

(where $M_B=\dfrac{\sigma\,\bar{R}_0^{\,2}\,\bar{B}^{\,2}}{\bar{\mu}_B}$ is the Hartmann number for the core region , i.e. for the Bingham plastic fluid)

and , $$\alpha_N^2\frac{\partial U_N}{\partial t}=4f(t)-\frac{2}{r}\frac{\partial}{\partial r}(r\,\tau_N)-M_N U_N\ ; \tag{13}$$

where , $M_N=\dfrac{\sigma\,\bar{R}_0^2\,\bar{B}^2}{\bar{\mu}_N}$ is the Hartmann number for peripheral region , i.e. for Newtonian fluid

and , $$f(t)=1+e\cos t+B\cos(\omega t+\phi) \tag{14}$$

Using non-dimensional variables, equations (5) and (6) reduce to

$$\tau_B=\theta-\frac{1}{2}\frac{\partial U_B}{\partial r}\ ,\ \ if\ \ \tau_B\ge\theta,\ R_p\le r\le R_1(z)\ ; \tag{15}$$

$$\frac{\partial U_B}{\partial r}=0\ ,\ \ if\ \ \tau_B\le\theta,\ 0\le r\le R_p\ ; \tag{16}$$

and , $$\tau_N=-\frac{1}{2}\frac{\partial U_N}{\partial r}\ ,\ \ if\ R_1(z)\le r\le R(z)\ ; \tag{17}$$

The boundary conditions in the non-dimensional form are given by

(i) τ_B is finite at $r=0$ (18)

(ii) $U_N=U_S$ at $r=R(z)$ (19)

(iii) $\tau_B=\tau_N$, $U_B=U_N$ at $r=R_1(z)$ (20)

The volumetric flow rate is given by:

$$\bar{Q}(\bar{t}) = Q(t)\,\frac{\pi(\bar{R}_0)^4 A_0}{8\,\bar{\mu}_B} \quad , \tag{21}$$

where, the non-dimensional form becomes, $Q = 4\int_0^{R(z)} r\,U(r,z,t)\,dr \quad ;$ (22)

following Sankar and Lee (2010) .

The effective viscosity $\bar{\mu}_e$ is defined in dimensional form as :

$$\bar{\mu}_e = \frac{\pi\left(-\frac{\partial \bar{p}}{\partial \bar{z}}\right)\left(\bar{R}(\bar{z})\right)^4}{\bar{Q}(\bar{t})} \quad (23$$

The effective viscosity in non-dimensional form can be express as:

$$\mu_e = \left(R(z)\right)^4 (1 + e\cos t)/Q(t) \quad (24$$

METHOD OF SOLUTION:

Since $\alpha^2{}_B$ and α_N^2 are time dependent , so we will expand (12), (13) and (15) to (17) in perturbation series about α_B^2 and α_N^2 and further we will use Bessels function for solution whenever necessary.

We put:

$$U_p = U_{0p}(z,t) + \alpha_B^2 U_{1p}(z,t) + --------- \tag{25}$$

$$R_p = R_{0p}(z,t) + \alpha_B^2 R_{1p}(z,t) + --------- \tag{26}$$

$$U_N = U_{0N}(z,r,t) + \alpha_N^2 U_{1N}(z,r,t) + ------ \tag{27}$$

$$U_B = U_{0B}(z,r,t) + \alpha_B^2 U_{1B}(z,r,t) + ------- \tag{28}$$

From equation (13) we get:

$$4f(t) + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial U_{0N}}{\partial r}\right)\right] - M_N U_{0N} = 0 \tag{29}$$

and, $$\frac{\partial U_{0N}}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial U_{1N}}{\partial r}\right)\right] - M_N U_{1N} \tag{30}$$

From equation (12) we get:

$$4f(t) - \frac{2}{r}\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial U_{0B}}{\partial r}\right)\right] - M_B U_{0B} = 0 \tag{31}$$

and, $$\frac{\partial U_{0B}}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial U_{1B}}{\partial r}\right)\right] - M_B U_{1B} \tag{32}$$

From equation (31) we get:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_{0B}}{\partial r}\right)-M_B U_{0B}= f(t)\left(\frac{2k}{r}-4\right) \tag{33}$$

where , $\dfrac{\theta}{f(t)}=k$; (33.1)

Now , using (27) , (28) in (19) , (20) , we have :

$U_{0B}=U_{0N}$ and $U_{1B}=U_{1N}$ at $r=R_1$; (34)

$U_{0N}=U_s$ and $U_{1N}=0$ at $r=R$; (35)

Now on the axis, i.e. on $r = 0$, the velocity as well as the shear stress are finite; (36)

Solving the system of equations from (29) to (33), subject to the conditions (34), (35) and (36), we obtain the following solutions:

$$U_{0N} = \left[\zeta_1 J_0(r\,i\sqrt{M_N}) + \frac{4}{M_N}\right] f(t) \ , \tag{37}$$

$$U_{0B} =\left[2k\,J_0(r\,i\sqrt{M_B})\int_0^r \bar{\phi}(r)\,dr + \frac{4}{M_B} + J_0(r\,i\sqrt{M_B})\zeta_2\right] f(t) \ ; \tag{38}$$

where, we define :

$$\zeta_1 = \frac{\left\{\dfrac{U_s}{f(t)}-\dfrac{4}{M_N}\right\}}{J_0(R\,i\sqrt{M_N})} \ , \quad \zeta_2 = \left\{\frac{\zeta_1 J_0(R_1 i\sqrt{M_N}) + \dfrac{4}{M_N} - \bar{B}_1}{J_0(R_1 i\sqrt{M_B})}\right\}$$

$$\bar{B}_1 = \left\{2k\,J_0(R_1 i\sqrt{M_B})\int_0^{R_1} \bar{\phi}(r)\,dr + \frac{4}{M_B}\right\} \ ,$$

and $$\bar{\phi}(r) = \frac{\int_0^r J_0(r\,i\sqrt{M_B})\,dr}{r J_0^2(r\,i\sqrt{M_B})} \ ; \tag{39}$$

We also obtain:

$$U_{1B} = \left(J_0(r\,i\sqrt{M_B})Z_1(r) - \frac{4}{M_B^{\,2}}\right) f'(t) \ , \tag{40}$$

$$U_{1N} =\left(\zeta_1 J_0(r\,i\sqrt{M_N})\, Z_2(r) - \frac{4}{M_N^{\,2}}\right) f'(t) \ ; \tag{41}$$

where, we define :

$$Z_1(r) = \int_0^r \bar{\phi}_2(r)\,dr \ , \quad \bar{\phi}_2(r) = \frac{\int_0^r \bar{G}(r)\,r\,J_0^2(r\,i\sqrt{M_B})\,dr}{r\,J_0^{\,2}(r\,i\sqrt{M_B})} \ ,$$

$$\bar{G}(r) = \left[2k \int_0^r \bar{\phi}(r)\, dr + \zeta_2 \right] \ ,$$

$$Z_2(r) = \left[-\frac{ri}{2\sqrt{M_N}} \frac{J_1(ri\sqrt{M_N})}{J_0(ri\sqrt{M_N})} + \bar{k}_1 \int_{R_1}^{r} \frac{dr}{r J_0^{\,2}(ri\sqrt{M_N})} + \bar{k}_2 \right] \ ,$$

$$\bar{k}_2 = \frac{1}{\zeta_1 J_0(R_1 i\sqrt{M_N})} \left[J_0(R_1 i\sqrt{M_B}) \int_0^{R_1} \bar{\phi}_2(r)\, dr - \frac{4}{M_B^2} + \frac{4}{M_N^2} + \frac{\zeta_1 R_1 i J_1(R_1 i\sqrt{M_N})}{2\sqrt{M_N}} \right] \ ,$$

$$\bar{k}_1 = \left(\frac{\zeta_4 - \bar{k}_2}{\zeta_3} \right) \ , \quad \zeta_3 = \int_{R_1}^{R} \frac{dr}{r J_0^2(ri\sqrt{M_N})} \ ,$$

$$\zeta_4 = \frac{4}{M_N^{\ 2} \zeta_1 J_0(Ri\sqrt{M_N})} + \frac{Ri}{2\sqrt{M_N}} \frac{J_1(Ri\sqrt{M_N})}{J_0(Ri\sqrt{M_N})} . \tag{42}$$

Now the non – dimensional velocity field is determined by:

$$U_{0p} = \left(U_{0B}\right)_{r=R_p} \quad \text{and} \quad U_{1p} = \left(U_{1B}\right)_{r=R_p} \ .$$

Therefore , $U_p = \left(U_{0p} + \alpha_B^2 U_{1p}\right)$, (43)

$U_B = \left(U_{0B} + \alpha_B^2 U_{1B}\right)$, and $U_N = \left(U_{0N} + \alpha_N^2 U_{1N}\right)$; (44)

where, U_{0B} , U_{0N} , U_{1N} , U_{1B} are defined in the expressions of (38), (37), (41) and (40) respectively .

The wall shear stress $\bar{\tau}_w$ in the wall of the stenosis is given by

$\bar{\tau}_w$ = Shear stress in the Newtonian region = $\bar{\tau}_N$;

$$\Rightarrow \bar{\tau}_w = \bar{\tau}_N = \left(\bar{\mu}_N \frac{\partial \bar{U}_N}{\partial \bar{r}} \right)_{\bar{r} = \bar{R}} = \left(\frac{A_0 \bar{R}_0}{2} \right) \frac{1}{2} \left(\frac{\partial U_N}{\partial r} \right)_{r=R} \ ;$$

$$\Rightarrow \frac{\bar{\tau}_w}{\left(\frac{A_0 \bar{R}_0}{2} \right)} = \frac{1}{2} \left(\frac{\partial U_N}{\partial r} \right)_{r=R} \Rightarrow \tau_w = \frac{1}{2} \left(\frac{\partial U_N}{\partial r} \right)_{r=R} \ ;$$

where , τ_w is the non - dimensional wall shear stress .

$$\Rightarrow \tau_w = \frac{1}{2} \left[\frac{\partial U_{0N}}{\partial r} \right]_{r=R} + \frac{\alpha_N^2}{2} \left[\frac{\partial U_{1N}}{\partial r} \right]_{r=R} \ ; \tag{45}$$

where, we define:

$$\left[\frac{\partial U_{0N}}{\partial r}\right]_{r=R} = -\,\zeta_1 f(t)(i\sqrt{M_N})J_1(Ri\sqrt{M_N}) \quad ,$$

$$\left[\frac{\partial U_{1N}}{\partial r}\right]_{r=R} = f'(t)\zeta_1 J_0(Ri\sqrt{M_N})\left(\frac{\partial Z_2(r)}{\partial r}\right)_{r=R} - f'(t)\zeta_1\left[Z_2(r)i\sqrt{M_N}\; J_1(ri\sqrt{M_N})\right]_{r=R} \tag{46}$$

where , $$\left[\frac{\partial Z_2(r)}{\partial r}\right]_{r=R} = \left[\frac{r}{2}\left\{1+\frac{J_1^2(ri\sqrt{M_N})}{J_0^2(ri\sqrt{M_N})}\right\} + \frac{\bar{k}_1}{rJ_0^2(ri\sqrt{M_N})}\right]_{r=R} .$$

The volumetric flow rate Q is given from (22) by

$$Q = 4\int_0^{R_{0p}} r\left(U_{0p}+\alpha_B^2 U_{1P}\right)dr + 4\int_{R_{0P}}^{R_1} r\left(U_{0B}+\alpha_B^2 U_{1B}\right)dr + 4\int_{R_1}^{R} r\left(U_{0N}+\alpha_N^2 U_{1N}\right)dr \tag{47}$$

$$= Q_1 + Q_2 + Q_3 \quad ;$$

where , we define:

$$Q_1 = 4\int_0^{R_{0P}} r\left(U_{0P}+\alpha_B^2 U_{1P}\right)dr = 4\left(U_{0P}+\alpha_B^2 U_{1P}\right)\left[\frac{r^2}{2}\right]_{r=0}^{r=R_{0P}} = 2\left(U_{0P}+\alpha_B^2 U_{1P}\right)R_{0P}^2 \quad ;$$

$$Q_2 = 4\left[\begin{array}{l}\left\{\begin{array}{l} -\dfrac{R_1 i}{\sqrt{M_B}}\rho_1(R_1)J_1(R_1 i\sqrt{M_B}) + \dfrac{R_{0P}i}{\sqrt{M_B}}\rho_1(R_{0P})J_1(R_{0P}i\sqrt{M_B}) + \\ \dfrac{2ki}{\sqrt{M_B}}\displaystyle\int_{R_{0P}}^{R_1}\bar{\varphi}(r)\,r\,J_1(ri\sqrt{M_B})dr - \dfrac{4iR_1^2 J_1(R_1 i\sqrt{M_B})}{M_B\sqrt{M_B}} + \\ \dfrac{4iR_{0P}^2 J_1(R_{0P}i\sqrt{M_B})}{M_B\sqrt{M_B}} + \dfrac{4i}{M_B\sqrt{M_B}}\displaystyle\int_{R_{0P}}^{R_1} r\,J_1(ri\sqrt{M_B})dr \end{array}\right\} f(t) + \\ \qquad \alpha_B^2\left\{\displaystyle\int_{R_{0P}}^{R_1} r\,J_0(ri\sqrt{M_B})Z_1(r)dr - \dfrac{2}{M_B^2}(R_1^2 - R_{0P}^2)\right\} f'(t)\end{array}\right] ;$$

$$Q_3 = 4\left[\left\{-\frac{\zeta_1 R i}{\sqrt{M_N}}J_1(Ri\sqrt{M_N}) + \frac{2}{M_N}\left(R^2-R_1^2\right) + \frac{\zeta_1 R_1 i}{\sqrt{M_N}}J_1(R_1 i\sqrt{M_N})\right\} f(t)\right]$$

$$+4\alpha_N^2\left[\begin{array}{l}\zeta_1\left\{\dfrac{R_1^2}{2M_N}J_2(R_1 i\sqrt{M_N}) - \dfrac{R^2}{2M_N}J_2(Ri\sqrt{M_N}) + \dfrac{\bar{k}_1}{M_N J_0(Ri\sqrt{M_N})} - \dfrac{\bar{k}_1}{M_N J_0(R_1 i\sqrt{M_N})}\right\} \\ +\zeta_1\left\{\dfrac{\rho_2(R_1)R_1 i J_1(R_1 i\sqrt{M_N})}{\sqrt{M_N}} - \dfrac{\rho_2(R)Ri J_1(Ri\sqrt{M_N})}{\sqrt{M_N}}\right\} - \dfrac{2}{M_N^2}\left(R^2-R_1^2\right)\end{array}\right] f'(t);$$

where , $$\rho_2(r) = \left\{\bar{k}_1\int_{R_1}^{r}\frac{dr}{rJ_0^2(ri\sqrt{M_N})} + \bar{k}_2\right\} ,$$

and , $$\rho_1(r) = 2k\int_0^r \bar{\varphi}(r)\,dr + \zeta_2 \quad .$$

The numerical results concerning the expressions U_N , U_B , τ_w and Q are computed by using Wolfram Mathematica software .

The expression for effective viscosity μ_e can be obtained from the expressions of (24) and (47).

RESULTS AND DISCUSSION:

This model has been developed to analyse the magnetic effect on velocity profile, wall shear stress on the wall of the stenosis, volumetric flow rate and effective viscosity of blood in a two layer blood flow model under various flow parameters assuming the core region as Bingham plastic and the peripheral layer as Newtonian fluid. Here, we will discuss graphically the velocity field and wall shear stress under various flow parameters.

Figure 1 and Figure 2 depict the variation of peripheral layer (Newtonian fluid) velocity U_N and core region velocity (Bingham plastic) U_B versus radial distance r for various values of M_B (Hartman number for Bingham plastic) with fixed values of $B = 1$, $e=1$, $U_S = 0.05$ and $M_N = 0.2$ (Hartman number for Newtonian fluid). In these figures we observe that both U_N and U_B decrease with increase of M_B.

Figure 3 and Figure 4 show the behaviour of U_N and U_B versus r for various values of M_N with fixed values of $U_S = 0.05, B=1, e=1, M_B = 0.2$. In these figures, we note that with increase of M_N, the U_N increases whereas the U_B decreases.

Thus from Figure 1, Figure 2, Figure 3 and Figure 4 , we see that with increase of M_B, both U_N and U_B decrease, whereas U_N increases but U_B decreases with increase of M_N.

Figure 5 and Figure 6 portray the variation of U_N and U_B versus r for various values of body acceleration parameter B with fixed values of $U_S = 0.05$, $M_B = 0.2$, $M_N = 0.2$, $e = 1$. Here we observed that both U_N and U_B increase with a rise in the body acceleration parameter B .

Figure 7 and Figure 8 depict the variation of U_B and U_N versus r for various values of slip velocity U_s with fixed values of $B=1$, $M_B=0.2$, $M_N=0.2$, $e=1$. Here we see that both U_B and U_N increase with increase in slip velocity U_s .

Figure 9 and Figure 10 portray the variation of U_N and U_B versus r for various values of pressure gradient parameter e with the values of $U_S = 0.05,\ M_B = 0.2,\ M_N = 0.2,\ B = 1$ fixed. Here we note that both U_N and U_B increase with increase in pressure gradient parameter e.

Figure 11 depicts the behaviour of wall shear stress τ_w versus t for various values of M_B with fixed values of $U_S=0.05, B=1, e=1, M_N=0.5$. Here, we observe that magnitude of τ_w decreases with increase of M_B.

Figure 12 depicts the variation of wall shear stress τ_w versus t for various values of M_N with fixed values of $U_S=0.05,\ B=1,\ e=1, M_B=0.5$. Here also, we observe that the magnitude of τ_w decreases with increase of M_N.

Hence, from figure 11 and figure 12 we observe that shear stress τ_w decreases in magnitude with increase in M_B and M_N.

Figure 13 shows the variation of wall shear stress τ_w versus t for various values of body acceleration parameter B with fixed values of $U_S = 0.05,\ e = 1,$ $M_B = 0.5,\ M_N = 0.5$. Here we note that the magnitude of τ_w increases with increase in B.

Figure 14 depicts the variation of wall shear stress τ_w versus t for various values of pressure gradient parameter e with fixed values of $U_S = 0.05,\ B = 1,$ $M_B = 0.5,\ M_N = 0.5$. Here also, we observe that the magnitude of τ_w increases with increase in e.

Figure 15 shows the variation of wall shear stress τ_w versus t for various values of slip velocity U_S with fixed values of $e = 1,\ B = 3,\ M_B = 0.5,$ $M_N = 0.5$. Here we see that the magnitude of τ_w decreases with increase of U_S.

CONCLUSIONS:

From the above discussion, we may come to the following conclusions:

i) The speed of blood flow in stenosed artery is greatly influenced by the strength of the magnetic field, because in presence of magnetic field, the velocity in the peripheral layer (Blood plasma) and the velocity in core region (erythrocytes) mostly decrease. So, patients with coronary artery diseases should stay away from strong magnetic fields to maintain the normal flow of blood. But, a magnetic field of appropriate strength may be beneficial for patients with coronary artery diseases to keep control on the speed of blood flow.

ii) Under the influence of magnetic field, with increase of body acceleration, both the velocities of blood plasma as well as erythrocytes (represented by Bingham plastic) increase. So periodic body acceleration needs to be controlled for patients with coronary artery disease, whenever they are under a magnetic field.

iii) Under the influence of magnetic field, with increase of slip velocity the velocity of blood increases whereas the wall shear stress decreases. Hence, drugs that increase the slip velocity may be beneficial in increasing the flow of blood and in decreasing the wall shear stress which may help in minimizing the harmful effects of cardiovascular diseases to a great extent.

iv) The speed of blood flow (both in peripheral and core region) under magnetic field increases with increase in pressure gradient. Hence, to keep control over the flow of blood under magnetic effects, the pressure gradient must be maintained at appropriate levels.

v) The wall shear stress in the wall of stenosed artery decreases with increase in the strength of the applied magnetic field. So, to regulate the wall shear stress on the stenosed artery, a suitable magnetic field is desirable so that the risk due to cardiovascular diseases may be minimized.

vi) With increase in the body acceleration parameter as well as in the pressure gradient parameter, the wall shear stress in the wall of the stenosed artery increases. Hence, the body acceleration and pressure gradient should be well maintained for the safety of the patient with coronary artery diseases whenever they are under the influence of a magnetic field.

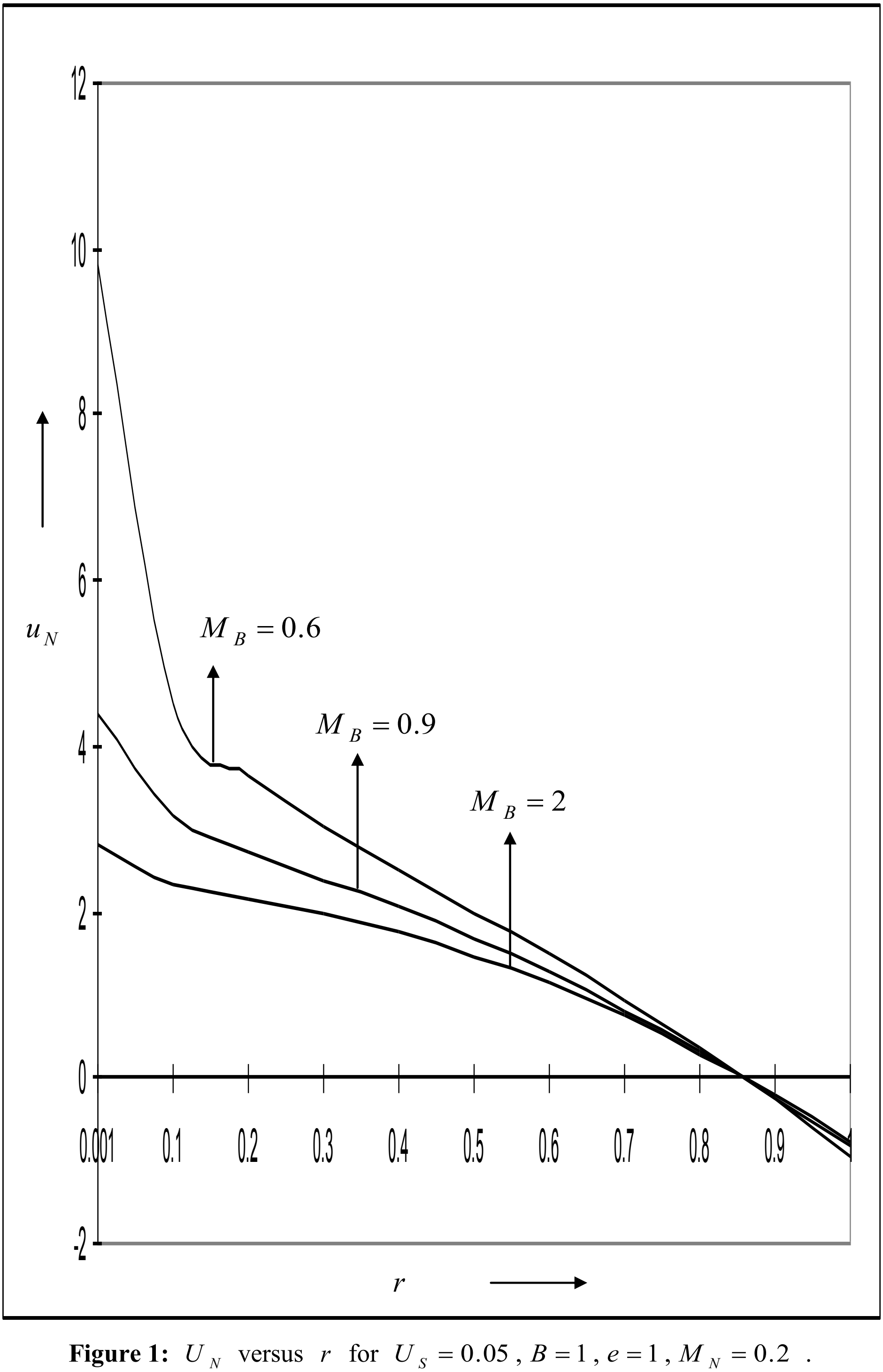


Figure 1: U_N versus r for $U_S = 0.05$, $B = 1$, $e = 1$, $M_N = 0.2$.

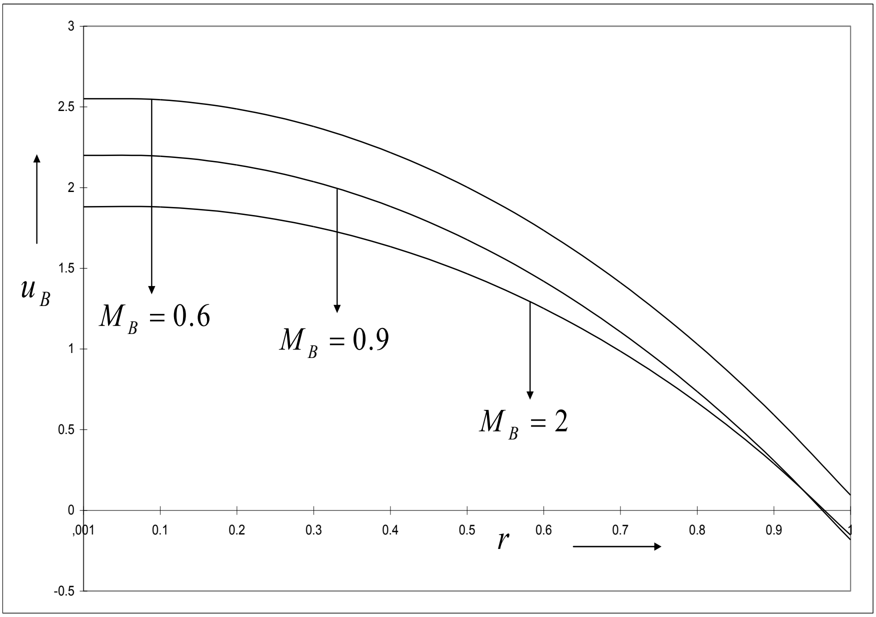


Figure 2: U_B versus r for $U_S = 0.05$, $B = 1$, $e = 1$, $M_N = 0.2$.

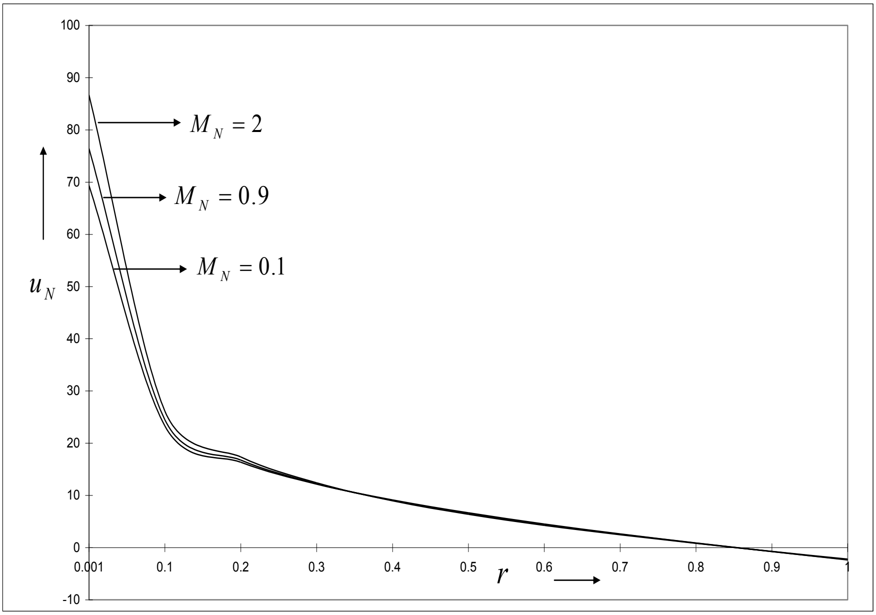


Figure 3: U_N versus r for $U_S = 0.05$, $B = 1$, $e = 1$, $M_B = 0.2$.

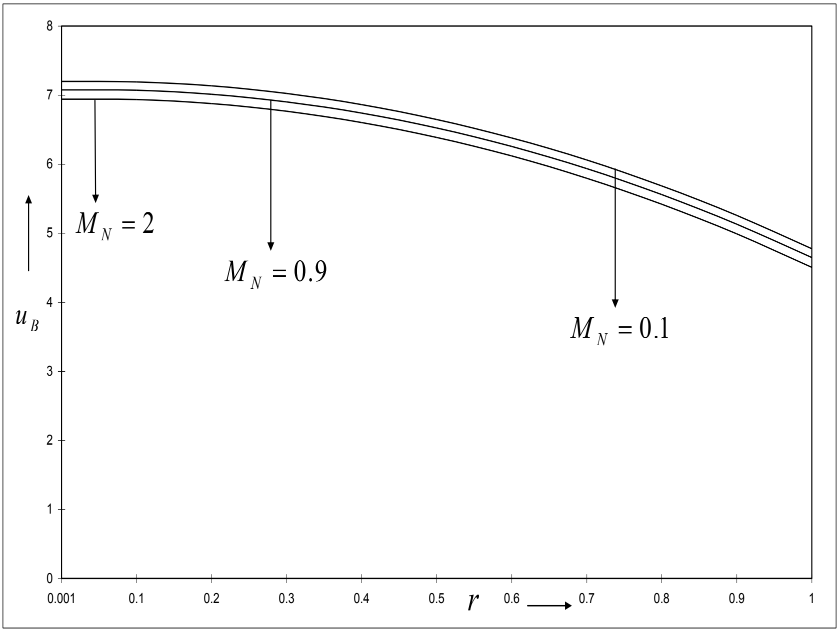


Figure 4: U_B versus r for $U_S = 0.05$, $B = 1$, $e = 1$, $M_B = 0.2$.

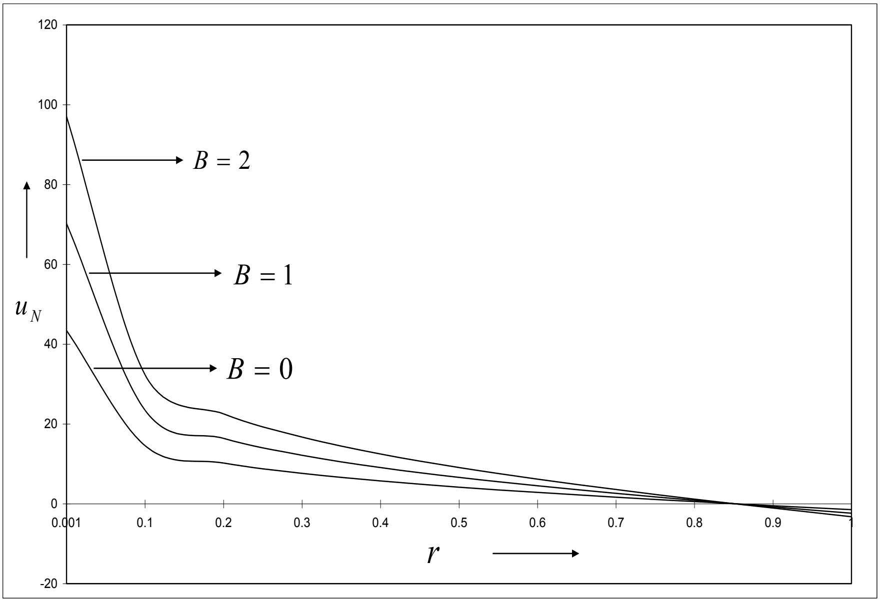


Figure 5: U_N versus r for $U_S = 0.05$, $M_N = 0.2$, $e = 1$, $M_B = 0.2$.

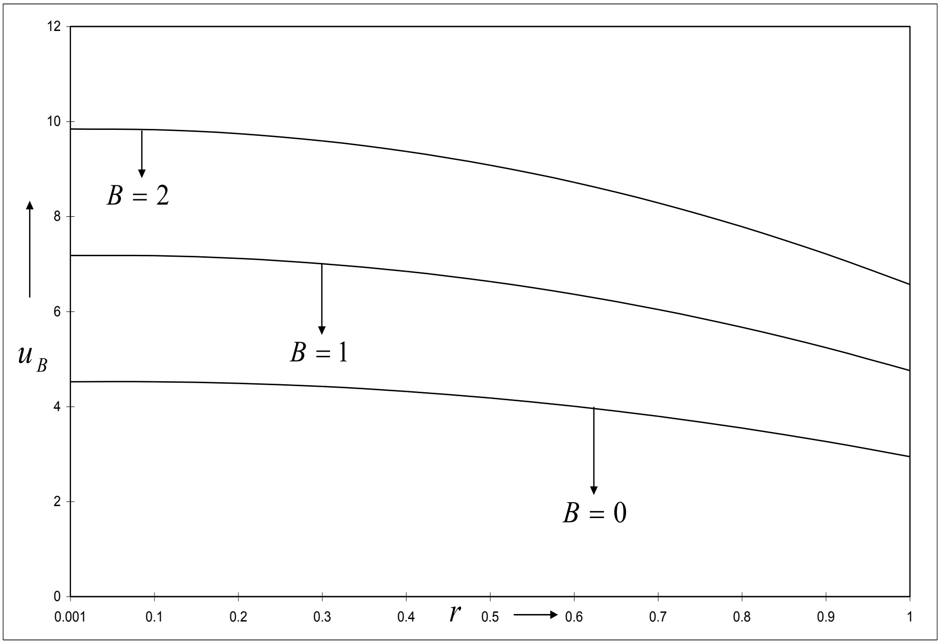


Figure 6: U_B versus r for $U_S = 0.05$, $M_N = 0.2$, $e = 1$, $M_B = 0.2$.

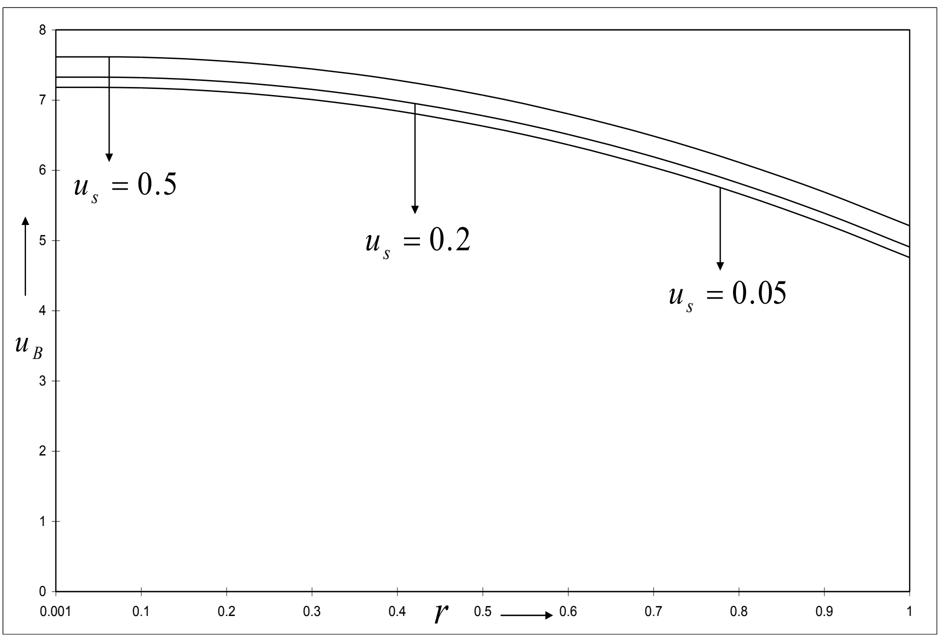


Figure 7: U_B versus r for $B = 1$, $M_N = 0.2$, $e = 1$, $M_B = 0.2$.

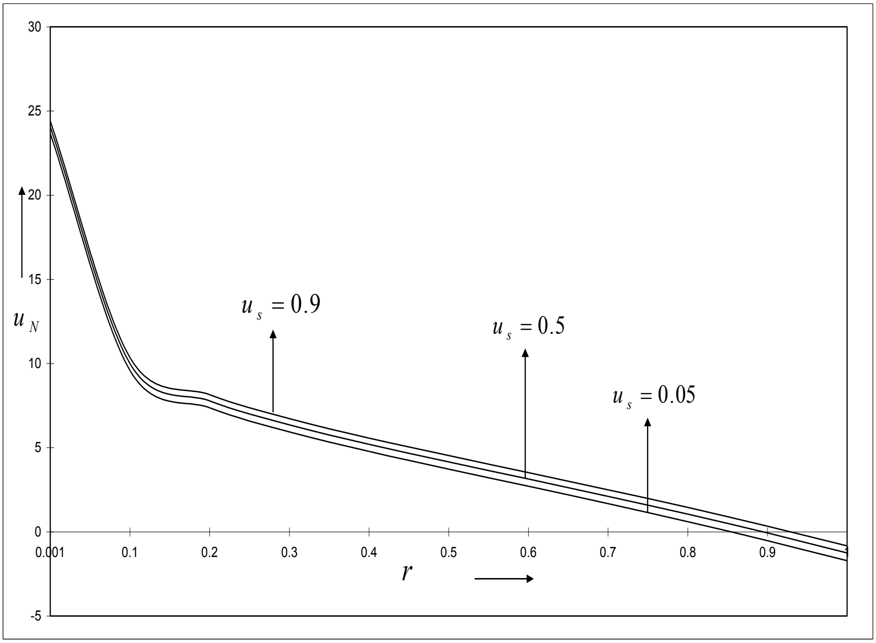


Figure 8: U_N versus r for $B = 3$, $M_N = 0.5$, $e = 1$, $M_B = 0.5$.

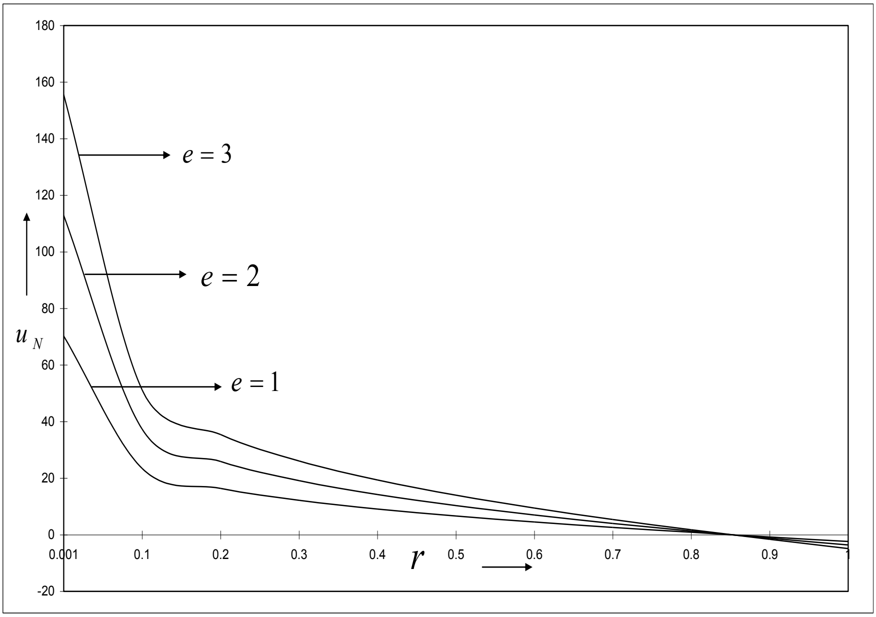


Figure 9: U_N versus r for $B = 1$, $M_N = 0.2$, $U_S = 0.05$, $M_B = 0.2$.

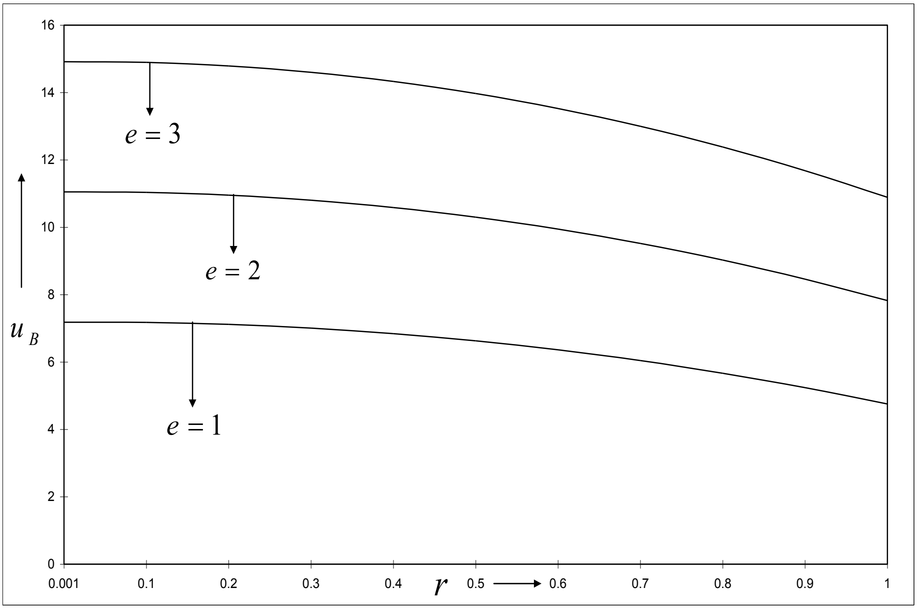


Figure 10: U_B versus r for $B = 1$, $M_N = 0.2$, $U_S = 0.05$, $M_B = 0.2$.

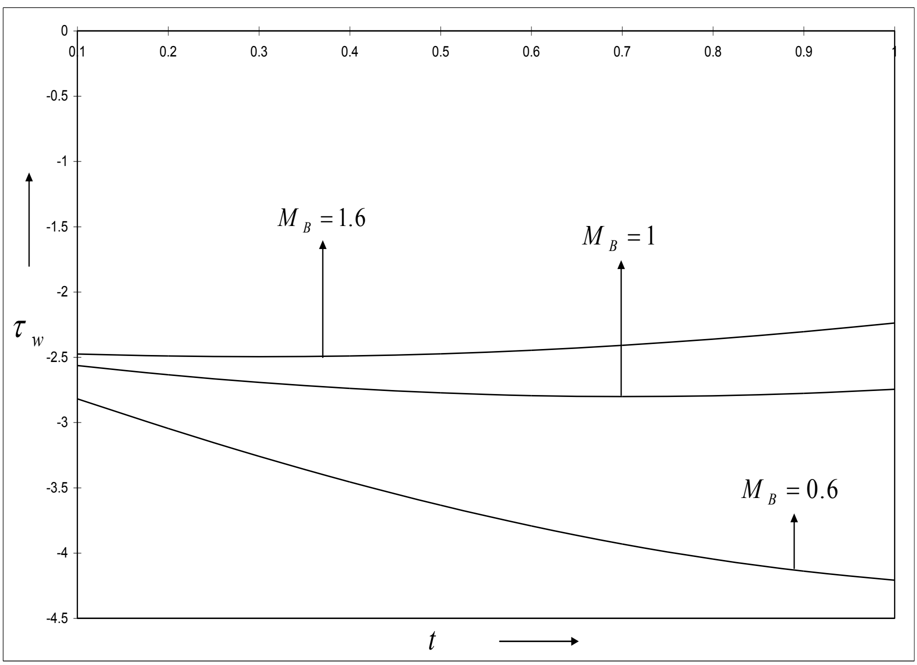


Figure 11: τ_w versus t for $B = 1$, $M_N = 0.5$, $U_S = 0.05$, $e = 1$.

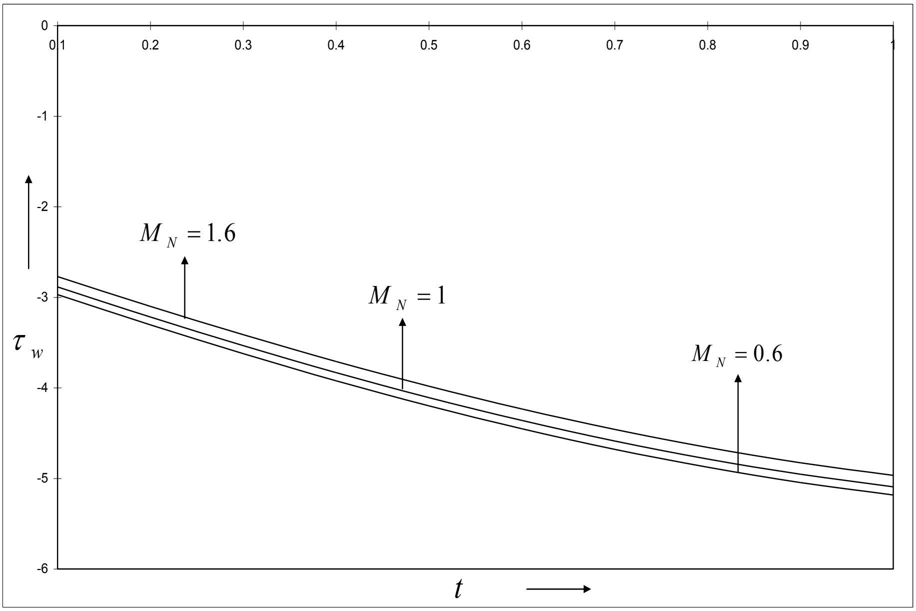


Figure 12: τ_w versus t for $B = 1$, $M_B = 0.5$, $U_S = 0.05$, $e = 1$.

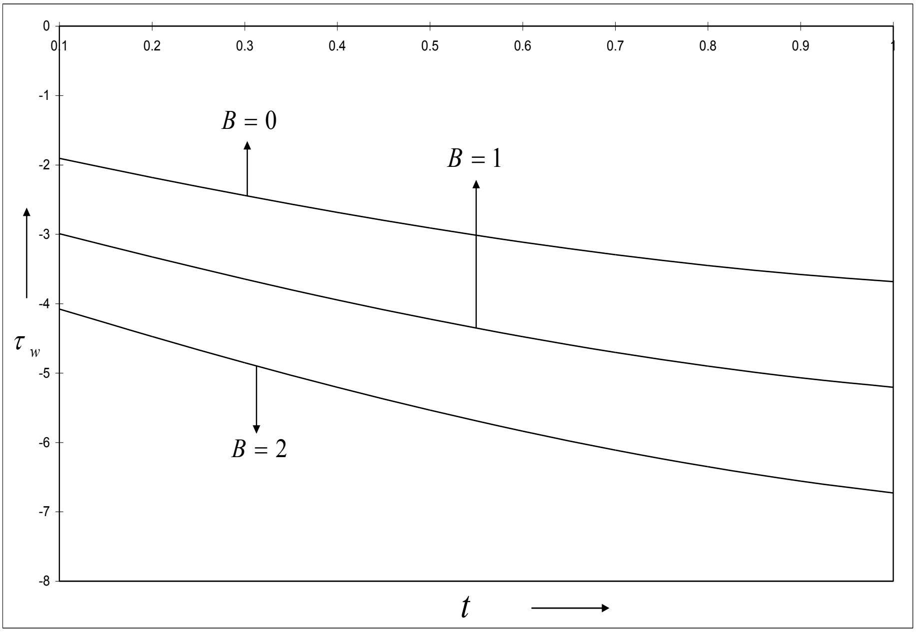


Figure 13: τ_w versus t for $M_N = 0.5$, $M_B = 0.5$, $U_S = 0.05$, $e = 1$.

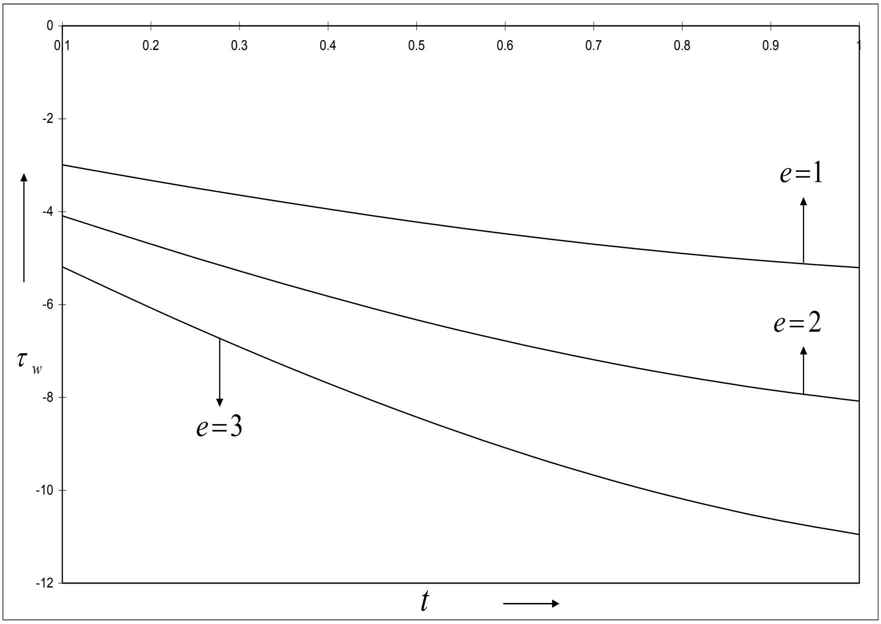


Figure 14: τ_w versus t for $M_N = 0.5$, $M_B = 0.5$, $U_S = 0.05$, $B = 1$.

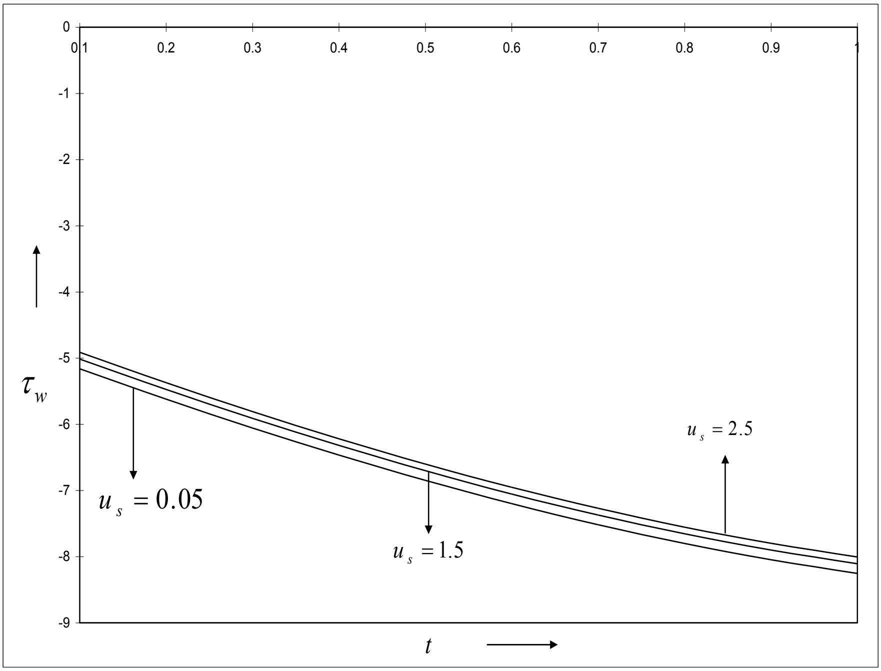


Figure 15: τ_w versus t for $M_N = 0.5$, $M_B = 0.5$, $e = 1$, $B = 3$.

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