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PROPERTIES ON INTUITIONISTIC FUZZY JOIN SEMI L-QUOTIENT FILTER

Abstract: *This paper contains the definition of Intuitionistic fuzzy relation, Intuitionistic fuzzy equivalence relation, Intuitionistic fuzzy join semi L-coset filter, Intuitionistic fuzzy join semi L-quotient filter and some properties are established.*

Keywords: *Intuitionistic fuzzy relation, Intuitionistic fuzzy equivalence relation, Intuitionistic fuzzy join semi L-coset filter, Intuitionistic fuzzy join semi L-quotient filter*

INTRODUCTION

L. A. Zadeh in 1965 introduced the notion of fuzzy set to describe vagueness mathematically in its very abstractness and tried to solve such problems by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. Intuitionistic fuzzy sets were introduced in 1983 by K.T. Atanassov[6]. P.Burillo and H. Bustince [20,21] have introduced the intuitionistic fuzzy relations and their properties. D. K. Basnet and N.K. Sarma[7] discuss the Intuitionistic fuzzy equivalence relation. K.V.Thomas and Latha S. Nair[2], discuss the Intuitionistic fuzzy equivalences and congruences of a Lattice . In this paper, we discuss some properties of intuitionistic fuzzy join semi L-quotient filter.

Definition : An Intuitionistic fuzzy relation(IFR) R on L is called (s, t) reflexive in $\mu(x, x) = s$ and $\gamma(x, x) = t$ for all $x \in L$.

Definition : An intuitionistic fuzzy (s, t) reflexive relation is called (s, t) equivalence relation (IFER) if R is Symmetric ie)$R^{-1} = R$ and R is transitive (ie) $RoR \subseteq R$.

Definition : $A(s, t)$ equivalence Relation R on L is called (s, t) congruence if for all $a, b, c, d \in L$

$\mu(a\vee c, b\vee d) \geq \min\{\mu(a, b), \mu(c, d)\}$

$\gamma (a\vee c, b\vee d) \leq \max \{\gamma (a. b), \gamma (c, d)\}$

Note : If R is an intuitionistic fuzzy Equivalence relation (IFER) then $\mu(x, x) \geq \sup_{x,y\in L} \mu(x, y)$ and $\gamma (x, x) \leq \inf_{x,y\in L} \gamma(x, y)$

Theorem

Let $A, B \in$ IFR of L with $\sup_{x,y\in L}\mu_A (x, y)? = s_1$, $\sup_{x,y\in L} \mu_B (x, y) = s_2$ and $\inf_{x,y\in L}\gamma_A (x, y) = t_1$, $\inf_{x,y\in L}\gamma_B (x, y) = t_2$. Then $A\subseteq AoB \Rightarrow s_1 \leq s_2$ and $t_1 \geq t_2$.

Proof

Suppose $s_1 \nleq s_2$ (or) $t_1 \ngeq t_2$. then

$\sup_{x,y\in L}\mu_B (x, y) < \sup_{x,y\in L}\mu_A (x, y)$ (or)

$\inf_{x,y\in L} \gamma_A (x, y) < \inf_{x,y\in L}\gamma_B (x, y)$

If $\sup_{x,y\in L}\mu_B (x, y) < \sup_{x,y\in L}\mu_A (x, y)$

Then $\sup_{x,y\in L} \mu_B (x, y) < \sup_{x,y\in L} \mu_A (x_0, y_0)$ for some $x_0, y_0 \in L$

$$
\begin{aligned}
\mu_{\mathrm{AoB}} (x_0, y_0) &= \sup_{z\in L} \{\min\{\mu_A (x_0, z), \mu_B (z, y_0)\}\} \\
&\leq \sup_{z\in L}\mu_B (z, y_0) \\
&< \sup_{z,y\in L} \mu_B (z, y) \\
&< \mu_A (x_0, y_0) \\
\mu_{\mathrm{AoB}} (x_0, y_0) &< \mu_A(x_0, y_0)
\end{aligned}
$$

This is a contradiction to $A\subseteq AoB$

Now, $\inf_{x,y\in L}\gamma_A (x, y) < \inf_{x,y\in L}\gamma_B (x, y)$

$\Rightarrow \gamma_A(x_1, y_1) < \inf_{x,y\in L} \gamma_B (x, y)$ for some $x_1, y_1 \in L$

$$
\begin{aligned}
\gamma_{AoB}(x_1, y_1) &= \inf_{z\in L} \{\max \{\gamma_A (x_1, z), \gamma_B (z, y_1)\}\} \\
&\geq \inf_{z\in L} \gamma_B (z, y_1) \\
&\geq \inf_{z,y\in L} \gamma_B (z, y) \\
&> \gamma_B(x_1, y_1) \\
\gamma_{\mathrm{AoB}}(x_1, y_1) &\geq \gamma_B(x_1, y_1)
\end{aligned}
$$

This is a contradiction to $A \subseteq$ AoB

Hence $s_1 \leq s_2$ & $t_1 \geq t_2$.

Note

Let A, $B \in$ IFR of L with $\sup_{x,y\in L} \mu_A(x, y) = s_1$, $\sup_{x,y\in L} \mu_B(x, y) = s_2$ and $\inf_{x,y\in L} \gamma_A(x, y) = t_1$, $\inf_{x,y\in L} \gamma_B(x, y) = t_2$. Then A, $B \subseteq$ AoB => $s_1 = s_2$ and $t_1 = t_2$.

Definition

Let $s, t \in [0,1]$ with $s + t \leq 1$. Then the sub collection $\mathrm{Pr}_{(s,t)}$ of IFS of L is called a (s,t) - partition of L, if the following are satisfied.

(i) For each $A \in Pr_{(s,t)}$, $\mu_A(x) = s$, $\gamma_A(x) = t$ for at least one $x \in L$.

(ii) For each $x \in L$, there exist only one $A \in Pr_{(s,t)}$ satisfying $\mu_A(x) = s$, $\gamma_A(x) = t$.

(iii) If A, $B \in Pr_{(s,t)}$ such that $\mu_A(x) = \mu_B(y) = s$,

$\gamma_A(x) = \gamma_B(y) = t$, for all $x, y \in L$,

then $\mu_A(y) = \mu_B(x)$ $= \sup_{z\in L}\{\min\{\mu_A(z), \mu_B(z)\}\}$ and $\inf_{z\in L}\{\max\{\gamma_A(z), \gamma_B(z,)\}\}$

$= \gamma_B(x)$

$= \gamma_A(y)$

Let $\mathrm{Pr}_{(s,t)}$ be a (s,t) - partition of L and $x \in L$. Then the unique member of $\mathrm{Pr}_{(s,t)}$ which takes the value (s, t) at x is denoted by $[x]_{pr}$

Definition

For a (s, t) equivalence relation R on L, we call the set $\mathrm{Pr}_{(s,t)} = \{ [x]_R / x \in L\}$ a quotient set with respect to R and denoted by L/R. The members of L/R are called (s,t) equivalence classes of L.

Definition

Let A be an intuitionistic fuzzy join semi lattice (IFJSL) and $<\mu, \gamma>$ be any intuitionistic fuzzy join semi L-filter (IFJSLF) of A. Then the intuitionistic fuzzy join semi L- filter (IFJSLF) $< \mu_y^*, \gamma_y^*>$ of A, where $y \in A$ defined by $\mu_y^*(z) = \mu(z \vee y)$,

$\gamma_y^*(z) = \gamma(z \vee y)$ for all $z \in A$ is termed as the intuitionistic fuzzy join semi L- coset filter (IFJSLCF) determined by y.

Theorem

Let $<\mu, \gamma>$ be any IFJSLF of a IFJSL A. Let $<\mu_y^*, \gamma_y^*>$ for all $y \in A$ be an IFJSLCF of $<\mu, \gamma>$ of A. Then $<\mu_y^*, \gamma_y^*>$ is also a IFJSLF of A.

Proof

Let $<\mu, \gamma>$ be any IFJSLF of a IFJSL A.

For all $x, z \in A$,

$$\begin{aligned}
\mu_y^*(x \vee z) &= \mu((x \vee z) \vee y) \\
&= \mu((x \vee y) \vee (z \vee y)) \\
&\leq \max\{\mu(x \vee y), \mu(z \vee y)\} \\
&= \max\{\mu_x^*, \mu_z^*\} \\
\mu_y^*(x \vee z) &\leq \max\{\mu_x^*, \mu_z^*)
\end{aligned}$$

similarly

$$\begin{aligned}
\gamma_y^*(x \vee z) &= \gamma(x \vee z) \vee y \\
&= \gamma((x \vee y) \vee (z \vee y)) \\
&= \min\{\gamma(x \vee z), \gamma(z \vee y)\} \\
&= \min\{\gamma_x^*, \gamma_z^*\},
\end{aligned}$$

Hence IFJSLCF is an IFJSLF of A.

Definition

Let $<\mu, \gamma>$ be any IFJSLF of a IFJSL A. Then intuitionistic fuzzy join semi L- quotient filter (IFJSLQF) $<\mu^*, \gamma^*>$ of $<A_\mu, A_\gamma> = <A/\mu_t, A/\gamma_t>$ is defined by $\mu^*(x \vee \mu_t) = \mu(x)$ for all $x \in A$ and $\gamma^*(x \vee \gamma_t) = \gamma(x)$ for all $x \in A$ where $\mu_t = \{x/\mu(1) = \mu(x) = t\}$ and $\gamma_t = \{x/\gamma(0) = \gamma(x) = t\}$.

Theorem

It $<\mu, \gamma>$ be an IFJSLF of a IFJSL A. Then $\mu_y^* = \mu_1^*$ & $\gamma_y^* = \gamma_0^*$ if $\mu(y) = \mu(1)$ and $\gamma(y) = \gamma(0)$ for all $y \in A$.

Proof

Let $\mu(y) = \mu(1)$. => (1)

Then for all $x \in A$,

$\mu(x) \geq \mu(1)$ => (2)

From (1),(2) we get $\mu(y) \leq \mu(x)$.

Case (i)

If $\mu(y) < \mu(x)$.

Then $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

$= \mu(x)$

If $\mu(y) = \mu(x)$

Then $x, y \in \mu_t$ where $t = \mu(1)$.

$\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

$= \mu(y) = \mu(1)$

$\mu(x \vee y) = \mu(y) = \mu(1) = \mu(x)$.

Thus in either case

$\mu(x \vee y) = \mu(x)$ for all $x \in A$,

(ie) $\mu_y^*(x) = \mu(x) = \mu_1^*(x)$

Hence $\mu_y^* = \mu_1^*$

Also, let $\gamma(y) = \gamma(0)$ => (3),

Then for all $x \in A$,

$\gamma(x) \geq \gamma(0)$ => (4),

From (3)& (4) we get $\gamma(y) \leq \gamma(x)$

If $\gamma(y) < \gamma(x)$ then $\gamma(x \vee y) \geq \min\{\gamma(x), \gamma(y)\}$

$= \gamma(y)$

If $\gamma(y) = \gamma(x)$, then $x, y \in \mu_t$ where $t = \gamma(0)$

$\gamma(x \vee y) \geq \min\{\gamma(x), \gamma(y)\}$

$= \gamma(y) = \gamma(0)$

Hence $\gamma(x \vee y) = \gamma(y) = \gamma(0) = \gamma(x)$

Thus $\gamma(x \vee y) = \gamma(x)$ for all $y \in A$

(ie) $\gamma_{y}^{*}(x) = \gamma(x) = \gamma_{0}^{*}(x)$

$\gamma_{y}^{*} = \gamma_{0}^{*}$

Hence if $\gamma(y) = \gamma(0)$, $\mu(y) = \mu(1)$ then $\mu_{y}^{*} = \mu_{1}^{*}$ and $\gamma_{y}^{*} = \gamma_{0}^{*}$

Theorem

If $<\mu, \gamma>$ is an IFJSL of a IFJSL A, then $\mu(y) = \mu(1)$, $\gamma(y) = \gamma(0)$ iff $\mu_{y}^{*} = \mu_{1}^{*}$, $\gamma_{y}^{*} = \gamma_{0}^{*}$ for all $y \in A$,

Theorem

If $< \mu, \gamma >$ is an IFJSLF of a IFJSL A, then $A/\mu_{t} \cong A\mu$,where $t = \mu(1)$ and $A/\gamma_{t} \cong A\gamma$ where $t = \gamma(0)$.

Proof

Let A be an IFJSL and $<\mu, \gamma >$ be IFJSLF of A.

To prove that $A/\mu_{t} \cong A\mu$.

First we prove that g:A => Aμ is a map defined by $g(y) = \mu_{y}^{*}$ for all $y \in A$ is an into intuitionistic Fuzzy join semi L-filter homomorphism (IFJSLFM).

$\mu_{y \vee z}^{*}(x) = \mu((y \vee z) \vee x) \quad = \mu(y \vee x) \vee \mu(z \vee x)$

$= \mu_{y}^{*} \vee \mu_{z}^{*}$

Hence g is an IFJSLFH

Now $g(y) = \mu_{y}^{*}$ if f $\mu_{y}^{*} = \mu_{1}^{*}$

IFF $\mu(y) = \mu(1)$

This shows that kernel of g is equal to μ_{t}.

$A/\mu_{t} \cong A\mu$

Next we prove that g:A => A γ is a map defined by $g(y) = \gamma_{y}^{*}$ for all $y \in A$ is an IFJSLFH

Now $\gamma_{yvz}^{*}(x) = \gamma((y \vee z) \vee x)$

$= \gamma(y \vee x) \vee \gamma(z \vee x)$

$= \gamma_y^{*} \vee \gamma_z^{*}$

G is an IFJSLFH.

$g(y) = \gamma_y^{*}$ iff $\gamma_y^{*} = \gamma_0^{*}$

iff $\gamma(y) = \gamma(0)$ This shows that kernel of g is equal to γt.

$A/\gamma_y \cong A\gamma$

Theorem

If <μ, γ > is an IFJSLF of a IFJSL A, then the intuitionistic fuzzy join subset IFJSS μ^{*} of Aμ and γ * of $A\gamma$ by $\mu^{*}(y \vee \mu_t) = \mu(x)$ and $\gamma^{*}(yv\gamma_t) = \gamma(y)$ where $y \in A$ is a IFJSLF of < Aμ, A γ>

Proof

Let < μ, γ> be any IFJSLF of a IFJSLA.

T.P.T IFJSS μ^{*} of Aμ defined by $\mu^{*}(y \wedge \mu_t) = \mu(y)$

Where $y \varepsilon A$ is an IFJSLF.

Let $x, y \in A$. Then $\mu^{*}((x \vee \mu_t) \vee (y \vee/ \mu_t) = \mu^{*}(x \vee y \vee \mu_t)$

$= \mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

$= \max\{\mu^{*}(x \vee \mu_t), \mu^{*}(y \vee \mu_t)\}$

$\mu^{*}((x \vee \mu_t) \vee (y \vee \mu_t)) \leq \max\{\mu^{*}(x \vee \mu_t), \mu^{*}(y \vee \mu_t)\}$

$\gamma((x \vee \mu_t) \vee (y \vee \mu_t) = \gamma^{*}(x \vee y\,\mu_t) = \gamma(x \vee y)$

$\geq \min\{\gamma(x), \gamma(y)\}$

$= \min\{\,*(x \vee \gamma t), \gamma^{*}(y \vee \gamma_t)\}$

$\gamma * ((x \vee \mu_t) \vee (y \vee \mu_t)) \geq \min\{\gamma^{*}(x \vee \gamma_t), \gamma^{*}(y \vee \gamma_t)\}$

Hence < μ^{*}, γ * > is an IFJSLF of < Aμ, A^{γ}>

Theorem

Let < μ, γ> be any IfJSLF of a IFJSLA . Let <μ^*, γ^*> be any IFJSLF of the quotient IFJSL A/F, where F is any IFSS of A. Then Correspondence to < μ^*, γ^*> in A/F, there exist a IFJSLF of A.

Proof

Let < μ^*, γ^*> be any IFJSLF of A/F. Define IFSS < θ, θ * >of A by $\theta(y) = \mu^*(y \vee F)$ and $\theta^*(y) = \gamma^*(y \vee F)$ for all $Y \in A$.

$$\begin{aligned} \theta(y \vee z) &= \mu^* (y \vee z \vee F) \\ &= \mu^* (y \vee F) \text{ v } (z \vee F) \\ &\leq \max\{ \mu^* (y \vee F), \mu^* (Z \vee F)\} \\ &= \max \{\theta(y), \theta(z)\} \\ \theta(y \vee z) &\leq \max\{\theta(y), \theta(z)\} \end{aligned}$$

$$\begin{aligned} \text{Also } \theta *(y \vee z) &= \gamma^* (y \vee z) \vee F \\ &= \gamma^* (y \vee F) \vee (z \vee F) \\ &\geq \min \{ \gamma^* (y \vee F), \gamma^* (z \vee F)\} \end{aligned}$$

Hence $\theta^*(y \vee z) \geq \min \{ \theta^*(y), \theta^*(z)\}$

Hence < θ, θ * > is an *IFJSLF* of A.

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