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A NEW RANKING METHOD BASED ON ARITHMETIC MEAN OPERATION OF FUZZY NUMBERS

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Abstract

In this paper, we provide a new method for the ranking of Trapezoidal Fuzzy Numbers (TFNs) based on the arithmetic mean operation. Some results are also discussed in terms of expected interval, expected value, value, ambiguity and width of fuzzy numbers. We also check the convexity of the newly formed TFN.

Keywords: *Ranking of fuzzy numbers, Trapezoidal fuzzy numbers, Arithmetic mean.*

1. INTRODUCTION

Most of the real life problems are complex in nature because of the indistinctness and impreciseness of the available data. In 1970 Bellman and Zadeh proposed the concept of fuzzy sets and fuzzy models to effectively handle these imprecise data. To solve such real world problems, we can develop fuzzy expert systems by seeking the help of experts who have knowledge in that particular area. There may be many factors that influence a certain problem. While developing expert system, one has to rank these factors based on the experts' judgements. Usually experts' opinion is obtained as linguistic variables which can easily be converted into fuzzy numbers. For arriving at conclusions, we need to compile the experts' judgements, which in turn need better ranking methods of fuzzy numbers.

Ranking of fuzzy numbers plays a very significant role in linguistic multi-criteria decision making problems. Several fuzzy ranking methods have been proposed since 1976. The linguistic terms are represented quantitatively using fuzzy sets and then fuzzy optimal alternative is calculated which gives the relative merit of each alternative. Bass and H. Kwakernaak [4] proposed a method in 1977

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consisting of computing weighted final ratings for each alternative and comparing the final weighted rating. Final evaluation of the alternative consists of degree of membership in the fuzzy set. Dubois and Prade [7] in 1978 extended the normal algebraic operations to fuzzy numbers by the use of a fuzzification principle for ordering the fuzzy numbers. Shan-Huo Chen [5] in 1985 introduced the concept of maximizing and minimizing set to find the ordering value of each fuzzy number and determined the order of $'n'$ fuzzy numbers. Adamo 1980 [1] introduced the concept of $\alpha -$ level set for ordering of fuzzy numbers. Tian-shy Liou and Mao-Jiun J Wang [9] in 1992 proposed a method with integral values and this method is independent of the type of membership functions used and normality of the functions. Philippe Fortemps and Mare Roubens [8] in 1996 introduced a procedure based on area compensation to compare fuzzy numbers and to induce a ranking of fuzzy numbers. Chung-Tsen Tsao 2002 [6] proposed a ranking method with the area between the centroid point and original point.

The Arithmetic mean operation of TFNs is introduced here, which will be more advantageous in ranking procedure, as this method is easier to compile the variety of experts' judgements. This paper is organized as follows: Section 2 presents the preliminaries; Section 3 depicts the method of finding Arithmetic Mean of TFNs with illustrations and Section 4 concludes the work.

2. PRELIMINARIES

Definition 1: A fuzzy set A in a universe of discourse X is defined as the set of pairs, $A = \{(x, \mu_A(x)): x \in X\}$, where $\mu_A(x): X \to [0,1]$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition 2: The set $A(\alpha)$ is a convex set if $x, y \in A(\alpha) \Rightarrow \lambda x + (1 - \lambda) y \in A(\alpha)$ where $\lambda \in [0,1]$.

Definition 3: For a fuzzy set A of X, the support of A, denoted by $supp(A)$ is the crisp subset of X which contains elements having nonzero membership grades in A.

i.e., $$supp(A) = \{x \in X: \mu_A(x) > 0\}$$

Definition 4 [2]: A fuzzy number A is a fuzzy subset of the real line; $A: R \to [0,1]$ satisfying the following properties:

(i) A is normal (i.e. there exists $x_0 \in R$ such that $A(x_0) = 1$);

(ii) A is fuzzy convex ;

(iii) A is upper semi continuous on $R. ie;$ $\forall \varepsilon > 0, \exists\, \delta > 0$ such that

$A(x) - A(x_0) < \varepsilon$ whenever $|x - x_0| < \delta$;

(v) The closure , $cl(supp(A))$ is compact.

Definition 5 [2]: The α-cut, $\alpha \in (0,1]$ of a fuzzy number A is a crisp set defined as $A(\alpha) = \{x\epsilon R: A(x) \geq \alpha\}$. Every A_α is a closed interval of the form $[A_L(\alpha), A_U(\alpha)]$.

Definition 6 [3]: A Trapezoidal fuzzy number denoted by A is defined as (l, m, n, u) where the membership function is given by

$$\mu_A(x) = \begin{cases} 0, x \leq l \\ \frac{x-l}{m-l}, l \leq x \leq m \\ 1, m \leq x \leq n \\ \frac{u-x}{u-n}, n \leq x \leq u \\ 0, x \geq u \end{cases}$$

Definition 7 [2]: The expected interval $EI(A)$, expected value $EV(A)$, ambiguity $amb(A)$, value $val(A)$, width $width(A)$, left-hand ambiguity $amb_L(A)$, right-hand ambiguity $amb_U(A)$ of a fuzzy number A are denoted and defined as the follows:

$$EI(A) = [l_e(A), u_e(A)] = \left[\int_0^1 A_L(\alpha)d\alpha \, , \int_0^1 A_U(\alpha)d\alpha\right]$$

$$EV(A) = \frac{1}{2}[l_e(A) + u_e(A)]$$

$$amb(A) = \int_0^1 \alpha\big(A_U(\alpha) - A_L(\alpha)\big)\, d\alpha$$

$$val(A) = \int_0^1 \alpha\big(A_U(\alpha) + A_L(\alpha)\big)\, d\alpha$$

$$width(A) = \int_0^1 \big(A_U(\alpha) - A_L(\alpha)\big)\, d\alpha$$

$$amb_L(A) = \int_0^1 \alpha\big(EV(A) - A_L(\alpha)\big)\, d\alpha$$

$$amb_U(A) = \int_0^1 \alpha(A_U(\alpha) - EV(A))d\alpha$$

3. ARITHMETIC MEAN OPERATION OF TRAPEZOIDAL FUZZY NUMBERS

Consider the Trapezoidal Fuzzy Numbers:

$A_1 = (a_1, a_2, a_3, a_4), A_2 = (b_1, b_2, b_3, b_4), \ldots, A_n = (n_1, n_2, n_3, n_4)$ with membership functions,

$$\mu_{A_1}(x_1) = \begin{cases} 0\ ,\ x_1 \le a_1 \\ \frac{x_1-a_1}{a_2-a_1},\ a_1 \le x_1 \le a_2 \\ 1\ ,\ a_2\ \le x_1\ \le a_3 \\ \frac{a_4-x_1}{a_4-a_3},\ a_3 \le x_1 \le a_4 \\ 0\ ,\ x_1 \ge a_4 \end{cases}$$

$$\mu_{A_2}(x_2) = \begin{cases} 0\ ,\ x_2 \le b_1 \\ \frac{x_2-b_1}{b_2-b_1},\ b_1 \le x_2 \le b_2 \\ 1\ ,\ b_2\ \le x_2\ \le b_3 \\ \frac{b_4-x_2}{b_4-b_3},\ b_3 \le x_2 \le b_4 \\ 0\ ,\ x_2 \ge b_4 \end{cases}$$

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$$\mu_{A_n}(x_n) = \begin{cases} 0\ ,\ x_n \le n_1 \\ \frac{x_n-n_1}{n_2-n_1},\ n_1 \le x_n \le n_2 \\ 1\ ,\ n_2\ \le x_n\ \le n_3 \\ \frac{n_4-x_n}{n_4-n_3},\ n_3 \le x_n \le n_4 \\ 0\ ,\ x_n \ge n_4 \end{cases}$$

Or,

$$\mu_{A_1}(x_1) = max\left[min\left(\frac{x_1-a_1}{a_2-a_1}, 1, \frac{a_4-x_1}{a_4-a_3}\right), 0\right]$$

$$\mu_{A_2}(x_2) = max\left[min\left(\frac{x_2-b_1}{b_2-b_1}, 1, \frac{b_4-x_2}{b_4-b_3}\right), 0\right]$$

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$$\mu_{A_n}(x_n) = max\left[min\left(\frac{x_n-n_1}{n_2-n_1}, 1, \frac{n_4-x_n}{n_4-n_3}\right), 0\right]$$

α- cuts of these fuzzy numbers are given by:

$$A_1(\alpha) = [A_{1L}(\alpha), A_{1U}(\alpha)] = [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$$

$$A_2(\alpha) = [A_{2L}(\alpha), A_{2U}(\alpha)] = [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]$$

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$$A_n(\alpha) = [A_{nL}(\alpha), A_{nU}(\alpha)] = [n_1 + \alpha(n_2 - n_1), n_4 - \alpha(n_4 - n_3)]$$

Then we define the Arithmetic Mean of these fuzzy numbers as follows:

Let $A_V = \frac{A_1 + A_2 + \cdots + A_n}{n} = \left(\frac{a_1+b_1+\cdots+n_1}{n}, \frac{a_2+b_2+\cdots+n_2}{n}, \frac{a_3+b_3+\cdots+n_3}{n}, \frac{a_4+b_4+\cdots+n_4}{n}\right)$

with membership function,

$$\mu_{A_V}(X) = \begin{cases} sup\left[min\left(\frac{x_1-a_1}{a_2-a_1}, \frac{x_2-b_1}{b_2-b_1}, \ldots, \frac{x_n-n_1}{n_2-n_1}\right): \frac{x_1+x_2+\cdots+x_n}{n} = X\right] \\ if\ a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2, \ldots, n_1 \le x_n \le n_2 \\ 1\ if\ a_2 \le x_1 \le a_3, b_2 \le x_2 \le b_3, \ldots, n_2 \le x_n \le n_3 \\ sup\left[min\left(\frac{a_4-x_1}{a_4-a_3}, \frac{b_4-x_2}{b_4-b_3}, \ldots, \frac{n_4-x_n}{n_4-n_3},\right): \frac{x_1+x_2+\cdots+x_n}{n} = X\right] \\ if\ a_3 \le x_1 \le a_4, b_3 \le x_2 \le b_4, \ldots, n_3 \le x_n \le n_4 \end{cases}$$

That is,

$$\mu_{A_V}(X) = \begin{cases} \frac{X-\left(\frac{a_1+b_1+\cdots+n_1}{n}\right)}{\left(\frac{a_2+b_2+\cdots+n_2}{n}\right)-\left(\frac{a_1+b_1+\cdots+n_1}{n}\right)}\ if\ \left(\frac{a_1+b_1+\cdots+n_1}{n}\right) \le X \le \left(\frac{a_2+b_2+\cdots+n_2}{n}\right) \\ 1\ if\ \left(\frac{a_2+b_2+\cdots+n_2}{n}\right) \le X \le \left(\frac{a_3+b_3+\cdots+n_3}{n}\right) \\ \frac{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right)-X}{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right)-\left(\frac{a_3+b_3+\cdots+n_3}{n}\right)}\ if\ \left(\frac{a_3+b_3+\cdots+n_3}{n}\right) \le X \le \left(\frac{a_4+b_4+\cdots+n_4}{n}\right) \\ 0\ otherwise \end{cases}$$

The α- cut of A_V is:

$$A_V(\alpha) = [A_{VL}(\alpha), A_{VU}(\alpha)]$$

$$= \begin{bmatrix} \left\{\left(\frac{a_1+b_1+\cdots+n_1}{n}\right) + \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right\}, \\ \left\{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right)\right\} \end{bmatrix}$$

Result 1:

1. $EI(A_V) = \left[\frac{l_e(A_1)+l_e(A_2)+\cdots+l_e(A_n)}{n}, \frac{u_e(A_1)+u_e(A_2)+\cdots+u_e(A_n)}{n}\right]$
2. $EV(A_V) = \frac{1}{n}[EV(A_1) + EV(A_2) + \ldots + EV(A_n)]$
3. $val(A_V) = \frac{val(A_1)+val(A_2)+\cdots+val(A_n)}{n}$
4. $amb(A_V) = \frac{amb(A_1)+amb(A_2)+\cdots+amb(A_n)}{n}$
5. $width(A_V) = \frac{width(A_1)+width(A_2)+\cdots+width(A_n)}{n}$

Proof: The Arithmetic mean of n fuzzy numbers $A_1, A_2, \ldots, A_n$ is defined by $A_V = \frac{A_1 + A_2 + \cdots + A_n}{n}$

1. The Expected Interval for the fuzzy number A_1 is $EI(A_1) = [l_e(A_1), u_e(A_1)]$ where

$$l_e(A_1) = \int_0^1 [a_1 + \alpha(a_2 - a_1)]\, d\alpha = \frac{a_1+a_2}{2}$$

And $$u_e(A_1) = \int_0^1 [a_4 - \alpha(a_4 - a_3)]\, d\alpha = \frac{a_4+a_3}{2}$$

$\therefore$ $$EI(A_1) = \left[\frac{a_1+a_2}{2}, \frac{a_4+a_3}{2}\right]$$

Similarly $EI(A_2) = \left[\frac{b_1+b_2}{2}, \frac{b_4+b_3}{2}\right], \ldots, EI(A_n) = \left[\frac{n_1+n_2}{2}, \frac{n_4+n_3}{2}\right]$

The Expected interval $EI(A_V) = [l_e(A_V),\ u_e(A_V)]$ where

$$l_e(A_V) = \int_0^1 \left[\left(\frac{a_1+b_1+\cdots+n_1}{n}\right) + \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right] d\alpha$$

$$= \frac{1}{2n}[a_1 + b_1 + \cdots + n_1 + a_2 + b_2 + \cdots + n_2] \text{and}$$

$$u_e(A_V) = \int_0^1 \left[\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right)\right] d\alpha$$

$$= \frac{1}{2n}[a_3 + b_3 + \cdots + n_3 + a_4 + b_4 + \cdots + n_4]$$

$$\therefore\ EI(A_V) = \left[\frac{1}{2n}[a_1 + \cdots + n_1 + a_2 + \cdots + n_2], \frac{1}{2n}[a_3 + \cdots + n_3 + a_4 + \cdots + n_4]\right]$$

$$= \left[\frac{l_e(A_1)+l_e(A_2)+\cdots+l_e(A_n)}{n}, \frac{u_e(A_1)+u_e(A_2)+\cdots+u_e(A_n)}{n}\right]$$

2. Expected value $EV(A_1) = \frac{1}{2}\left[\frac{a_1+a_2}{2} + \frac{a_4+a_3}{2}\right] = \frac{1}{4}[a_1 + a_2 + a_3 + a_4]$

Similarly $EV(A_2) = \frac{1}{4}[b_1 + b_2 + b_3 + b_4], \ldots, EV(A_n) = \frac{1}{4}[n_1 + n_2 + n_3 + n_4]$

$$\therefore\quad EV(A_V) = \frac{1}{2}\left\{\begin{bmatrix} \frac{1}{2n}[a_1 + \cdots + n_1 + a_2 + \cdots + n_2] \\ + \frac{1}{2n}[a_3 + \cdots + n_3 + a_4 + \cdots + n_4] \end{bmatrix}\right\}$$

$$= \frac{1}{n}[EV(A_1) + EV(A_2) + \ldots + EV(A_n)]$$

3. Value of the fuzzy number $A_1 = (a_1, a_2, a_3, a_4)$ is denoted and defined by

$$val(A_1) = \int_0^1 \alpha\, [A_{1U}(\alpha) + A_{1L}(\alpha)] d\alpha$$

$$= \int_0^1 \alpha\, [a_4 - \alpha(a_4 - a_3) + a_1 + \alpha(a_2 - a_1)] d\alpha$$

$$= \frac{a_1}{6} + \frac{a_2}{3} + \frac{a_3}{3} + \frac{a_4}{6}$$

Similarly $val(A_2) = \frac{b_1}{6} + \frac{b_2}{3} + \frac{b_3}{3} + \frac{b_4}{6}, \ldots, val(A_n) = \frac{n_1}{6} + \frac{n_2}{3} + \frac{n_3}{3} + \frac{n_4}{6}$

$$val(A_V) = \int_0^1 \alpha\,[A_{VU}(\alpha) + A_{VL}(\alpha)]d\alpha$$

$$= \int_0^1 \alpha\left[\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right) + \left(\frac{a_1+b_1+\cdots+n_1}{n}\right) + \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right] d\alpha$$

$$= \frac{\frac{a_1}{6}+\frac{a_2}{3}+\frac{a_3}{3}+\frac{a_4}{6}+\frac{b_1}{6}+\frac{b_2}{3}+\frac{b_3}{3}+\frac{b_4}{6}+\ldots+\frac{n_1}{6}+\frac{n_2}{3}+\frac{n_3}{3}+\frac{n_4}{6}}{n}$$

$$= \frac{val(A_1)+val(A_2)+\cdots+val(A_n)}{n}$$

4. Ambiguity of the fuzzy number $A_1 = (a_1, a_2, a_3, a_4)$ is given by

$$amb(A_1) = \int_0^1 \alpha\,[A_{1U}(\alpha) - A_{1L}(\alpha)]d\alpha$$

$$= \int_0^1 \alpha\,[a_4 - \alpha(a_4 - a_3) - a_1 - \alpha(a_2 - a_1)]d\alpha$$

$$= -\frac{a_1}{6} - \frac{a_2}{3} + \frac{a_3}{3} + \frac{a_4}{6}$$

Similarly $amb(A_2) = -\frac{b_1}{6} - \frac{b_2}{3} + \frac{b_3}{3} + \frac{b_4}{6}, \ldots, amb(A_n) = -\frac{n_1}{6} - \frac{n_2}{3} + \frac{n_3}{3} + \frac{n_4}{6}$

$$amb(A_V) = \int_0^1 \alpha\,[A_{VU}(\alpha) - A_{VL}(\alpha)]d\alpha$$

$$= \int_0^1 \alpha\left[\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right) - \left(\frac{a_1+b_1+\cdots+n_1}{n}\right) - \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right] d\alpha$$

$$= \frac{\frac{a_4}{6}+\frac{a_3}{3}-\frac{a_2}{3}-\frac{a_1}{6}+\frac{b_4}{6}+\frac{b_3}{3}-\frac{b_2}{3}-\frac{b_1}{6}+\ldots+\frac{n_4}{6}+\frac{n_3}{3}-\frac{n_2}{3}-\frac{n_1}{6}}{n}$$

$$= \frac{amb(A_1)+amb(A_2)+\cdots+amb(A_n)}{n}$$

5. Width of the fuzzy number $A_1 = (a_1, a_2, a_3, a_4)$ is given by

$$width(A_1) = \int_0^1 [A_{1U}(\alpha) - A_{1L}(\alpha)]\,d\alpha$$

$$= \int_0^1 [a_4 - \alpha(a_4 - a_3) - a_1 - \alpha(a_2 - a_1)]d\alpha$$

$$= \frac{a_4}{2} + \frac{a_3}{2} - \frac{a_2}{2} - \frac{a_1}{2}$$

Similarly $width(A_2) = \frac{b_4}{2} + \frac{b_3}{2} - \frac{b_2}{2} - \frac{b_1}{2}, \ldots, width(A_n) = \frac{n_4}{2} + \frac{n_3}{2} - \frac{n_2}{2} - \frac{n_1}{2}$

$$width(A_V) = \int_0^1 [A_{VU}(\alpha) - A_{VL}(\alpha)]\, d\alpha$$

$$= \int_0^1 \left[\left(\frac{a_4+b_4+\cdots+n_4}{n}\right) - \alpha\left(\frac{a_4+b_4+\cdots+n_4}{n} - \frac{a_3+b_3+\cdots+n_3}{n}\right) - \left(\frac{a_1+b_1+\cdots+n_1}{n}\right) - \alpha\left(\frac{a_2+b_2+\cdots+n_2}{n} - \frac{a_1+b_1+\cdots+n_1}{n}\right)\right] d\alpha$$

$$= \frac{\frac{a_4}{2}+\frac{a_3}{2}-\frac{a_2}{2}-\frac{a_1}{2}+\frac{b_4}{2}+\frac{b_3}{2}-\frac{b_2}{2}-\frac{b_1}{2}+\ldots+\frac{n_4}{2}+\frac{n_3}{2}-\frac{n_2}{2}-\frac{n_1}{2}}{n}$$

$$= \frac{width(A_1)+width(A_2)+\cdots+width(A_n)}{n}$$

Result 2: α- cut, $A_V(\alpha)$ is a convex set.

Proof: Since $A_1(\alpha), A_2(\alpha), \ldots, A_n(\alpha)$ are the $\alpha -$ level sets associated with the Trapezoidal fuzzy numbers, they are all convex sets.

Let $x_i, y_i \in A_i(\alpha)$, then $\lambda x_i + (1-\lambda) y_i \in A_i(\alpha);\ i = 1, \ldots, n$

Let $X, Y \in A_V(\alpha)$ where $\frac{x_1 + x_2 + \ldots + x_n}{n} = X$ & $\frac{y_1 + y_2 + \ldots + y_n}{n} = Y$.

We have

$$a_1 + \alpha(a_2 - a_1) < \lambda x_1 + (1-\lambda) y_1 < a_4 - \alpha(a_4 - a_3)$$

$$b_1 + \alpha(b_2 - b_1) < \lambda x_2 + (1-\lambda) y_2 < b_4 - \alpha(b_4 - b_3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$n_1 + \alpha(n_2 - n_1) < \lambda x_n + (1-\lambda) y_n < n_4 - \alpha(n_4 - n_3)$$

Combining all these equations;

$$(a_1+b_1 + \ldots + n_1) + \alpha\,(a_2+b_2 + \ldots + n_2 - a_1 - b_1 - \ldots - n_1)$$

$$< \lambda(x_1 + x_2 + \ldots + x_n) + (1-\lambda)(y_1 + y_2 + \ldots + y_n)$$

$$< (a_4+b_4 + \ldots + n_4) - \alpha(a_4+b_4 + \ldots + n_4 - a_3 - b_3 - \ldots - n_3)$$

$$\Rightarrow \frac{(a_1 + b_1 + \ldots + n_1) + \alpha\,(a_2 + b_2 + \ldots + n_2 - a_1 - b_1 - \ldots - n_1)}{n}$$

$$< \frac{\lambda(x_1 + x_2 + \ldots + x_n) + (1-\lambda)(y_1 + y_2 + \ldots + y_n)}{n}$$

$$< \frac{(a_4 + b_4 + \ldots + n_4) - \alpha(a_4 + b_4 + \ldots + n_4 - a_3 - b_3 - \ldots - n_3)}{n}$$

$$\Rightarrow \quad A_{VL}(\alpha) < \lambda X + (1-\lambda) Y < A_{VU}(\alpha)$$

$$\Rightarrow \quad \lambda X + (1-\lambda) Y \in A_V(\alpha)$$

$\Rightarrow A_V(\alpha)$ is a convex set.

Illustration: Consider a problem with n factors. Then comparison matrix (a_{ij}) can be formed where $i = 1,2,..,n$ & $j = 1,2,\ldots,n$ using some prescribed scaling.

For example:

	Linguistic variables	**Trapezoidal fuzzy scaling**
1.	Extremely insignificant (strongly inferior)	(0.00, 0.05, 0.15, 0.25)
2.	Insignificant (slightly inferior)	(0.15, 0.25, 0.35, 0.45)
3.	Equally significant (equal contribution)	(0.35, 0.45, 0.55, 0.65)
4.	Moderately significant (slightly favour)	(0.55, 0.65, 0.75, 0.85)
5.	Extremely significant (strongly favour)	(0.75, 0.85, 0.95, 1.00)

We construct the judgement matrix of the form:

$$A_k = (a_{ijk}) = \begin{bmatrix} l_{ij1} & m_{ij1} & n_{ij1} & u_{ij1} \\ l_{ij2} & m_{ij2} & n_{ij2} & u_{ij2} \\ \cdots & \cdots & \cdots & \cdots \\ l_{ijk} & m_{ijk} & n_{ijk} & u_{ijk} \end{bmatrix}$$

where $1 \le j \le n$, $1 \le i \le n$ and k is the number of experts.

Arithmetic mean operation is used for aggregation.

$$(l_{ij}, m_{ij}, n_{ij}, u_{ij}) = \left(\frac{l_{ij1} + \cdots + l_{ijk}}{k}, \frac{m_{ij1} + \cdots + m_{ijk}}{k}, \frac{n_{ij1} + \cdots + n_{ijk}}{k}, \frac{u_{ij1} + \cdots + u_{ijk}}{k}\right)$$

where $1 \le j \le n$, $1 \le i \le n$ and k is the number of experts.

Comparison between factors is done by finding the degree of possibility.

4. CONCLUSION

Arithmetic mean operation preserves linearity with respect to Expected interval, Expected value, Ambiguity, Value and Width. We can prove linearity with respect to left and right ambiguity also. This new method does not require much computational effort in the ranking procedure and it depends only on the experts' judgments. Hence the arithmetic mean of fuzzy numbers can be effectively and easily used for compiling data while developing fuzzy expert systems.

Reference

[1] Adamo, J. M. *"Fuzzy decision trees."*, Fuzzy sets and systems, 4.3, 207-219, Elsevier (1980).

[2] Ban, Adrian I., and Lucian Coroianu. *"Existence, uniqueness and continuity of trapezoidal approximations of fuzzy numbers under a general condition."* Fuzzy sets and Systems, 257, 3-22, Elsevier (2014).

[3] Banerjee, Sanhita, and Tapan Kumar Roy. *"Arithmetic operations on generalized trapezoidal fuzzy number and its applications."* Turkish Journal of Fuzzy Systems, 3.1, 16-44, tjfs-journal.org, (2012).

[4] Bass, Sjoerd M., and Huibert Kwakernaak. *"Rating and ranking of multiple-aspect alternatives using fuzzy sets"*, Automatica, 13.1, 47-58, Elsevier (1977).

[5] Chen, Shan-Huo. *"Ranking fuzzy numbers with maximizing set and minimizing set."* Fuzzy sets and Systems, 17.2, 113-129, Elsevier (1985).

[6] Chu, Ta-Chung, and Chung-Tsen Tsao. *"Ranking fuzzy numbers with an area between the centroid point and original point",* Computers & Mathematics with Applications, 43.1, 111-117, Elsevier (2002).

[7] Dubois, Didier, and Henri Prade. "*Operations on fuzzy numbers"*, International Journal of systems science, 9.6, 613-626, Taylor and Francis (1978).

[8] Fortemps, Philippe, and Marc Roubens. *"Ranking and defuzzification methods based on area compensation."* Fuzzy sets and systems, 82.3, 319-330, Elsevier (1996).

[9] Liou, Tian-Shy, and Mao-Jiun J. Wang. *"Ranking fuzzy numbers with integral value."*, Fuzzy sets and systems, 50.3, 247-255, Elsevier (1992).