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Modification of Difference-Iterative Algorithms

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Abstract : Extension of possibilities of known difference-iterative algorithms has been carried out with the aim of class extension of realizable on microprocessors functional information processing for the local automation systems of technological objects and processes. The basis of such extensions are mathematical models of differential-iterative algorithms. For this we have introduced the concept of the function $y = sign\ x$, limit values, iterable values, conditions for the convergence of the iterative process. The expansion of the class of mathematical functions adaptive to their implementation on microprocessors ensures its wider application in the automation of complex technological processes. This will simplify the design of algorithmic support of such microprocessor-based systems because of the integrality of difference-iterative algorithms.
Keywords : Microprocessors, integer algorithms, production automatization.

1. INTRODUCTION

Difference-iteractive methods are non-analytic computation methods providing digital modelling on the basis of final increments calculation of iterative magnitudes covering iterative processes to desired calculated values [1].

At first, difference-iterative algorithms (DIA) were applied for hardware implementation [2], [3]. After the introduction of microprocessors (MP) they were applied for software implementation [4], [5]. They received special development in the algorithmic software of MP due to their speed and absence of operations, such as: multiplication, division and other so-called "long-operations" in their command set [6].

The Methodological Basis of the Study

The structure of DIA is that the right-hand sides of the recursion equations contain only addition, subtraction, arithmetic shifts (left or right) of its different additive members.

For example, the function $z = \dfrac{(x-y)^2}{y}$ can be calculated according to DIA [7]:

$$
\begin{aligned}
q_{i-1} &= \text{sign } Z_{i-1} = \begin{cases} +1, \text{if } Z_{i-1} \geq 0; \\ -1, \text{if } Z_{i-1} < 0; \end{cases} \\
Z_0 &= y - x, \\
Z_i &= Z_{i-1} - q_{i-1} \cdot y \cdot 2^{-i}, \\
Z_n &\to 0; \\
Y_0 &= y - x, \\
Y_i &= Y_{i-1} - q_{i-1} \cdot x \cdot 2^{-i} \\
Y_n &\to \frac{(x-y)^2}{y}
\end{aligned} \tag{1}
$$

Here and then i is an iteration number, = 1, 2…, n, n is digit bit of arguments.

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The synthesis of new DIA is conducted by a heuristic method because the DIA theory is not sufficiently developed [8]. We have constructed mathematical models of the main types of DIA [9] that allows modifying them and significantly expanding the class of functions realizable by them.

Here there are a number of such DIA modifications.

The First Modification of DIA

The DIA, given in [10], computes $z = \dfrac{x^2 + y^2}{x + y} (x > 0,\ y > 0)$:

$$
\begin{aligned}
q_{i-1} &= \operatorname{sign}(X_{i-1} - Y_{i-1})\,; \\
X_0 &= x, \\
X_i &= X_{i-1} - q_{i-1} \cdot y \cdot 2^{-i}, \\
X_n &\to \frac{x^2 + y^2}{x + y} \\
Y_0 &= y, \\
Y_i &= Y_{i-1} + q_{i-1} \cdot x \cdot 2^{-i}, \\
Y_n &\to X_n.
\end{aligned}
\tag{2}
$$

On the basis of analysis of the mathematical model of DIA [5] we have made its modification: y and x, standing in the right-hand sides of the recursion correlation are replaced by $2w$ and $2u$, respectively. Besides, it is assumed that arguments x, y, u, w may have any sign (+ or –).

The modified DIA will have the following form :

$$
\begin{aligned}
q_{i-1} &= \operatorname{sign}(X_{i-1} - Y_{i-1}) \cdot \operatorname{sign}(u + w), \\
X_0 &= x, \\
X_i &= X_{i-1} - q_{i-1} \cdot w \cdot 2^{1-i}, \\
X_n &\to \frac{xu + yw}{u + w} \\
Y_0 &= y, \\
Y_i &= Y_{i-1} + q_{i-1} \cdot u \cdot 2^{1-i}, \\
Y_n &\to X_n.
\end{aligned}
\tag{3}
$$

The condition of convergence is $\left|\dfrac{x - y}{u + w}\right| < 2.$

DIA (3) allows calculating function of four arguments x, y, u, w. This function is weighted average of two values x, y taken with weights u and w respectively

$$F(x, y, u, w) = \frac{xu + yw}{u + w}.$$

The virtue of the DIA (3) does not end there. Suppose values x, y, u, w are taken as equal to

$$
\begin{aligned}
x &= k_x \cdot t + m_x, \\
u &= k_u \cdot t + m_u, \\
y &= k_y \cdot t + m_y, \\
w &= k_w \cdot t + m_w,
\end{aligned}
\tag{4}
$$

where $k_x,\ k_y,\ k_u, k_w$ are natural powers $\dfrac{1}{2}$, taken with the sign + or –.

This provides the calculations (4) with the arithmetic shifts *t* and subsummation of numbers m_x, m_y, m_u, m_w. Then DIA (3) will calculate a function of one variable t [11]

$$F(t) = \frac{At^2 + Bt + C}{Dt + C}. \tag{5}$$

Coefficients in (5) along with the coefficients of representations (4) can be selected such that (5) will approximate the required function (including the given one tabularly) on the preassigned (working) interval of t variations. For example, $f(t) = \sin\sqrt{t}$ or $f(t) = \sqrt{\sin t}$ and so on.

The interval for t is set with the real domain of MP application in devices of local automatization.

We have developed [12] a computer program of obtaining coefficients in the expression (4) in accordance with the required function $f(t)$ (or setting table).

The Second Modification of DIA

The DIA [7] of multiplication and division without restoration (reproduction) of remainder are well known.

$$\begin{aligned} q_{i-1} &= \operatorname{sign} W_{i-1}; \\ W_0 &= w, \\ W_i &= W_{i-1} - q_{i-1} \cdot y \cdot 2^{1-i}, \\ W_n &\to 0; \\ V_0 &= v, \\ V_i &= V_{i-1} + q_{i-1} \cdot x \cdot 2^{1-i}, \\ V_n &\to v + \frac{w}{y}. x. \end{aligned} \tag{6}$$

In order to increase the degrees of numerator and denominator polynomials in (5), that increases approximation of function $f(t)$, we modify DIA (6). For this purpose it will be presented as 'two-storied'. Then it will be able to calculate

$$F(s, y, x, w, u) = s + \frac{y}{x}.\left(w + \frac{y}{x}.u\right) \tag{7}$$

Such DIA calculates the expression in brackets on the first “floor” and on the second it calculates the expression (7) wholly. Moreover, in order to accelerate, the partial result is passed into each iteration in brackets to the process of calculating expression, written to the left of the bracket. Deduce DIA for calculation expression (7) by the method of mathematical induction.

We represent the fraction $\frac{y}{x}$ only by three summands for reduction of records

$$\frac{y}{x} = q_0 . 2^{-1} + q_1 . 2^{-2} + q_2 . 2^{-3}, \tag{8}$$

and then the result will be spread onto *n* summands.

Substitute the expression (8) into the right-hand part of the expression (7).

We have

$$\begin{aligned} S &= s + (q_0 . 2^{-1} + q_1 . 2^{-2} + q_2 . 2^{-3}). [z + q_0 . 2^{-1} + q_1 . 2^{-2} + q_2 . 2^{-3}). \\ &= s + q_0 . 2^{-1} + q_1 . 2^{-2} z + q_1 . 2^{-2} . z + q_2 . 2^{-3}. z + (q_0 . 2^{-1} + q_1 . 2^{-2} + q_2 . 2^{-3})2. u \\ &= s + q_0 . 2^{-1} + z + q_1 . 2^{-2} . z + q_2 . 2^{-3}. z + (q_0{}^2 . 2^{-2} + u + q_2{}^2 . 2^{-6}. u \\ &\quad + 2_{q0} . q_1 .. 2^{-3}. u + 2_{q0} . q_1 .. 2^{-3} . u + 2q_0 . q_2 . 2^{-4} . u + 2q_1 . q_2 . 2^{-5}. u \end{aligned}$$

We group together the last summands in the following way :

$$S = s + q_0 \cdot 2^{-1} \cdot z + 2^{-2} \cdot u + q_1 \cdot 2^{-2} \cdot z + 2q_0 \cdot q_1 \cdot 2^{-2} \cdot u + 2^{-4} \cdot u + q_2 \cdot 2^{-3} \cdot z + 2q_0 \cdot q_2 \cdot 2^{-4} \cdot u + 2q_1 \cdot q_2 \cdot 2^{-5} \cdot u + 2^{-6} \cdot u. \quad (9)$$

It is taken into account that $q_0^{\,2} = 1$, $q_1^{\,2} = 1$, $q_2^{\,2} = 1$. Further we convert the expressions by introducing a new iteration value.

$$R_0 = z, \qquad R_i = R_{i-1} + q_{i-1} \cdot u \cdot 2^{-i}. \quad (10)$$

Then its first three members will be

$$\begin{aligned} R_0 &= z, \\ R_1 &= R_0 + q_0 \cdot u \cdot 2^{-1} = z + q_0 \cdot u \cdot 2^{-1}; \\ R_2 &= R_0 + q_0 \cdot u \cdot 2^{-1} + q_1 \cdot u \cdot 2^{-2} \\ &= z + q_0 \cdot u \cdot 2^{-1} + q_1 \cdot u \cdot 2^{-2} \end{aligned} \quad (11)$$

Substituting three expressions for R_i ($i = 0, 1, 2$) (11) into grouping on lines components (9), we obtain more compact notation:

$$S = s + (2R_0 - z) \cdot q_0 \cdot 2^{-1} + 2^{-2} \cdot u + (2R_1 - z) \cdot q_1 \cdot 2^{-2} + 2^{-2} \cdot u + (2R_2 - z) \cdot q_2 \cdot 2^{-3} + 2^{-6} \cdot u \quad (12)$$

We notice that the expression $2R_0 - z = z$ has already been taken into account.

As far as in expression (12), R_i has a coefficient which equals 2, therefore to exclude this multiplication, it is necessary to add 1 to power 2^{-i}, we get 2^{-i+1}. As $2z$ is present in the expression $2R_i$, then one z will be subtracted (see (12)), and the second z will be transferred into the initial value $R_0 = z$.

So the final DIA has the following form (we extend the number of iterations to n).

$$\begin{aligned} q_{i-1} &= \operatorname{sign} W_{i-1}; \\ W_0 &= y, \\ W_i &= W_{i-1} - q_{i-1} \cdot x \cdot 2^{-i}, \\ W_n &\to 0; \\ R_0 &= w, \\ R_i &= R_{i-1} + q_{i-1} \cdot u \cdot 2^{1-i}, \\ R_n &\to w + 2\frac{y}{x} \cdot u; \\ S_0 &= s, \\ S_i &= S_{i-1} + R_{i-1} \cdot q_{i-1} \cdot 2^{1-i} + 2^{-2i} \cdot u, \\ S_n &\to s + \frac{y}{x} \cdot \left(w + \frac{y}{x} \cdot u \right) \end{aligned} \quad (13)$$

DIA (13) allows calculating rational functions with the usage of substitutions as in expression (4).

$$F(t) = \frac{At^3 + Bt^2 + Ct + D}{Et^2} \quad (14)$$

Such functions (14) allow to approximate the necessary functions $f(t)$ at the given interval t more exactly.

The Third Modification of DIA

The same RIA (6) can be modified to compute functions of the form:

$$F(p, w, y, v, z) = p + \frac{w}{y} \cdot \frac{v}{z} \quad (15)$$

It makes sense to consider two variants (with the right shift and with more accurate left one). The initial stages of DIA (6) and such correlation which helps avoid multiplication operation and so called dynamic multiplication are on the basis [13], [14].

$$(\alpha_{i-1} + \gamma_{i-1}) \,.\, (\beta_{i-1} - \mu_{i-1}) = (\beta_{i-1} + \mu_{i-1}) + \beta_{i-1} \,.\, \gamma_{i-1} + \alpha_{i-1} \,.\, \mu_{i-1} + \gamma_{i-1} \,.\, \mu_{i-1},$$

where γ_{i-1} and μ_{i-1} are values which equal 0 or ± 1.

We can demonstrate a modified DIA (1 variant).

$$\begin{aligned}
q_{i-1} &= \operatorname{sign} W_{i-1} \,.\, \operatorname{sign} y; \\
W_0 &= w, \\
W_i &= W_{i-1} - q_{i-1} \,.\, x.\, 2^{1-i} \,.\, y, \\
W_n &\to 0; \\
\varepsilon_{i-1} &= \operatorname{sign} V_{i-1} \,.\, \operatorname{sign} z; \\
V_0 &= v, \\
V_i &= V_{i-1} - \varepsilon_{i-1} \,.\, z \,.\, 2^{1-i}, \\
V_n &\to 0; \\
S_0 &= 0, \\
S_i &= S_{i-1} + q_{i-1} \,.\, 2^{1-i}, \\
S_n &\to \frac{w}{y}; \\
T_0 &= 0, \\
T_i &= T_{i-1} + \varepsilon_{i-1} \,.\, 2^{1-i}, \\
T_n &\to \frac{v}{z}; \\
P_0 &= p, \\
P_i &= P_{i-1} + q_{i-1} \,.\, 2^{1-i}.\, T_{i-1} + \varepsilon_{i-1} \,.\, 2^{1-i},\, S_{i-1} \\
&\qquad + q_{i-1} \,.\, \varepsilon_{i-1} \,.\, 2^{-2(1-i)} \\
P_n &\to p + \frac{w}{y} . \frac{v}{z}.
\end{aligned} \tag{17}$$

The advantage of DIA (17) is the possibility of two divisions implementations and one multiplication in the integer arithmetic (only due to addition/subtraction, arithmetic shifts and branches).

The disadvantage is a great number of arithmetic shifts to the right and low accuracy, unless a binary bit network (hardware or software) is used.

Therefore we demonstrate one more (the second) variant of the modified algorithm (6). It is based on the principle of synchronous zooming (some interactive values are multiplied by a variable scaled multiplier 2^(i-1)). Thereby we can avoid significant portion of shifts to the right [15].

$$\begin{aligned}
q_{i-1} &= \operatorname{sign} W_{i-1} \,.\, \operatorname{sign} y; \\
W_0 &= \frac{w}{2}, \\
W_i &= 2 \,.\, W_{i-1} - q_{i-1} \,.\, y, \\
W_n &\to 0; \\
\varepsilon_{i-1} &= \operatorname{sign} V_{i-1} \,.\, \operatorname{sign} z; \\
V_0 &= \frac{v}{2}, \\
V_i &= 2.\, V_{i-1} - \varepsilon_{i-1} \,.\, z,
\end{aligned}$$

$$
\begin{aligned}
&V_n \to 0;\\
&S_0 = 0,\\
&S_i = S_{i-1} + q_{i-1} \,.\, 2^{1-i},\\
&S_n \to \frac{w}{y};\\
&T_0 = 0,\\
&T_i = T_{i-1} + \varepsilon_{i-1} \,.\, 2^{1-i},\\
&T_n \to \frac{v}{z};\\
&P_0 = \frac{p}{2},\\
&P_i = 2P_{i-1} + q_{i-1} \,.\, T_{i-1} + \varepsilon_{i-1} \,.\, S_{i-1} + q_{i-1} \,.\, \varepsilon_{i-1} \,.\, 2^{-2(1-i)};\\
&P_n \to \left(p + \frac{w}{y}.\frac{v}{z}.\right).\, 2^{-n}.
\end{aligned}
\tag{18}
$$

Convergence of both variants is ensured at $\left|{}^{w}\!/_{y}\right| < 2$ and $\left|{}^{v}\!/_{z}\right| < 2$ simultaneously.

Additional advantages of the second variant of DIA (18) are increased operation and accuracy. The disadvantage is the necessity of the double bit network existence (hardware or software).

2. RESULTS

Table 1. Left part of table of absolute errors (in units of lower binary bits).

p = 200		***v***	
		400	***–550***
z	–500	0.556	0.619
	200	0.800	0.857
y	1350	0.499	0.820
	–425	0.875	0.950
p = 200		***–750***	***–1400***
		w	

Table 2. Right part of table of absolute errors (in units of lower binary bits).

w		***p = 200***	
–1650	***1450***		
0.495	0.908	600	***y***
0.909	0.392	–1200	
0.510	0.942	650	**z**
0.851	0.889	–200	
1300	805	***p = 200***	
v			

The computer realization has been made for DIA (18) accuracy checking. The exact value has been calculated in accordance with the expression (15) and algorithm (18) in the format of integers.

Absolute errors are given (shown) in the table 1 and table 2 (for $n = 15$ and $p = 200$) for a number of arbitrary sets w, y, v, z.

3. DISCUSSIONS

The proposed algorithms for computing complex functions belong to the class of integer algorithms. These algorithms operate on integers and display the result also in the form of integers. They are very convenient to implement on microprocessors simplified architecture. The latter circumstance provides the simplicity of their design and hardware high-speed processing of information with simple commands (not harder "add-subtract").

All of these integer algorithms are based on the laws and properties of integer arithmetic. In the process of their creation are used methods of digital interpolation of geometric objects (straight line, circle, exponential, etc.).

More complex geometric objects (perpendiculars to curved lines, the intersection of two straight lines, crossing with other curves, etc.) are based on their model properties and information from analytic geometry. This allows to solve a number of complex, for example, the kinematic task in the simulation of mechanisms and machines to replace real physical objects by modules, software implemented on microprocessors .

Further studies will be carried out towards the establishment of new iterative processes to empower the so-called non-analytic algorithms, e.g. for solving the inverse problem for robot manipulators [20]. Such algorithms are very effective for use in e-kinematics and mechatronics [16], [17].

4. CONCLUSION

All this creates the preconditions for wide and successful use of microprocessors in the implementation of the tasks of the so-called e-kinematics [16], mechatronics and manipulation robots [18].

We consider that DIA, especially modified ones will receive a wide application in microprocessor devices for the local automatization of technological objects and processes with the aim of ensuring broader classes of realizable functional information processing [19], [20], [21].

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