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HIGHER ORDER NONLINEAR EFFECT IN DUSTY PLASMA

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Abstract: The combined effects of dust ions and higher order nonlinearity are taking into account with the dust charge variation. The model of dust charge variation, taken here, is of the form $I_e + I_i = 0$, I_e and I_i being the electronic and ionic currents. The basic set of fluid equations for plasma consisting of dust cold ions, cold ions and hot electrons (nonisothermal and isothermal) reduces to modified Korteweg-de Vries equation to linear homogeneous equation for second order potential and ultimately highlights new features in plasma. Using reductive perturbation method stationary solutions of the coupled equations are obtained in the case of dust ions, retaining terms up to the third order. The numerical analysis for the stationary solutions is also discussed.

Keywords: Ion-acoustic waves, Dusty plasma, Modified Korteweg-de Vries equation, Hgher order nonlinearity.

1. INTRODUCTION

Dusty plasmas are ionized gases containing small particle of solid matter. These particles become electrically charged. The dust grain can be charged both positively and negatively [1]. Often their charges are varying large, so that they repel one another strongly. These charge particles can be suspended in plasma, levitated against by any modest electric field that might be in the plasma.

Substantial attention has been given to study the wave propagation and nonlinear structure in dusty multi ion plasmas because of it vital role for understanding different types of collective process in space and theoretical works in last few years. More recently there has been a considerable amount of theoretical as well as experimental work on waves in dusty plasma because of its relevance in various astrophysical scenarios, beginning with the work Bliokh *et al.*, [2]. At the out set of last decade of the last century, Rao *et al.*, [3] theoretically demonstrated the existence of very-low – velocity DA wave in unmagnetized dusty plasma, whereas Shkula *et al.*, [4] reported the existence of dust ion acoustic wave in dusty plasma. Both these waves are low-frequency modes exited in dusty plasma.

Various authors have derived the Korteweg-de Vries (KdV) Kadomtsev-Petviashvili (K-P) [5, 6] and equation which represents propagation of nonlinear waves in multicomponent plasma in diverse situation including in presence of dust. In this paper modified KdV equation is derived by using the reductive perturbation method analysis and numerical analysis is discuss using different standard values of various parameters.

2. THE MODEL EQUATIONS

The basic sets of model equation for dust acoustic (DA) wave in unmagnetized plasma, which includes variable dust charge and non thermal ions are given by

$$\left.\begin{aligned} \frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) = 0 \\ \frac{\partial u_\alpha}{\partial t} + u_\alpha \frac{\partial u_\alpha}{\partial x} = \frac{\partial \phi}{\partial x} \end{aligned}\right\}. \quad \text{(For ions)} \tag{1}$$

$$\left.\begin{aligned} \frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \\ \frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = -\frac{1}{Q}\frac{\partial \phi}{\partial x} \end{aligned}\right\}. \quad \text{(For dust)} \tag{2}$$

Together with Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + z_d n_d - n_i \,. \tag{3}$$

The electron density of the electron is given by as [7]

$$n_e = \exp(\phi) \tag{4}$$

The dust charge Q_d on a grain changes because of the currents according to the equation as [8] is $\frac{dQ_d}{dt} = \sum_\alpha I_\alpha$ and for dust charge we have taken the model

$$I_e + I_i = 0. \tag{5}$$

The electron and ion current are as [10] where $I_e = A_1 n_e \exp(n_o z_d)$ and $I_i = A_2 n_e \exp(1 - y_o z_d)$, where $x_0 = -\frac{e^2 z_{d0}\beta}{a_d T_i}$, $y_0 = -\frac{e^2 z_{d0}}{a_d T_i}$ and $\beta = \frac{T_i}{T_e}$. Also $A_1 = \frac{\pi a_d^2 n_{d0}}{w_{pd}}(\frac{8T_e}{\pi m_e})^{\frac{1}{2}}$ and $A_1 = \frac{\pi a_d^2 n_{d0}}{w_{pd}}(\frac{8T_e}{\pi m_e})^{\frac{1}{2}}$. Here a_d is the radius of the dust particle and w_{pd}^{-1} is the dust plasma period.

3. NORMALIZATION

With the purpose of under studying the parameter space, which limits the existence DA solutions, we normalizing all the parameters. According to [9] the dust particle number density n_d is normalized to n_{d0}, dust particle velocity u_d is normalized to $c_d = (\frac{z_d T_i}{m_d})^{\frac{1}{2}}$, where ϕ, z_d and m_d are the electrostatic potential, number of charge and mass of dust particles respectively. z_d is normalized to z_{d0}. n_i is the number of density of ions which is normalized to n_{i0} and n_e is the number of electrons of which is normalized to n_{e0}. Q is the ratio of dust mass m_d to ion mass m_i.Total charge neutrability at equilibrium in as [9] is $n_{e0}+n_{d0}z_{d0}=n_{i0}$.

4. REDUCTIVE PERTURBATIVE ANALYSIS

To derive the KdV type equation for law frequency DA wave, we use the following expression in ε about the equilibrium states as

$$\left.\begin{aligned} n_i &= n_{i0}+\varepsilon n_{i1}+\varepsilon^2 n_{i2}+\varepsilon^3 n_{i3}+\ldots\ldots\ldots\ldots \\ n_d &= n_{d0}+\varepsilon n_{d1}+\varepsilon^2 n_{d2}+\varepsilon^3 n_{d3}+\ldots\ldots\ldots\ldots \\ u_i &= \varepsilon u_{i1}+\varepsilon^2 u_{i2}+\varepsilon^3 u_{i3}+\ldots\ldots\ldots\ldots\ldots\ldots \\ u_d &= \varepsilon u_{d1}+\varepsilon^2 u_{d2}+\varepsilon^3 u_{d3}+\ldots\ldots\ldots\ldots \\ .z_d &= 1+\varepsilon z_{d1}+\varepsilon^2 z_{d2}+\varepsilon^3 z_{d3}+\ldots\ldots\ldots\ldots \\ \phi &= \varepsilon\phi_1+\varepsilon^{\frac{3}{2}}\phi_2+\varepsilon^2\phi_3+\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\}. \tag{6}$$

The stretching coordinate in this case as [7] are

$$\xi=\varepsilon^{\frac{1}{2}}(x-vt), \qquad \tau=\varepsilon^{\frac{3}{2}}vt\,. \tag{7}$$

Here ν is the phase velocity of the wave and the relative dust-ion contradiction as [11] is $\varepsilon=\frac{n_d}{n_i}$, a small dimensionless expansion parameter $(0<\varepsilon<1)$ measuring the weakness of the dispersion.

Substituting (6) and (7) in (1)-(5) and equating the coefficient of different powers of ε, we have the following results.

The coefficient of $\varepsilon^{\frac{1}{2}}$ and ε gives

$$\left.\frac{\partial n_{i0}}{\partial\xi}=0 \quad \text{and} \quad \frac{\partial n_{d0}}{\partial\xi}=0\right\} \tag{8}$$

i.e., n_{i0} and n_{d0} are constant

The coefficient of $\varepsilon^{\frac{3}{2}}$ gives

$$\left.\begin{aligned} n_{i1} &= -\frac{n_{i0}\phi_1}{v^2}, \quad u_{i1}=-\frac{\phi_1}{v}, \\ n_{d1} &= \frac{n_{d0}\phi_1}{Qv^2}, \quad u_{d1}=-\frac{\phi_1}{Qv} \end{aligned}\right\}. \tag{9}$$

The above expression are derived under the assumption that wave frame is moving under the sound velocity

$$v^2 = n_{i0} + \frac{n_{d0}}{Q}. \tag{10}$$

The coefficient of ε^2 gives

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 + \frac{1}{3}\phi_1^2 + n_{d2} + z_{d1}n_{d1} + z_{d2}n_{d0} - n_{i2} \tag{11}$$

The coefficient of $\varepsilon^{\frac{5}{2}}$ gives

$$v\frac{\partial n_{i1}}{\partial \tau} - v\frac{\partial n_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{i0}u_{i2} + n_{i1}u_{i1}) = 0 \tag{12}$$

and

$$v\frac{\partial u_{i1}}{\partial \tau} - v\frac{\partial u_{i2}}{\partial \xi} + u_{i1}\frac{\partial u_{i1}}{\partial \xi} = -\frac{\partial \phi_2}{\partial \xi}. \tag{13}$$

The coefficient of ε^3 gives

$$\frac{\partial^2 \phi_2}{\partial \xi^2} = \phi_3 + \phi_1\phi_2 + \frac{1}{6}\phi_1^3 + n_{d3} + z_{d1}n_{d2} + z_{d2}n_{d1} - n_{i3}. \tag{14}$$

The coefficient of $\varepsilon^{\frac{7}{2}}$ gives

$$v^2\frac{\partial n_{i3}}{\partial \xi} + n_{i0}\frac{\partial \phi_3}{\partial \xi} = v^2\frac{\partial n_{i2}}{\partial \tau} + vn_{i0}\frac{\partial n_{i2}}{\partial \tau} + \frac{\partial}{\partial \xi}(n_{i0}u_{i1}u_{i2} + n_{i1}u_{i2} + n_{i2}u_{i1}) = 0 \tag{15}$$

and

$$v^2\frac{\partial n_{d3}}{\partial \xi} - \frac{n_{d0}}{Q}\frac{\partial \phi_3}{\partial \xi} = v^2\frac{\partial n_{d2}}{\partial \tau} + vn_{d0}\frac{\partial n_{d2}}{\partial \tau} + \frac{\partial}{\partial \xi}(n_{d0}u_{d1}u_{d2} + n_{d1}u_{d2} + n_{d2}u_{d1}) = 0. \tag{16}$$

Now from (12) and making use of (8) and (9) we have for ions

$$v^2\frac{\partial n_{i1}}{\partial \tau} - v^2\frac{\partial n_{i2}}{\partial \xi} + n_{i0}\frac{\partial \phi_2}{\partial \xi} + n_{i0}\frac{\partial \phi_1}{\partial \tau} + \frac{3}{2}n_{i0}\frac{\partial \phi_1^2}{\partial \xi} = 0. \tag{17}$$

For the dust, from the same order equation in ε, we obtained

$$v^2 \frac{\partial n_{d1}}{\partial \tau} - v^2 \frac{\partial n_{d2}}{\partial \xi} - \frac{n_{d0}}{Q}\frac{\partial \phi_2}{\partial \xi} - \frac{n_{d0}}{Q}\frac{\partial \phi_1}{\partial \tau} + \frac{3}{2}\frac{n_{d0}}{Q}\frac{\partial \phi_1^2}{\partial \xi} = 0 . \qquad (18)$$

Also from (5) and comparing the coefficient of powers of ε, we have the following results.

The coefficient of ε gives

$$z_{d1} = C_1 \phi \qquad (19)$$

The coefficient of ε^2 gives

$$z_{d2} = f_1 \phi_2 + f_2 \phi_1^2 \qquad (20)$$

where

$$C_1 = \frac{\mu n_{i0}(1-y_0) + \lambda(1+x_0)}{(\lambda x_0 - \mu n_{i0} y_0 + \lambda x_0^2) v^2}, \qquad f_1 = 1 + x_0 ,$$

and

$$f_2 = -\left(\frac{\mu n_{i0} C_1}{v^2} + \lambda x_0 C_1 + \frac{x_0^2 C_1}{2} + \frac{1}{2}(1+x_0) \right).$$

Using the value of z_{d1} and z_{d2} in (11) we have

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 + \left(\frac{1}{2} + \frac{C_1 n_{d0}}{Q v^2} + f_2 \right) \phi_1^2 + n_{d2} - n_{i2} . \qquad (21)$$

Eliminating z_{d2} and n_{d2} from (14) and (18) and using (19) and (21) we get

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \qquad (22)$$

where

$$A = \frac{1}{2}\left\{ \frac{3}{2v^4}\left(n_{io} - \frac{n_{d0}}{Qv} \right) - \left(\frac{1}{2} + \frac{C_1 n_{d0}}{Q v^2} + f_2 \right) \right\} .$$

The second order quantities n_{i2}, n_{d2} u_{i2} and u_{d2} can be expressed in terms of ϕ_1 and ϕ_2 as

$$\left.\begin{aligned} n_{i2} &= \frac{n_{i0}}{v^2}\left[\phi_2 + \left(\frac{3}{2} - A\right)\phi_1^2 - \frac{\partial^2\phi_1}{\partial\xi^2}\right] \\ n_{d2} &= \frac{n_{d0}}{Qv^2}\left[-\phi_2 + \left(\frac{3}{2} + A\right)\phi_1^2 + \frac{\partial^2\phi_1}{\partial\xi^2}\right] \\ u_{i2} &= \frac{1}{v}\left[-\phi_2 + \frac{1}{2}(1 - A)\phi_1^2 - \frac{\partial^2\phi_1}{\partial\xi^2}\right] \\ u_{d2} &= \frac{n_{d0}}{2Qv}\left[\phi_2 + \left(\frac{1}{v} + A\right)\phi_1^2 + \frac{\partial^2\phi_1}{\partial\xi^2}\right] \end{aligned}\right\}. \tag{23}$$

5. CONTRIBUTION TO THE mKdV EQUATION

Eliminating n_{d3}, n_{i3} and ϕ_2 from (14)-(16) and using (23) we get

$$\frac{\partial\phi_2}{\partial\tau} + H\frac{\partial}{\partial\xi}(\phi_1\phi_2) + G\frac{\partial^3\phi_2}{\partial\xi^3} = S(\phi_1). \tag{24}$$

The source term is a function of ϕ_1 given by

$$S(\phi_1) = L_1\phi_1\frac{\partial^3\phi_1}{\partial\xi^3} + L_2\phi_1^2\frac{\partial\phi_1}{\partial\xi} + L_3\frac{\partial\phi_1^3}{\partial\xi} + L_4\frac{\partial}{\partial\xi}\left(\frac{\partial\phi_1}{\partial\xi}\right)^2 + L_5\frac{\partial^2\phi_1^2}{\partial\xi^2} + L_6\frac{\partial^5\phi_1}{\partial\xi^5}. \tag{25}$$

Where

$$H = \frac{1}{v + n_{i0} - n_{d0}}\left[\frac{n_{d0}}{Qv^2}\left\{\frac{1}{Qv^2}(v^2 + v - n_{d0}) - v(f_2 - 1)\right\} - \frac{n_{i0}}{v^2} - v\right],$$

$$G = \frac{n_{d0}}{Qv(v + n_{i0} - n_{d0})}$$

$$L_1 = -\frac{1}{v + n_{i0} - n_{d0}}\left[\frac{1}{2}A\left(\begin{aligned} &2f_2 + n_{d0} - n_{i0} + 1 + \frac{2C_1 n_{d0}}{Qv^2} \\ &+ \frac{1}{2v^4}\left\{\frac{n_{d0}}{Q}(v^2 + v - n_{d0}) - n_{i0}v(v+3)\right\}\end{aligned}\right)\right]$$

$$L_2 = \frac{1}{v + n_{i0} - n_{d0}} \left[\frac{3(v+1)}{v^3} \left\{ n_{d0} \left(A + \frac{1}{v} \right) + \frac{1}{2}(n_{d0} - n_{i0})(1 - A)A \right\} - 2A \left(\frac{1}{2} + \frac{C_1 n_{d0}}{Qv^2} + f_2 \right) \right]$$

$$L_3 = \frac{1}{Q(v + n_{i0} - n_{d0})} \left[\frac{1}{6} + C_1 \left(A + \frac{3}{2} + f_2 \right) \right]$$

$$L_4 = \frac{1}{4(v + n_{i0} - n_{d0})} \left[\frac{2n_{d0}}{Q^2}(v^2 + v - n_{d0}) - n_{i0} v (v + 3) + \frac{A}{4}(n_{d0} - n_{i0}) \right]$$

$$L_5 = \frac{1}{2(v + n_{i0} - n_{d0})} \quad \text{and} \quad L_6 = -\frac{n_{i0} - n_{d0} + \frac{1}{2}}{Q(v + n_{i0} - n_{d0})}.$$

6. STATIONARY SOLUTION

Many authors solve the modified KdV equation including [12]. In order to solve to solve (22) and (24), we use the renormalizing method of Kodamas and Taniuti [13], according to which (22) and (24) are modified as

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \phi_1}{\partial \xi^3} + \delta\lambda \frac{\partial \phi_1}{\partial \xi} = 0 \tag{26}$$

$$\frac{\partial \phi_2}{\partial \tau} + H \frac{\partial}{\partial \xi}(\phi_1 \phi_2) + G \frac{\partial^3 \phi_2}{\partial \xi^3} + \delta\lambda \frac{\partial \phi_2}{\partial \xi} = S(\phi_1) + \delta\lambda \frac{\partial \phi_1}{\partial \xi}. \tag{27}$$

In (26) and (27) the parameter $\delta\lambda$ is introduce in such a way that the resonant term in $S(\phi_1)$ is cancelled by the term $\delta\lambda \frac{\partial \phi_1}{\partial \xi}$ in (27).

Let us obtain the stationary solutions by the new variables η as

$$\eta = \xi - (\lambda + \delta\lambda)\tau. \tag{28}$$

Under this transformation (26) and (27) become

$$\frac{\partial^3 \phi_1}{\partial \eta^3} + \frac{1}{4} A \frac{\partial \phi_1^2}{\partial \eta} - \lambda \frac{\partial \phi_1}{\partial \eta} = 0 \tag{29}$$

$$G \frac{\partial^3 \phi_1}{\partial \eta^3} + \frac{\partial}{\partial \eta}(H\phi_1 - \lambda)\phi_2 = S(\phi_1) + \delta\lambda \frac{\partial \phi_1}{\partial \xi}. \tag{30}$$

Using the boundary conditions

$$\phi_1 = \phi_2 = \frac{\partial \phi_1}{\partial \eta} = \frac{\partial \phi_2}{\partial \eta} = \frac{\partial^2 \phi_1}{\partial \eta^2} = \frac{\partial^2 \phi_2}{\partial \eta^2} = 0 \quad \text{as} \quad \eta \to \infty.$$

We can integrate (29) and (30) and obtain

$$\frac{\partial^2 \phi_1}{\partial \eta^2} + \left(\frac{A}{4} \phi_1 - \lambda \right) \phi_1 = 0 \tag{31}$$

and

$$G \frac{\partial^2 \phi_2}{\partial \eta^2} + (H\phi_1 - \lambda)\phi_2 = \int_{-\alpha}^{\eta} \left[S(\phi_1) + \delta\lambda \frac{\partial \phi_1}{\partial \xi} \right] d\eta. \tag{32}$$

The solitary wave solution of (31) is given by

$$\phi_1 = \phi_0 \sec h^2(a\eta) \tag{33}$$

where

$$\phi_0 = \frac{4\lambda}{A} \quad \text{and} \quad a = \frac{\sqrt{\lambda}}{2}. \tag{34}$$

Again

$$\int_{-\alpha}^{\eta} \left[S(\phi_1) + \delta\lambda \frac{\partial \phi_1}{\partial \xi} \right] d\eta = M_1 \sec h^6(a\eta) + M_2 \sec h^4(a\eta) + (M_3 - \delta\lambda) \sec h^2(a\eta). \tag{35}$$

In order to cancel the secular term associate with $S(\phi_1)$, we set the coefficient of $\sec h^2(a\eta)$ in (35) equal to zero. We therefore immediately get

$$\delta\lambda = M_3. \tag{36}$$

Because of (36), equation (32) becomes

$$G \frac{\partial^2 \phi_2}{\partial \eta^2} + (H\phi_1 - \lambda)\phi_2 = M_1 \sec h^6(a\eta) + M_2 \sec h^4(a\eta) \tag{37}$$

where

$$M_1 = 4a^2\phi_0^2 \left(-\frac{3}{2} L_1 + L_2 + L_3 - L_4 + 5L_5 + 30a^2 L_6 \right)$$

$$M_1 = 4a^2\phi_0\left(\frac{\phi_0}{2}L_1 + \phi_0 L_4 + 4\phi_0 L_5 - 30a^2 L_6\right) \quad \text{and} \quad M_3 = 16a^4\phi_0 L_6$$

In order to solve (37), we introduce a new variable [7]

$$\mu = \tan(a\eta). \tag{38}$$

Under this transformation (37) becomes

$$\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial\phi_2}{\partial\mu}\right] + \left[3(3+1) - \frac{2^2}{1-\mu^2}\right]\phi_2(\mu) = \tau(\mu) \tag{39}$$

where

$$\tau(\mu) = \frac{4}{\lambda}[M_1(1-\mu)^2 + M_2(1-\mu)]. \tag{40}$$

Equation (39) has two independent solutions in terms of associated Legendre function with the right hand side equal to zero and is given by

$$P_3^2 = -15(\mu^2 - \mu) \quad \text{and} \quad Q_3^2 = \frac{-8 + 25\mu^2 - 15\mu^4}{1-\mu^2} + \frac{15}{2}\mu(1-\mu^2)\log\left[\frac{1+\mu}{1-\mu}\right]. \tag{41}$$

We can obtain the particular integral of (5.14) by using the variation of parameter as

$$\phi_p(\mu) = u_1(\mu)P_3^2(\mu) + u_2(\mu)Q_3^2(\mu) \tag{42}$$

where $u_1(\mu)$ and $u_1(\mu)$ are given by

$$u_1(\mu) = -\int \frac{\tau(\mu)Q_3^2}{15\times 8}d\mu$$

$$\Rightarrow \quad u_1(\mu) = \frac{1}{120\lambda}\left[\frac{1}{2}(49M_1 + 44M_2)\mu + \frac{1}{6}(219M_1 + 160M_2)\mu^3 + \frac{5}{2}(11M_1 + 4M_2)\mu^5 \right.$$
$$- \frac{15}{2}M_1\mu^7 + \frac{5}{4}(3M_1 + M_2)\log(\mu^2 - 1)$$
$$\left. + \frac{5}{4}\log\left(\frac{1+\mu}{1-\mu}\right)\begin{pmatrix} -12M_1 + 18\mu^2 M_1 - 12\mu^4 M_1 + 3\mu^6 M_1 \\ -12M_2 + 12\mu^2 M_2 - 4\mu^4 M_2 \end{pmatrix}\right] \tag{43}$$

and

$$u_2(\mu) = -\int \frac{\tau(\mu)P_5^4}{15\times 8}d\mu$$
$$= \frac{1}{8\lambda}\left[-\frac{3}{2}(M_1 + M_2) - 2(M_1 + M_2)\mu^2 + (3M_1 + 2M_2)\mu^4 + \frac{M_1}{2\lambda}\mu^8 \right. \tag{44}$$

The constant of integration are put to zero using the boundary conditions. Substituting (43) and (44) into (42) gives the particular solution in the old variable as

$$\phi_p(\eta)=\frac{1}{96\lambda(\mu^2-1)}\Big[\mu\Big\{-6\mu(81-319\mu^2+492\mu^4-390\mu^6+170\mu^8-30\mu^8)M_1$$
$$-8\mu(57-205\mu^2+256\mu^4-140\mu^6+30\mu^8)M_2-15(3M_1-4M_2)(\mu^2-1)^2(\log(\mu-1)$$
$$+15(3M_1+4M_2)(\mu^2-1)^2\Big\{\log(1+\mu)+2\mu^2(-4+6\mu^2-4\mu^4+\mu^6)\log\left[\frac{1+\mu}{1-\mu}\right]$$
$$-\log(1+\mu)+2\nu^2(-3-3\mu^2+\mu^4)\log\left[\frac{1+\mu}{1-\mu}\right]\Big\}\Big\}\Big]. \tag{45}$$

Neglecting the higher orders, equation (46) ultimately reduces to

$$\phi_p(\eta)=\frac{1}{3072\lambda}[\sec h^6(a\eta)(\{12(1019-822\cosh[2a\eta]$$
$$+127\cosh[4a\eta])\sinh^2[2a\eta]\}M_1+\{16(600-1507\cosh[2a\eta]$$
$$+60\cosh[4a\eta]-69\cosh[6a\eta]+4\cosh[8a\eta])\}M_2\,] \tag{46}$$

Complementary function of (39) is given by

$$F_c=C_2P_3^2(\mu)+C_3Q_3^2(\mu)\,. \tag{47}$$

The secular term in the solution of the homogeneous equation associated with (39) can also be eliminated by the renormalizing the amplitude.

7. NUMERICAL ANALYSIS

Reductive perturbative method has been used to study the effect of nonlinear dust acoustic waves. Using stretch variables for space and time, the first order KdV soliton is obtained. To get higher order correction, a non linear partial differential equation in ϕ_2 has been solved. To support further, we have chosen some typical numerical values. For numerical analysis of electro acoustic soliton the Earth's magnetotial, if we put as [7] $\lambda = 0.1$; as [4] $n_{d0} = 0.53$, $Q = 0.1$, $a_d = 13.81\times10^{-5}$, $z_{d0} = 3\times10^4$: as [14] $T_e = 5.0$ eV, as [15] $T_i = 0.1$ eV, then we get $a = 2.8120$, $M_1 = -5.9413\times10^{24}$, $M_2 = -3.6014\times10^{14}$, We take $\frac{n_{d0}}{n_{i0}}=0.05$. With the help of these values the equation (33) and (46) becomes

$$\phi(\eta)=-4.0301\times10^{-15}\sec h^2(2.812\eta) \tag{50}$$

$$\phi_p(\eta) = .00326\,\sec h^6[0.1581\eta]\{-5.7622\times 10^{15}(600 - 1507\cosh[0.3162\eta] + 60\cosh[0.6325\eta] - 69\cosh[0.949487\eta] + 4\cosh[1.2649\eta]) - 7.1296\times 10^{25}(1019 - 822\cosh[0.3163\eta] + 127\cosh[0.6325\eta])\sinh^2[0.3163\eta]\}. \tag{51}$$

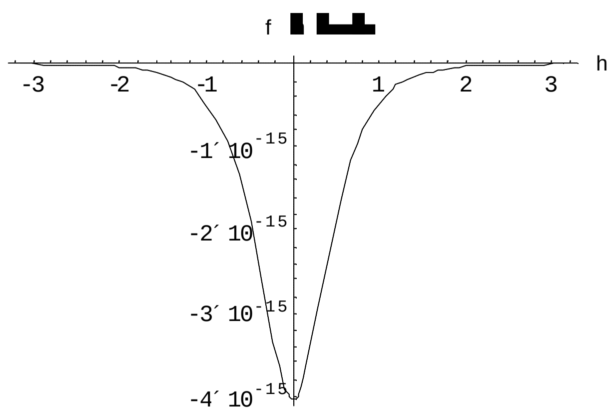

Figure [1]

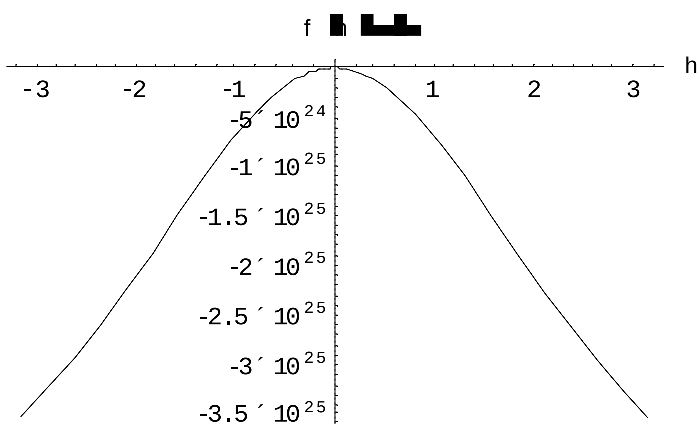

Figure [2]

Figure 1 shows the graphical representation of (50) and Fig. 2 that of (51).

8. CONCLUSION

The dust acoustic solitons have been studies taking into account the dust charge fluctuations. The second order correction to the KdV equation is derived. According to the normal form analysis by Kodama[16], the collision between the soliton is elastic only within the approximation to the KdV equation through the higher effect shows that the collision is inelastic, which leads to the change of soliton amplitude. In the case of dusty plasma, the effect of higher order non linearity is particularly significant for the parameters considered here. The second order nonlinearity is important in describing solitary waves in dusty plasma. The numerical result depicts distinctly that higher order non linearity is an important rule in the study of solitary wave in dusty plasma.

REFERENCES

[1] Shukla P. K., (2001), *Physics of Plasma*, **8**, 1791-1803.

[2] Bliokh P. V., and Yarroshenko, (1985), *Sov. Astron.*, **29**, 330.

[3] Rao N. N., Shukla P. K., and Yu M. Y., (1990), *Planet Space Science*, **38**, 543

[4] Shukla P. K., and Silin V. P., (1992), *Phys. Scr.*, **45**, 941.

[5] Das G. C., and Sarma J., (1998), *Chaos, Soliton & Fractals*, **9**(6), 901-911.

[6] Pakzad H. R., (2010), IAC, *J. of Phys.*, **74**(4), 605-614.

[7] Tagare S. G., and Reddy R. V., (1986), *J. Plasma Physics.,* **35**(2), 219-237.

[8] Goertz C. K., (1989), *Rev. Geophys.*, **27**, 271

[9] Forozani G., and Mohammadi M., Dust Acoustic Solitary Waves in Dusty Plasma with Variable Dust Charge, Ali Sina University, Hamdedan-Iran.

[10] Mitra S., Roychoudhary R., and Mondal K. K., (2005), *Czechoslovak J. Phys.*, **45,** 943-955.

[11] Melzer A., and Goree J., *Fundamentals of dusty Plasma*, 188.

[12] Sarma J., (2009), *Chaos, Soliton & Fractals*, **42**, 1599-1603.

[13] Kodamas Y. and Taniuti T., 1978, J. phys. Soc. Japan **45**, 298

[14] Gong J., and Du J., Secondary Electron Emission and Dust Charge Charging Currents in the Nonequilibrium Dusty Plasma Power Low Distributions, Tianjin University, China, **14**.

[15] El-Wakil S. A., El-Shewy E. K., Zahran M. A., Abelwahed, (2009), Effect of Different Size Dust Grains on the Properties of Solitary Waves in Space Environments, Mansoura University, Egypt.

[16] Y. Kodama, (1985), *Physica d*, **6**, 14

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