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Robust Computational HDE Technique For Optimal Approximation of Linear Systems

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ABSTRACT

In this article, optimal approximation of linear systems is simulated by using a mighty evolution algorithm HDE with search space expansion scheme, and by incorporating the different fitness functions like ITSE, ITAE, ISE, IAE. Approximating linear systems is illustrated by using couple of examples, one of the examples is a reactor / separator system, which is unstable system, the second example is a non-minimum phase system. The two examples were approximated by using HDE and time response and phase response respectively.

Keywords: Hybrid differential evolution algorithm(HDE), integral of squared error(ISE), integral of absolute error(IAE), integral of time multiplied squared error(ITSE), integral of time multiplied absolute error (ITAE).

1. INTRODUCTION

In control systems, the designing of the physical models are totally depends on the mathematical models. These mathematical models would resembles the exact characteristics of the physical model and it is a virtual implementation. This mathematical model is designed, because of the realistic model is very expensive, infeasible, and tedious or sometimes some simulation is impossible to conduct on the real model.Accurate model is difficult for understanding, optimizing, controlling and also study of the dynamics of the model. But the simplest model is easy to identify the nature of the model and optimization .This simplest model would be a lower order model, accurate model is not easy for the applications like hardware in the loop simulation and embedded model reference control. So that simplicity and accuracy are difficult for the system recognition, analysis, design of optimal controller.

The mathematical model is a physical system is leads to a large scale definitely a simplified engineering model. The analysis and design of controller for such a large scale engineering model is tedious and time consuming. For this type model approximation method is suitable. This model approximation is a mechanism that gives the low accuracy simple model over a high accuracy and complex model. This model approximation method is well-known in the designing of engineering works. It is necessary to have the simplest model from the complex model. Which gives the exact behaviour of the complex model. The model approximation method should satisfy the following properties a) The approximation error shall be small and be with in global error bound b) The procedure should be computationally stable and efficient c) System properties like passivity and stability should preserved in the approximated model.

There are many methods available in the literature to give the approximation of the systems. They are categorized in to two performance –oriented model approximation and non-performance oriented model

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approximation. These techniques can trust massively on a numerical depletion engender and an efficacious algorithm to classifying the performance criterion for retrieve optimal approximation models with diminishing H^{∞}-norm or L^2-norm. In the biography of approximation, the ecumenical solution is accomplished through detecting the foremost order optimality condition by a homotopy continuation method. However, it is unfit for unstable systems complexities are regularly encountered if an L^{∞}- norm or H^{∞}-norm choosen as the appearance index of model approximation because neither H^{∞}-norm or L^{∞}-norm is contiously defendable always and proficient numerical methods of critisizing and minimizing an H^{∞} or L^{∞} norm are still vanished. In order to overcome the limitations of the existing methods, in this paper a robust computational technique for approximation of linear systems is developed. This article is tabulated as follows.

2. PROBLEM STATEMENT

It is prominent in the control society that an intricate dynamic system should be competently expressed by a low –order transfer function. Hence, given a higher order rational or irrarational transfer function G(s), it is want to be an approximant mode in the form

$$H_m(s) = \frac{b_0 + b_1 s + \dots\dots\dots\dots + b_{m-1} s^{m-1}}{a_0 + a_1 s + \dots\dots\dots\dots + a_{m-1} + s^m} . e^{-\tau_d s} \tag{1}$$

So $H_m(s)$ enclose the thirst .... to have characteristics of the present system G(s). In this solicitation, we have intention to discover an optimal approximation model .

ISE:
$$\sum_{i=0}^{N} \left| G(j\omega_i) - H_m(j\omega_i) \right|^2 \tag{2}$$

IAE:
$$\sum_{0}^{N} | G(j\omega_i) - H_m(j\omega_i) | \tag{3}$$

ITSE:
$$\sum_{0}^{N} T \,|\, (G(j\omega_i) - H_m(j\omega_i)) \,|^2 \tag{4}$$

ITAE:
$$\sum_{0}^{N} T \left| (G(j\omega_i) - H_m(j\omega_i) \right| \tag{5}$$

Is diminished, the frequency values of φ_i, are ranging as φ_i, i = 1, 2, ….., N, and the integer N choosen a priori. In this concern the present system G(s) is asymptotically stable, and the constraint as follows

$$H_m(o) = G(o) \tag{6}$$

To a certain the steady state replications of the present system and the approximate model are identically clone for unit step input. The quandary of diminishing J given in (2) is an optimal parameter called quandary. The optimal parameter called quandary. The absolute parameter $a_i^{'}s$, $b_i^{'}s$ and τ_d of the proximate model (1) can be erect by a gradient search method or by a direct search optimization . In the present solicitation, we shall apply the direct search of hybrid differential evolution algorithm to find the paradigmatic parameters a_i, b_i = 1,….., m-1, and τ_d. It is noticed that because of the vanish of the exact erudition around the primises within which the optimal parameters locate, we blend a search space expansion scheme into hybrid differential evolution. A short and sweet description of such a hybrid differential evolution is provided in section III.

3. HYBRID DIFFERENTIAL EVOLUTION

3.1. Population Initialization

The HDE algorithm is commenced with engendering a population N_p of authentic valued n dimensional vectors

$$X_j = [x_{j,1}, x_{j,2,} ...., x_{j,n}]^T, j = 1, 2, ...N_p \tag{7}$$

The fundamental parameter values are choosen from within utilizer defined bounds

$$x_{j,k} \in [\underline{x_k}, \overline{x_k}],\ k = 1, 2, ..., \text{n}. \tag{8}$$

It is noted that the search region or interval $[\underline{x_k}, \overline{x_k}]$, set by a user for the parameter x_k may be more minute than, but with in, the allowable interval (x_k^-, x_k^+) in the case of $x_k^- = -\infty$ and/or $x_k^+ = +\infty$. Once the initial population is engendered, the cost of each population vector is evaluated and stacked for future reference.

3.1.1. Mutation

The summing of differential vector to a population vector is known as mutation. In this HDE algorithm, a population vector X_α is mutated into $z = [z_1, z_2, ...z_n]^T$ by summing to X_α the weighted difference of two desultorily called but .... different population vectors X_β and X_γ, i.e.,

$$Z = X_\alpha + F_s \cdot (\text{X}_\beta + X_\gamma) \tag{9}$$

Where F_s is a scaling factor in the interval (0, 2) [6]. The mutated vector z will be utilized as a donor vector for engendering a trail vector.

3.1.2. Crossover

Engendering a trail vector by superseding specific parameter of the target vector by the according parameter of arbitrarily engendered donor vector is called crossover. The crossover rate C_r justifies when a parameter should be subplant. The phenomenon of engendering a trail vector $X_t = (x_{t,1}, ..., x_{t,n})^T$ from the target vector $X = (x_1, ...x_n)^T$ and the donor vector Z is beginned with engendering a set of N arbitrary numericals which are distributed systematically in the interval (0, 1). Next, a set of N non uniform binary sequence is engendered by letting

$$b_i = \begin{cases} 1 & if \quad r_i \le C_r i = 1, 2..., n. \\ 0 & otherwise. \end{cases} \tag{10}$$

Then each element of the trail vector X_t is taken as

$$x_{t,k} = \begin{cases} x_k & \text{for b}_\text{k} = 1 \\ z_k & \text{for b}_\text{k} = 0. \end{cases} \tag{11}$$

Once the trail vector is fixed, its cost is estimate and collated with the coinciding target vector. The target vector is superseded by it in t_r in next engender. If its cost is more immensely colossal that of the trail vector.

3.2. Selection

The phenomena of engendering best progeny is called selection. The cost of each trail vector X_t is similitude with that of its parent target vector X_i. If the rate of the target vector X_i is diminished value that of the trail vector, the target is empowered in lead to the next offspring. contrastive, the target vector is superseded by the trail vector in the next off spring.

3.2.1. Migration if necessary

The next step after the selection in HDE is Migration. The main aim of migration is to branch out a population that failed in actual tolerance besides eluding from local optimal and averts premature conjunction. The incipient populations are predicated on the best individuals. The h^{th} genre of the i^{th} individuals as follows

$$X_{hi}^{G+1} = \begin{cases} X_{hb}^{G+1} + \rho_1(X_{h\min} - X_{hb}^{G+1}), \; if \;\; \rho_2 < \dfrac{X_{hi}^{G+1} - X_{h\min}}{X_{h\max} - X_{hb}^{G+1}} \\ X_{hb}^{G+1} + \rho_1(X_{h\max} - X_{hb}^{G+1}), \; otherwise \end{cases} \tag{12}$$

Where ρ_1, ρ_2 are desultorily engendered numerous uniformly distributed in the range of [0, 1]; h = 1, ….. , N_c. The migration in HDE is executed only if assesments fails to match the desired steadiness of population diversity. This quantification Is defines as follows.

$$\rho = \sum_{\substack{i=1 \\ i \neq b}}^{N_p} \sum_{j=1}^{n_c} X_{ji} \Big/ n_c(N_p - 1) < \varepsilon_1 \tag{13}$$

where

$$X_{ji} = \begin{cases} 1, \; if \;\; \left| \dfrac{X_{ji}^{G+1} - X_{jb}^{G+1}}{X_{jb}^{G+1}} \right| > \varepsilon_2 \\ 0, \; otherwise \end{cases} \tag{14}$$

The hybrid differential algorithm as follows

1. Adopt control variables of optimization process that are population size, scaling factor, crossover probability, convergence criterion, number of problem variables, lower and bounds of variables and maximum number of iterations. tablish an initial population entity with random positions.

$$X_{i0} = X_{\min} - X_{\max} - X * rand$$

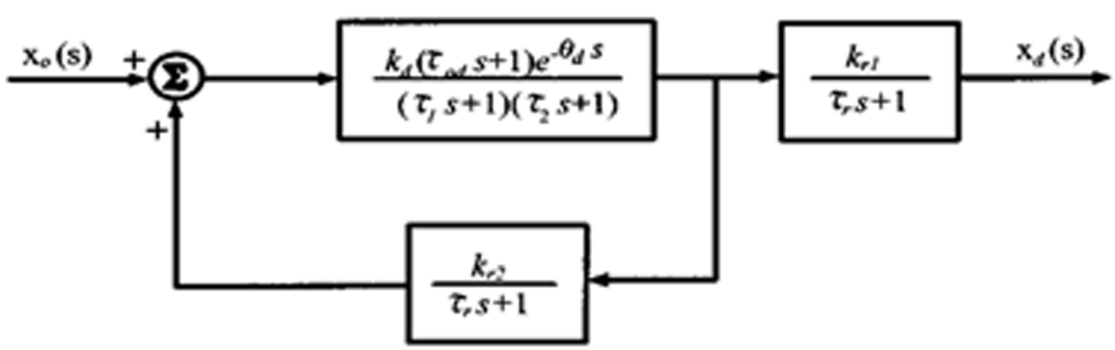

Figure 1: Block diagram of the system in Example1.

The value of fitness function is to be calculate for every parameter.

2. The fitness of each particle with personal best (Pbest) is to be correlated. If current result is better than Pbest then replace Pbest by current solution.
3. The fitness of all species with global best (Gbest) is distuinguished.If the fitness of any particle is better than Gbest, and then replace Gbest.
4. A new parameter is engendered with weighted difference between two vectors to a third vector Mutant vector is engendered based on the current individuals

$$Y_i^{G+1} = X_i^G + F((X_{r1}^G - X_{r2}^G) + (X_{r3}^G - X_{r4}^G))$$

5. A new vector called trailed vector is obtained by the combining of mutant vector and target vector the combining or mixing of parameter is called crossover Each generation of i^{th} individual is reproduced from mutant vector X_i^G and present individual Y_i^{G+1}

$$Y_i^{G+1} = \begin{cases} X_{hi}^G & \text{; if a random number} > \text{CR} \\ Y_{hi}^G & \text{; otherwise} \end{cases}$$

6. All the parents are have chance to select as a solutions in the population with the irrespective of their fitness value.
7. Crossover and mutation evaluates the children .If the performance or fitness of the offspring is compared with the fitness of parent vector. If offspring or children have best fitness value then the parent is replaced by its offspring.
8. To get desired fitness value rerun steps 2 to 7.

4. ILLUSTARTIVE EXAMPLES

This robust computational technique for optimal approximation of linear systems can be investigated by a couple of examples. The first example is a non-minimum phase response system, which is carried out from a reactor /separator system [2] for this time reponses are plotted for original order and reduced order. The second example is a unstable system[7][8] for this frequency response is plotted for analyzing the behavior of original order and approximated order.

Example 1: A reactor/separator system which is anon minimum phase response system and the transfer for the fig.1 is as shown below

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k_d k_{r1}(\tau_{0d}s+1)e^{-\theta_d s}}{(\tau_r s+1)(\tau_1 s+1)(\tau_2 s+1) - k_{r2}k_d(\tau_{0d}s+1)e^{-\theta_d s}} \tag{15}$$

The parameters in the above transfer function is given by

$$k_{r1} = 0.258, k_{r2} = 0.281, k_d = 1.4494, \theta_d = 0.2912,$$
$$\tau_r = 1.3498, \tau_{0d} = 0.3684, \tau_1 = 1.9624, \tau_2 = 0.43256.$$

Here it is desired to find the second- and third- order models

$$H_2(s) = \frac{k_{2,p}(s+\tau_{2,z})e^{-\tau_{2,d}s}}{s^2 + a_{2,1}s + a_{2,0}} \tag{16}$$

$$H_3(S) = \frac{k_{3,p}(S+\tau_{3,z})e^{-\tau_3 ds}}{S^3 + a_{3,2}S^2 + a_{3,1}S + a_{3,0}} \tag{17}$$

Such that the performance index given by (2) with values $\omega_i = 10^{-2+10i} \in [10^{-2}, 10^3]$, $i = 0,1, \cdots\cdots,$ $N = 50$ above mentioned original order is reduced to third order and second order in that again two unknowns $a_{2,0}$ and $a_{3,0}$ are as follows.

$$a_{2,0} = \frac{k_{2,p}(1-k_{r2}k_d)}{k_d k_{r1}}\tau_{2,z} \tag{18}$$

$$a_{3,0} = \frac{k_{3,p}(1-k_{r2}k_d)}{k_d k_{r1}}\tau_{3,z} \tag{19}$$

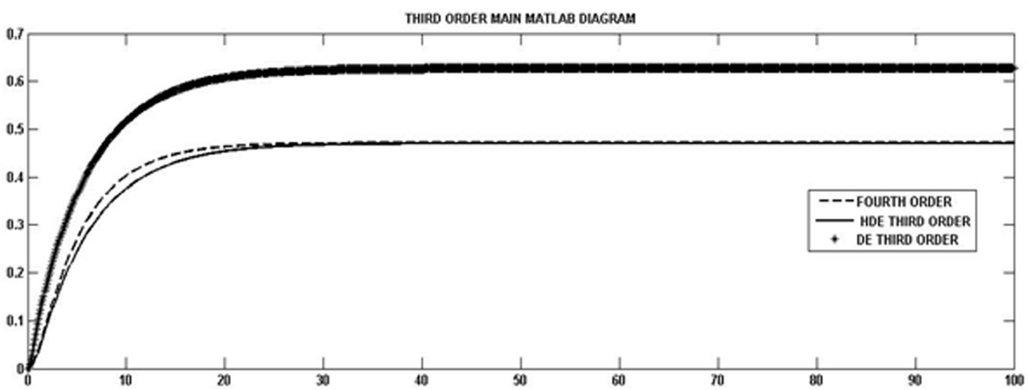

Figure 2(a): Comparision of step responses of Example 1 with given order and reduced order i.e $H_3(S)$ by using HDE.

$$H^*{}_3(s) = \frac{(0.5601s + 0.5113)e^{-0.2912s}}{s^3 + 5.259s^2 + 7.157s + 1.086} \tag{20}$$

$$H^*{}_{a,3}(s) = \frac{(0.1155s + 0.2668)e^{-0.2912s}}{s^3 + 3.46s^2 + 3.845s + 0.566} \tag{21}$$

$$H^*{}_{b,3}(s) = \frac{(0.5601s + 0.5113)e^{-0.2912s}}{s^3 + 5.259s^2 + 7.157s + 1.086} \tag{22}$$

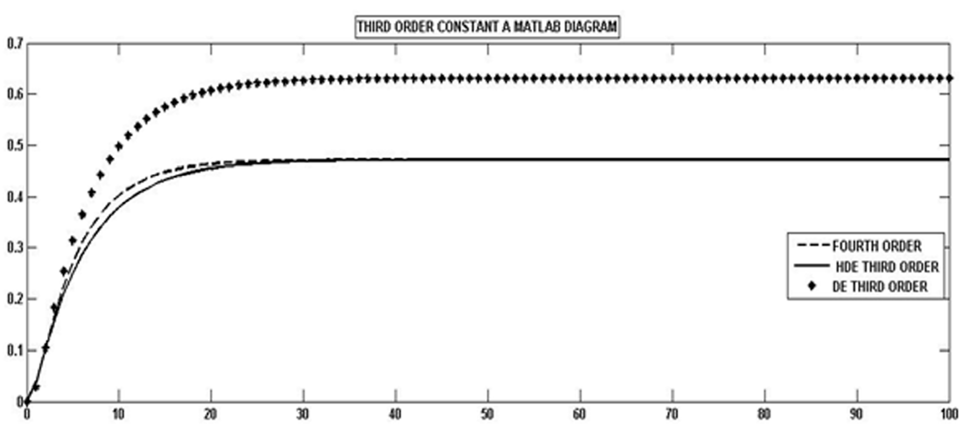

Figure 2(b): Comparision of step responses of Example 1 with given order and reduced order i.e $H_{3,a}^*(S)$ by using HDE.

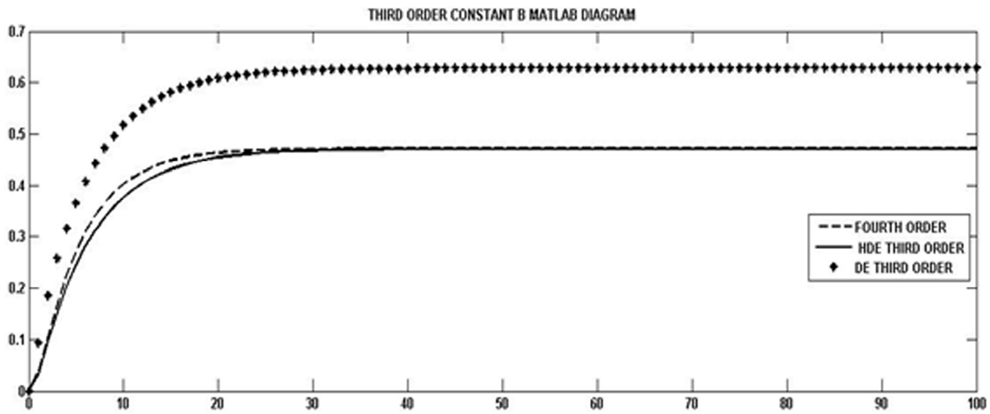

Figure 2(c): Comparision of step responses of Example 1 with given order and reduced order i.e $H^*_{b,3}(s)$ by using HDE

Fitness values of different performance indexes for example 1 is observed and tabulated in table II. Fig 2a, 2b are shows the step response of non-minimum phase response system by using DE and HDE. From the table the fitness function value for HDE by using ITSE is better than the DE by using ISE. Hence the HDE methodology is gives better optimall value than DE with the performance index ITSE.

In this examples two unknown are solved and $e^{-\theta_d s}$ is considered as constant until and unless of solving of $e^{-\theta_d s}$ the tentative work should not move forward for this the solving of $e^{\theta s}$ is taken from pade approximation [4]. Fig 2a shows third order time response of HDE fig 2b shows third order with a unknown $a_{2,0}$, fig 2c shows $a_{3,0}$. Hence this is solved and the behavior of original order and reduced order are analyzed. Different performance indexes also tried to get optimum value table II shows different performance index values. ITSE gives better value than DE.

Table 1
Collation Of Different Performance Indexes For Example 1

		ISE	IAE	ITSE	ITAE
DE	$H_3(s)$	0.0224	0.3397	0.0022	0.0340
	$H_{a,3}(s)$	0.0018	0.1211	0.00001809	0.1150
	$H_{b,3}(s)$	0.00087	0.0679	0.0000725	0.0068
HDE	$H^*_3(s)$	0.0268	0.4234	0.0016	0.0423
	$H^*_{a,3}(s)$	0.0294	0.4035	0.0029	0.0040
	$H^*_{b,3}(s)$	0.0268	0.4234	0.0027	0.0423

EXAMPLE 2: Given the fourth order unstable and non-minimum phase transfer function [7], [8]

$$G(\mathrm{s}) = \frac{60\mathrm{s}^3 + 25850s^2 + 685000s - 2500000}{s^4 + 105s^3 + 10450s^2 + 45000s - 500000} \tag{23}$$

It is want to approximate this transfer function by the second order model and the third order model

$$H_2(s) = \frac{c_{2,1}s + c_{2,0}}{s^2 + b_{2,1}s + b_{2,0}} \tag{24}$$

$$H_3(s) = \frac{c_{3,2}s^2 + c_{3,1}s + c_{3,0}}{s^2 + b_{3,1}s + b_{3,0}} \tag{25}$$

This is unstable system. to justify the behavior of original order and approximated order a frequency response is plotted and that to Nyquist is plotted for better understanding . This model is reduce to third order and second order models the frequency response of second order model is as shown in fig.3a. and for third order model is as shown in fig. 3b.

$$G^{*}{}_{3}(s)=\frac{198.5776s^{2}+2561.643s-10871}{s^{3}+57.784s^{2}+104.517s-2168.14} \tag{26}$$

$$G^{*}{}_{2}(s)=\frac{129.297s-445.711}{s^{2}+13.089s-91.7495} \tag{27}$$

Table 2
Comparsion of Different Performance Indexes of Example 2

		ISE	IAE	ITSE	ITAE
DE	Second order	2.02	2.32	1.148	1.258
	Third order	4.39	6.80	4.30	6.52
HDE	Second order	8.8	1.9	4.91	1.0
	Third order	3.122	2.65	3.78	2.12

For this model different performance index are tried to get better optimum value and are tabulated in table III which shows different optimum values for different performance index. For HDE ITAE gives

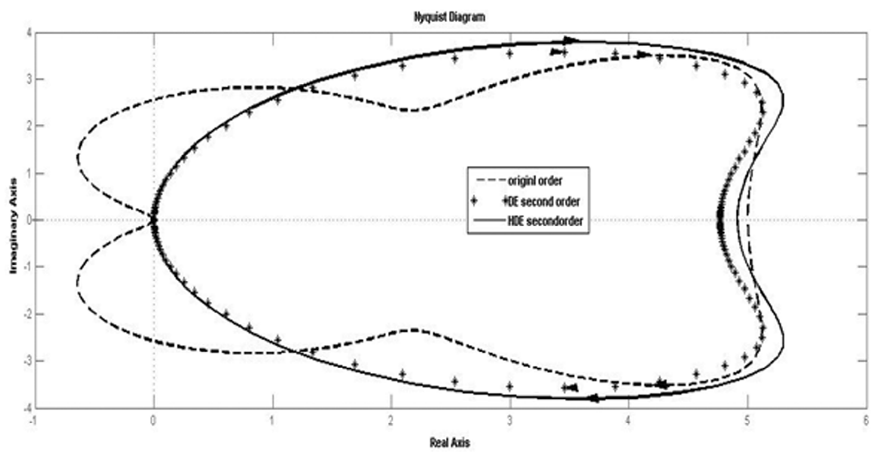

Figure 3(a): Comparision of nyquist of Example 2 with given order and reduced order i.e $G^{*}{}_{2}(s)$ by using HDE.

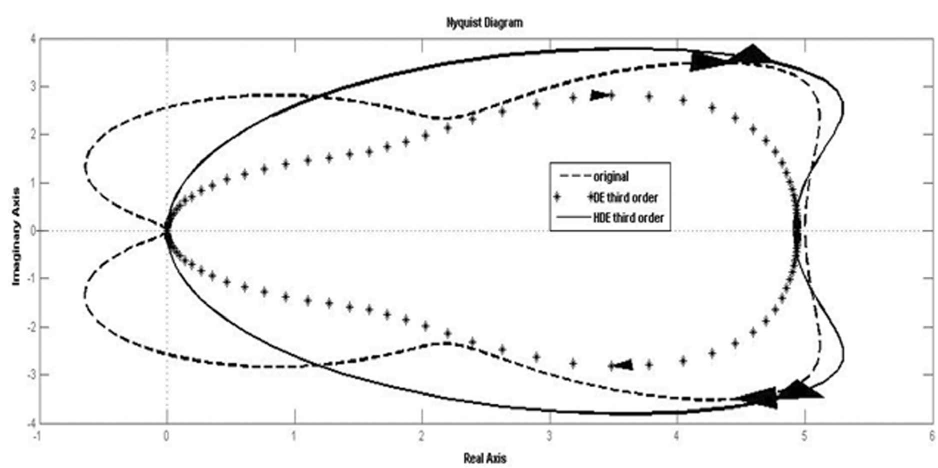

Figure 3(b): Comparision of nyquist responses of Example 2 with given order and reduced order i.e $G^{*}{}_{3}(s)$ by using HDE.

better value. Hence the frequency response of unstable system is as shown in fig 3a, fig 3b. Table III represents the different performance index values among the different performance or cost function ITAE gives the optimum value by using HDE whereas in DE optimum value obtained with ITSE the better optimal value can be obtained by HDE with performance index ITAE.

5. CONCLUSION

The DE algorithm gives the optimum value with ISE but in HDE the optimum value is better optimum value than the DE by using different performance indexes like IAE, ITSE, and ITAE. DE algorithm is used to approximate the linear systems (model order reduction).gives the optimum. But whereas HDE gives the better cost function value with different performance index. In example 1 ITSE gives the optimum value for both evolutionary algorithms whereas ISE gives better according to[1],but in second example the best optimum can be obtained by ITAE . From the results of example1 and example 2 HDE gives the robust values than DE so HDE is robust computational technique for optimal approximations of linear systems.

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