I J C T A, 9(2-A), 2016, pp. 779-790

Design of Discrete-Time High Gain Adaptive Observer for Nonlinear Systems with Application to a Bioreactor Process

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ABSTRACT

In this paper, the problem of adaptive observer for a class of nonlinear discrete-time systems is considered. A high-gain observer is used to estimate the unknown parameters and state variables. The formulation of the discrete adaptive observer design problem is realized via a high gain adaptive observer working with a continuous-time using the method of Euler approximation. The proposed algorithm is developed to guarantees the convergence of parameters and state variables to their true value. Based in a new formulation of the high-gain observer, sufficient conditions and assumptions to ensure asymptotic convergence are established. The algorithm is applied to highly nonlinear system of a typical bioreactor and simulations results are given to show the effectiveness of the proposed observer.

Keywords: Discrete-time nonlinear systems; High-gain adaptive observer; Euler discretization; A typical bioreactor.

1. INTRODUCTION

Many designs of discrete-time nonlinear observers have been paid a lot attention in recent years [1]-[7]. Our problem concerns the design of constrained state and parameters estimators for nonlinear discrete-time systems. The problem of simultaneous parameter and state estimation has attracted the attention of several researchers [8]-[10]. These observers are used in many practical problems such as fault detection, signal transmission or control.

Motivated by this interest a lot approaches have been proposed to simultaneously estimate the state vector and identify the unknown parameters, with one of them consisting of using the Extended Kalman Filter (EKF) [11]-[14], which was a great success because of their easy of implementation despite the complexity of nonlinear systems. More recently, an interesting study of EKF method, used for the joint parameter and state estimation of nonlinear discrete-time systems, for this case, a sufficient condition of the EKF for noisy systems used as a local asymptotic observer for the deterministic case have been developed by Song *et al*. [14]. However, this design is based on linearized approaches of the nonlinear systems, where robustness and the proof of convergence proprieties are difficult to prove.

Note that, in the state estimation, very few results for the class of nonlinear systems have been established, in which, the observer design problem has been solved via the (LMIs) technique involving the Lyapunov or Riccati equations. More specifically, the observer gain is obtained by solving the problem of linear matrix inequality (LMIs) [15]-[17]. We can mention also the technique proposed in [18] for the class of nonlinear systems with sampled measurements, it consists in using an impulsive observer. The observer convergence

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analysis as well as the determination of its gain is easily solved by standard convex algorithms using (LMIs) tools. However, the feasibility of such problem of LMI is considered generally not known a priori.

Another observer design approach has been devoted to estimate the state and the unknown parameters for a class of nonlinear systems. Indeed, adaptive high gain observers have been proposed by many researchers [19]-[24].

We can mention also, in [25], the authors proposed a discrete implementation of high gain observers to compensate the tracking errors due to the flexibility of robot manipulators. In the same theme, Farza *et al.* [26] proposed a high gain adaptive observer for particular classes of continuous-time systems with discrete-time outputs measurements, this observer is presented under the form of impulsive system. These methods have been established assuming the existence of some Lyapunov functions satisfying particular conditions.

The objective of this paper is the design of an adaptive high gain nonlinear discrete observer allowing estimating conjointly the unknown parameters and the immeasurable state vector of a dynamic systems described by nonlinear discrete-time state space mathematical model. This observer is obtained after descritization by using the Euler approximation of the continuous-time high gain adaptive observer proposed by Farza *et al.* [26].

The paper is organized as follows:

Section 2, is devoted to the problem statement. In section 3, the design of the continuous-time high gain adaptive observer proposed by Farza *et al.* [26] is briefly reminded. The main contribution of this paper is given in section 4, in which, a new-time discrete high gain adaptive observer, which can estimate conjointly the unknown parameters and the state variables of the considered nonlinear system is studied in this paragraph. In section 5, we propose a simulations result involving a typical bioreactor in order to show the effectiveness and the validity of our results. Finally, some conclusions are given in section 6.

2. PROBLEM FORMULATION

Consider the class of nonlinear system described by the following set of equations:

$$\begin{cases} \dot{x}(t) = Ax(t) + \Phi(u(t), x(t), \rho(t)) \\ y(t) = Cx(t) = x_1(t) \end{cases} \tag{1}$$

where $x(t) = [x_1(t) \cdots x_n(t)]^T \in \Re^n$ and $\rho(t) = [\rho_1(t) \cdots \rho_m(t)]^T \in \Re^m$ respectively denote the state and the unknown parameters, $u(t) \in \Re$ is the input of the system, $y(t) \in \Re$ denotes the system output and Φ is a nonlinear function with triangular structure in the state, i.e.

$$\Phi_i(u(t), x(t), \rho(t)) = \begin{pmatrix} \Phi_1(u(t), x_1(t), \rho(t)) \\ \Phi_1(u(t), x_1(t), x_2(t), \rho(t)) \\ \vdots \\ \Phi_i(u(t), x_1(t), \ldots, x_i(t), \rho(t)) \end{pmatrix}, \; i = 1, \ldots, n \tag{2}$$

A is the dynamic matrix of the system and *C* is vector, which is defined by:

$$A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}, \qquad \text{C} = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix} \tag{3}$$

where I_{n-1} is the $(n-1) \times (n-1)$ identity matrix.

The objective of this paper is the design of a high gain nonlinear discrete observer using Euler approximation in order to build the joint state and unknown parameters.

The nonlinear systems considered here of the form:

$$\begin{cases} x(k+1) = (I_n + T_e A)x(k) + T_e \Phi(u(k), x(k), \rho(k)) \\ y(k) = Cx(k) = x_1(k) \end{cases} \tag{4}$$

where $x(k) = [x_{1,k} \cdots x_{n,k}]^T \in \Re^n$ and $\rho(k) = [\rho_{1,k} \cdots \rho_{m,k}]^T \in \Re^m$ respectively denote the state and the unknown parameters at some time step k, $u \in \Re$ is the control input of the system at some time step k and T_e is the sampling time of the discrete-time system (4).

The choice of T_e is such that discrete time nonlinear system as stated above provides a good approximation of the corresponding continuous-time (1).

3. NONLINEAR CONTINUOUS-TIME HIGH-GAIN OBSERVER DESIGN

This part briefly summaries the high-gain nonlinear observer for a class of continuous-time systems, which is studied by Farza *et al.* [26].

The sampled output adaptive observer has the following form:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + \Phi(u(t), \hat{x}(t), \hat{\rho}(t)) - \theta\Delta_\theta^{-1}(S^{-1} + \Upsilon(t)P(t)\Upsilon^T(t))C^T C\tilde{x}(t) \\ \dot{\hat{\rho}}(t) = -\theta\Omega_\theta^{-1} P(t)\Upsilon^T(t)C^T C\tilde{x}(t) \\ \dot{\Upsilon}(t) = \theta(A - \theta S^{-1}C^T C)\Upsilon(t) + \theta\Phi(u(t), \hat{x}(t), \hat{\rho}(t)) \quad with \quad \Upsilon(0) = 0 \\ \dot{P}(t) = -\theta P(t)\Upsilon^T(t)C^T C\Upsilon(t) + \theta P(t) \quad with \quad P(0) = P^T(0) > 0 \end{cases} \tag{5}$$

where $\hat{x}(t) = (\hat{x}_1(t) \ldots \hat{x}_n(t))^T \in \Re^n$ and $\hat{\rho}(t) = (\hat{\rho}_1(t) \ldots \hat{\rho}_m(t))^T \in \Re^m$ respectively denote the state and parameter estimates, $\tilde{x}(t) = x(t) - \hat{x}(t)$ and $\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t)$ are respectively the state observation error and the parameter estimation error.

Let us introduce some useful notations related to the high-gain observer design:

- The diagonal matrix Δ_θ is defined by:

$$\Delta_\theta = diag\begin{bmatrix} 1 & \frac{1}{\theta} & \cdots & \frac{1}{\theta^{n-1}} \end{bmatrix} \tag{6}$$

  where θ is a positive scalar. One can easily check that the following identities hold.

$$\Delta_\theta A \Delta_\theta^{-1} = \theta A, \qquad C\Delta_\theta^{-1} = C \tag{7}$$

- The unique solution S of the algebraic Lyapunov equation:

$$S + A^T S + SA - C^T C = 0 \tag{8}$$

• Now, let Ω_θ be the following $m \times m$ diagonal matrix:

$$\Omega_\theta = diag\begin{bmatrix} \frac{1}{\theta^{\upsilon_1}} & \frac{1}{\theta^{\upsilon_2}} & \cdots & \frac{1}{\theta^{\upsilon_m - 1}} \end{bmatrix} \tag{9}$$

The m characteristic indices v_j associated to the m unknown parameters $\rho_j(k)$ are defined as follows: each component $\rho_j(k)$ has a characteristic index denoted by v_j which is equal to the smallest positive integer i such that $\partial \dot{x}_i(k)/\partial \rho_j(k) \neq 0$. That is, one has $\partial \dot{x}_i(k)/\partial \rho_j(k) = 0$ for $i = 1,\ldots,v_j - 1$ and $\partial \dot{x}_{v_j}(k)/\partial \rho_j(k) \neq 0$.

4. PREDICTIVE FORM OF DISCRETE HIGH GAIN OBSERVER

Here, we introduce our main contribution which consists in a new adaptive high gain observer. In follows, we present the proposed observer design using Euler approximation methods under some assumptions.

$$\begin{cases} \hat{x}(k+1) = (I_n + T_e A)\hat{x}(k) + T_e \Phi(u(k), \hat{x}(k), \hat{\rho}(k)) - T_e \theta \Delta_\theta^{-1}(S^{-1} + \Upsilon(k)P(k)\Upsilon(k)^T)C^T C\tilde{x}(k) \\ \hat{\rho}(k+1) = \hat{\rho}(k) - T_e \theta \Omega_\theta^{-1} P(k)\Upsilon(k)^T C^T C\tilde{x}(k) \\ \Upsilon(k+1) = (I_n + T_e \theta(A - S^{-1}C^T C))\Upsilon(k) + T_e \theta \Phi(u(k), \hat{x}(k), \hat{\rho}(k)) \\ P(k+1) = (I_n - T_e \theta P(k)\Upsilon(k)^T C^T C\Upsilon(k) + T_e \theta)P(k) \end{cases} \tag{10}$$

where $\hat{x}(k) = \left(\hat{x}_{1,k} \ldots \hat{x}_{n,k}\right)^T \in \Re^n$ and $\hat{\rho}(k) = \left(\hat{\rho}_{1,k} \ldots \hat{\rho}_{m,k}\right)^T \in \Re^m$ respectively denote the state and parameter estimates, $\tilde{x}(k) = x(k) - \hat{x}(k)$ and $\tilde{\rho}(k) = \rho(k) - \hat{\rho}(k)$ are respectively the state observation error and the parameter estimation error, A, C, Δ_θ, S, Ω_θ and $\Phi(u(k), \hat{x}(k), \hat{\rho}(k))$ are respectively given by (3), (6), (7), (8), (9) and (2) and is a positive scalar.

The nonlinear observer design requires some assumptions, namely

A1. The state $x(k)$ the control $u(k)$ and the unknown parameters $\rho(k)$ are bounded, i.e. $x(k) \in X$, $u(k) \in U$ and $\rho(k) \in \Omega$ where $X \subset \Re^n$, $U \subset \Re$ and $\Omega \subset \Re^m$ are compacts sets.

A2. The discrete nonlinear function $\Phi : \Re^n \times \Re^p \to \Re^n$, moreover, it is Lipchitz with respect to $x(k)$ and $\rho(k)$ and uniformly in $u(k)$, for all $(u(k), x(k), \rho(k)) \in U \times X \times \Omega$, its Lipchitz constant is denoted L_d.

A3. The function $\Phi_{vj}^j(u, x, \rho), j = 1,\ldots,m,$ satisfy the following condition:

$$\forall (u, \hat{x}) \in U \times X, \forall \left(\hat{\rho}, \rho\right) \in \Omega^2 : \left\| \Phi_{vj}^j(u, \hat{x}, \hat{\rho}) - \Phi_{vj}^j(u, \hat{x}, \rho) \right\| \leq T_e . \upsilon \sqrt{\lambda_M(P)/(n\lambda_M(S))} \tag{11}$$

where $\| . \|$ is the induced 2-norm, T_e is a sampling time, S and P are respectively given by (5) and (8) and v is a positive scalar satisfying $v < 1$.

Based on (11), the Lipchitz constant of the Euler approximation is $L_d = T_e.v$. Again, we assume $L_d < 1$. This is even less restrictive than in a continuous observer, because here T_e directly multiplies v and can be chosen sufficiently small.

A4. The input u is such that for any trajectory of system (4) starting from $(\hat{x}(0), \hat{\rho}(0)) \in X \times \Omega$, the matrix $C\Upsilon(t)$ is persistently exciting i.e.

$$\exists \delta_1, \delta_2, \exists T > 0; \forall t = k.T_e \geq 0$$

$$\delta_1 I_m \leq \int_{t=k.T_e}^{k.T_e+T} \Upsilon^T(\tau) C^T C \Upsilon(\tau) d(\tau) \leq \delta_2 I_m \tag{12}$$

5. SIMULATION EXAMPLE

In this section, an example concerning a typical bioreactor is provided to show performances of the proposed high gain observer design. Consider a simple microbial culture which involves a single biomass of concentration $x_2(t)$ growing on a single substrate of concentration $x_1(t)$. The bioprocess is supposed to be continuous with a scalar dilution rate $D(t)$ and an input substrate concentration S_{in} [26].

5.1. System description

The mathematical dynamical model of this process is hence constituted by the following two mass balance equations:

$$\begin{cases} \dot{x}_1(t) = -\mu^* x_1(t) x_2(t) / (k_c x_2(t) + x_1(t)) + D(t)(S_{in} - x_1(t)) \\ \dot{x}_2(t) = \mu^* x_1(t) x_2(t) / (k_c x_2(t) + x_1(t)) - D(t) x_2(t) \\ y(t) = x_1(t) \end{cases} \tag{13}$$

where $x_1(t)$ and $x_2(t)$ inhibitor respectively denote the concentration of the biomass and the substrate, μ^* and k_c are the Contois law parameters, S_{in} is the input substrate concentration and $D(t)$ is the dilution rate.

The objective of this paper is to estimate $x_2(t)$ together with the Contois law parameters μ^*, k_c and the input substrate concentration S_{in} at discrete-time T_e.

System (13) has been already considered in Farza *et al.* [26], where the transformation:

$$f : (x_1(t), x_2(t))^T \in X$$

$$\downarrow$$

$$Z : \left(z_1(t) = x_1(t), z_2(t) = \frac{-\mu^* x_1(t) x_2(t)}{(k_c x_2(t) + x_1(t))} \right)^T \tag{14}$$

where f is a diffeomorphism from X,

The system (13) can be input under the following form:

$$\begin{cases} \dot{z}_1(t) = z_2(t) + D(t)\left(\rho_1(t) - z_1(t)\right) \\ \dot{z}_2(t) = -\rho_3(z_2(t)/z_1(t))^2(z_2(t) + D(t)\left(\rho_1(t) - z_1(t)\right)) \\ +\rho_2(t)z_2(t)(1+\rho_3(t)(z_2(t)/z_1(t)))^2 - D(t)z_2(t)\left(1+\rho_3(t)(z_2(t)/z_1(t))^2\right) \\ y(t) = z_1(t) \end{cases} \tag{15}$$

where $\rho_1 = S_{in}$, $\rho_2 = \mu^*$ and $\rho_3 = k_c/\mu^*$.

After the transformation, the system (15) is described under the form:

$$\begin{cases} \dot{z}(t) = Az(t) + \Phi(u(t), z(t), \rho(t)) \\ y(t) = Cz(t) = z_1(t) \end{cases} \tag{16}$$

with

$$\Phi(u(t), z(t), \rho(t)) \overset{\Delta}{=} \begin{pmatrix} D(t)\left(\rho_1(t) - z_1(t)\right) \\ -\rho_3(t)(z_2(t)/z_1(t))^2(z_2(t) + D(t)\left(\rho_1(t) - z_1(t)\right)) \\ +\rho_2(t)z_2(t)(1+\rho_3(t)(z_2(t)/z_1(t)))^2 - D(t)z_2(t)\left(1+\rho_3(t)(z_2(t)/z_1(t))^2\right) \end{pmatrix} \tag{17}$$

where

$$\Delta(t) = \theta\Delta_\theta^{-1}(S^{-1} + \Upsilon(t)P(t)\Upsilon(t)^T)C^TC$$
$$\Omega(t) = \theta\Omega_\theta^{-1}P(t)\Upsilon(t)^TC^TC \tag{18}$$

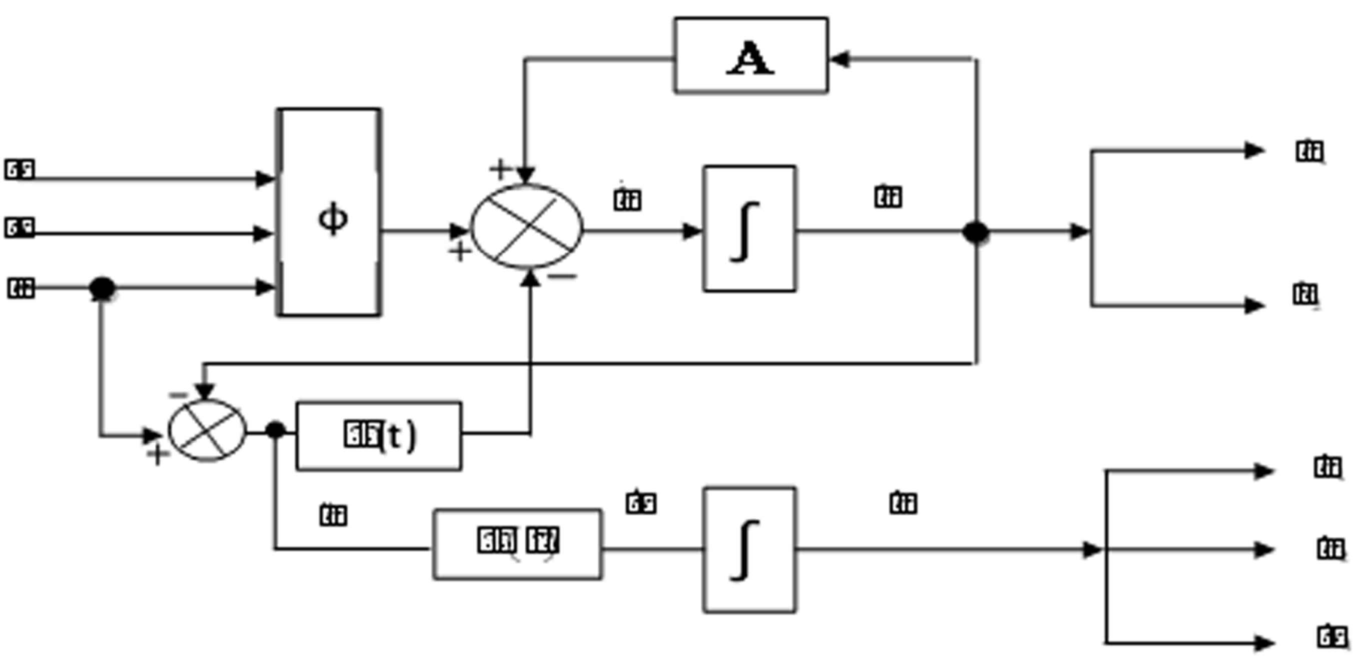

Figure 1: Block diagram of continuous-time high gain observer design

5.2. Discrete-time system

Consider the discrete-time model of the typical bioreactor, obtained by Euler descritization, we have:

$$\begin{cases} z_1(k+1) = z_1(k) + T_e z_2(k) + T_e D(k)\left(\rho_1(k) - z_1(k)\right) \\ z_2(k+1) = z_2(k) - T_e \rho_3(k)(z_2(k)/z_1(k))^2\left(z_2(k) + D(k)\left(\rho_1(k) - z_1(k)\right)\right) \\ +T_e \rho_2(k) z_2(k)(1 + \rho_3(k)(z_2(k)/z_1(k)))^2 - T_e D(k) z_2(k)\left(1 + \rho_3(k)(z_2(k)/z_1(k))\right) \\ y(k) = z_1(k) \end{cases} \tag{19}$$

The model (19) can be written as follows:

$$\begin{cases} z(k+1) = (I_n + T_e A) z(k) + T_e \Phi(u(k), z(k), \rho(k)) \\ y(k) = Cz(k) = z_1(k) \end{cases} \tag{20}$$

where

$$z(k+1) = \begin{pmatrix} z_1(k+1) \\ z_2(k+1) \end{pmatrix},\ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \text{C}=\begin{bmatrix} 1 & 0 \end{bmatrix} \tag{21}$$

and

$$\Phi(u(k), z(k), \rho(k)) \overset{\Delta}{=} \begin{pmatrix} D(k)\left(\rho_1(k) - z_1(k)\right) \\ \\ -\rho_3(k)(z_2(k)/z_1(k))^2(z_2(k) + D(k)\left(\rho_1(k) - z_1(k)\right)) \\ +\rho_2(k) z_2(k)(1 + \rho_3(k)(z_2(k)/z_1(k)))^2 \\ -D(k) z_2(k)\left(1 + \rho_3(k)(z_2(k)/z_1(k))^2\right) \end{pmatrix} \tag{22}$$

where

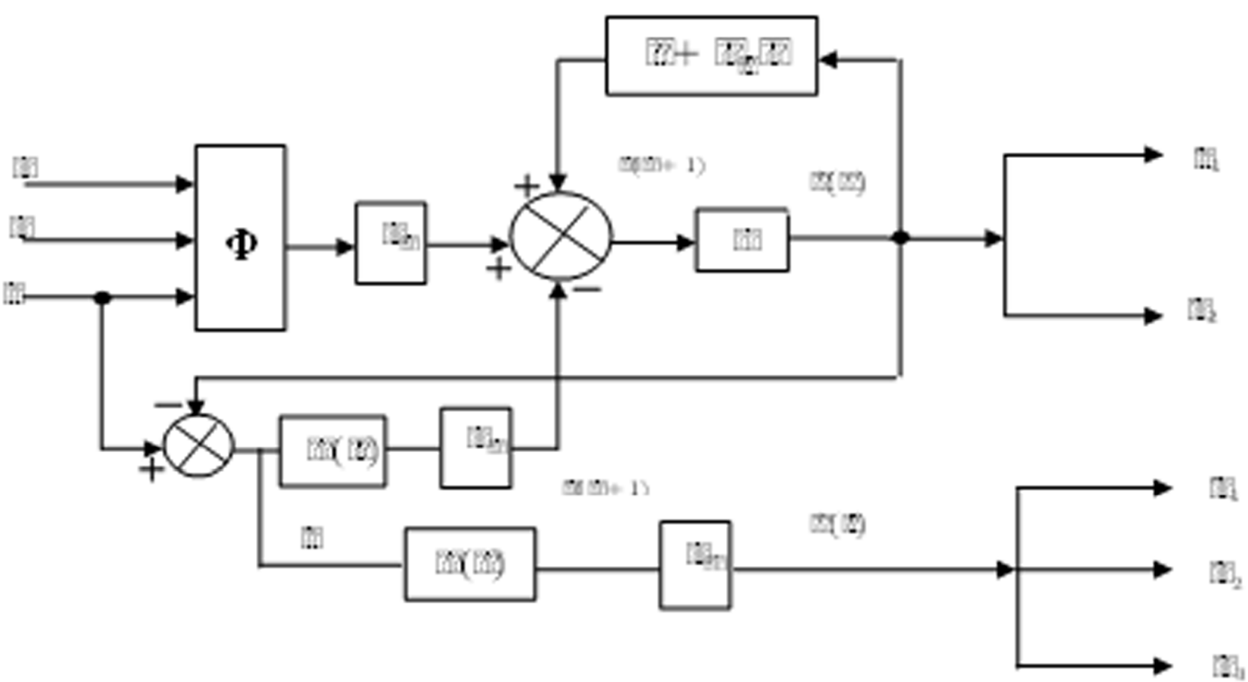

Figure 2: Block diagram of discrete-time high gain observer design

$$\Delta(k) = \theta\Delta_\theta^{-1}(S^{-1} + \Upsilon(k)P(k)\Upsilon(k)^T)C^T C$$
$$\Omega(k) = \theta\Omega_\theta^{-1}P(k)\Upsilon(k)^T C^T C \quad (23)$$

5.3. Simulation results

The observer (10) is used to estimate system states $z_1(k)$ and $z_2(k)$ and the unknown parameters $\rho_1(k)$, $\rho_2(k)$ and $\rho_3(k)$ in the typical bioreactor.

The data concerning this example are as follows:

- One can prove that the values of the characteristic indices associated to the unknown parameters are $v_1 = 1$ and $v_2 = v_3 = 2$ and hereafter the diagonal matrix Ω_θ is given by $\Omega_\theta = diag(1/\theta, 1/\theta^2, 1/\theta^2)$.
- The following values used in simulation are:

$$S_{in} = 20(g/L),\ \mu^* = 1.064(1/h) \text{ and } k_c = 4.39(g/g)$$

- The resulting values of the parameters are:

$$\rho_1 = 20(g/L),\ \rho_2 = 1.064(1/h) \text{ and } \rho_3 = 4.126h.$$

- The bioreactor was initialized as follows:

$$x_1(0) = 15(g/L),\ x_2(0) = 7.5(g/L)$$

- The observer was initialized with:

$$\hat{z}_1(0) = z_1(0),\ \hat{z}_2(0) = 1.5z_2(0),\ \hat{\rho}_1(0) = 0.5\rho_1 = 10(g/L) \text{ and } \hat{\rho}_2(0) = \hat{\rho}_3(0) = 0.$$

- All the entries of matrix $\Upsilon(0)$ were set to zero and $P(0)$ were set to 3×3 identity matrix.
- The simulations results obtained with $\theta = 10$ and a sampling time $T_e = 0.01h$. Indeed, the choice of is such that the discrete-time system as stated above provides a good approximation of the corresponding continuous-time dynamics of the bioreactor.
- The input $u(k) = D(k) = 0.3 + 0.5\sin(k)$

The evolution of the state variable $z_1(k)$ is show in Figure 3. Similarly, the unknown trajectory of $z_2(k)$ is shown in Figure 4. Figures 5, 6 and Figure 7 show the estimates of the three unknown parameters ρ_1, ρ_2 and ρ_3 and the parameter estimation errors $\tilde{\rho}_{i,i=1,2,3}(k) = \rho_i - \hat{\rho}_i(k)$.

Based on simulation results, one can conclude the following:

From Figure 3 and Figure 4, it is clearly seen that the state variables converges into the state estimate variables as fast as required and the estimation errors remain the neighbor of zero. The Figure 5, Figure 6 and Figure 7, it is clear that the parameter estimates converge to their true values and it is shown that the parameter estimation errors converge to a neighborhood around zero.

In the next, to illustrate the limits of the high gain observer, we suppose the estimation of the state variables $z_1(k)$ and $z_2(k)$ and the unknown parameters ρ_1, ρ_2 and ρ_3 by using three different values from the parameter of synthesis θ = 2, θ = 5 and θ = 10.

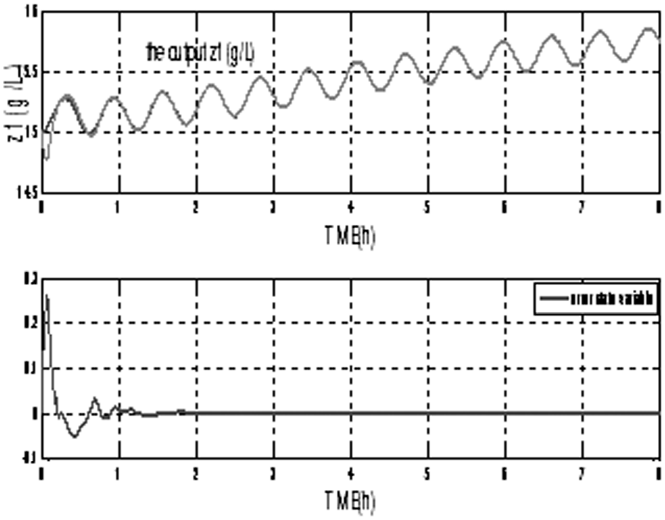

Figure 3: Evolution of the state variables $z_1(k)$, their estimates $\hat{z}_1(k)$ and their error

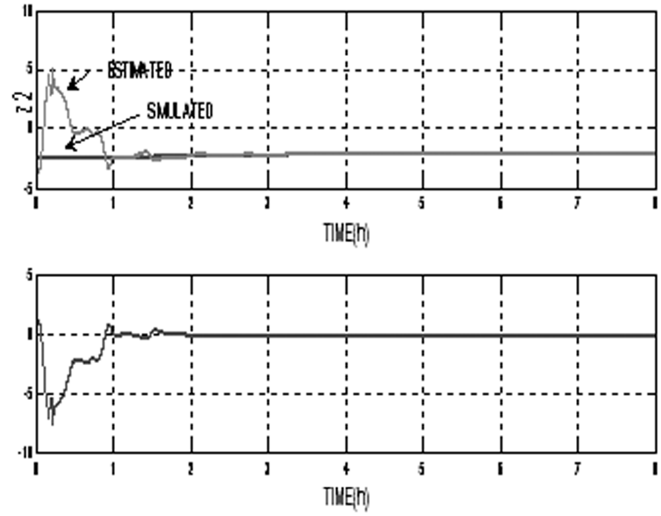

Figure 4: Evolution of the state variables $z_2(k)$, their estimates $\hat{z}_2(k)$ and their error

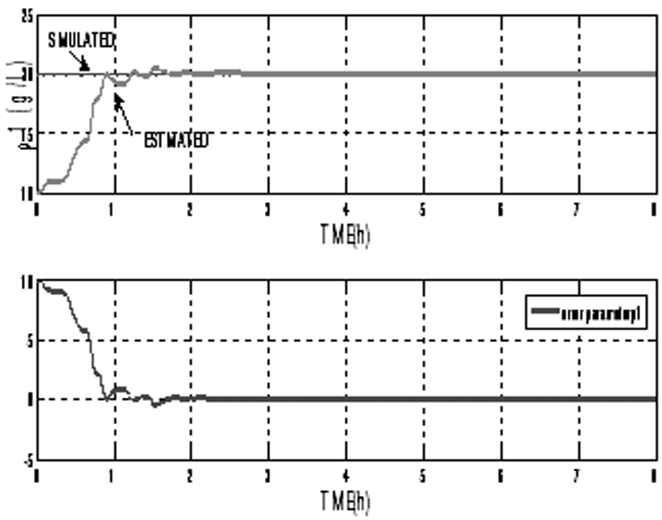

Figure 5: Estimation of the unknown parameter ρ_1, their estimates $\hat{\rho}_1(k)$ and their error

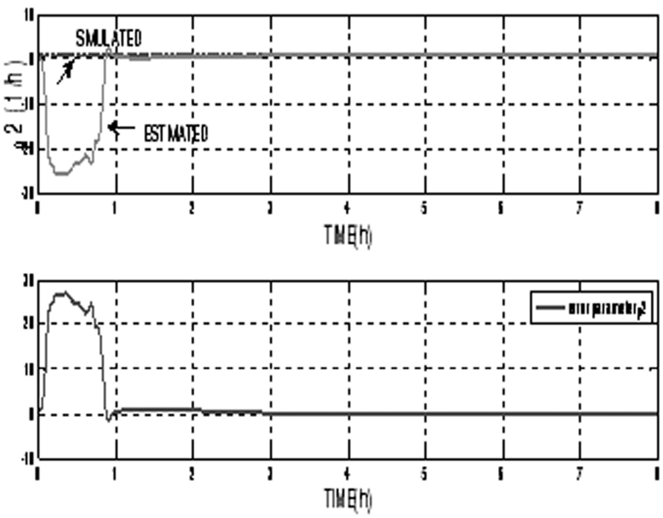

Figure 6: Estimation of the unknown parameter ρ_2, their estimates $\hat{\rho}_2(k)$ and their error

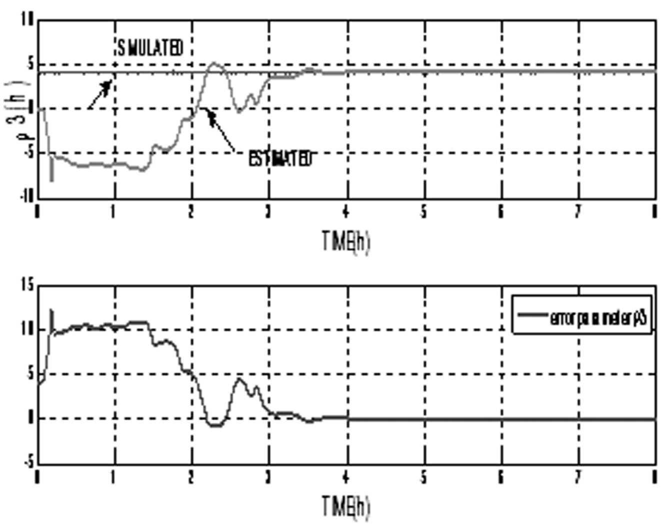

Figure 7: Estimation of the unknown parameter ρ_3, their estimates $\hat{\rho}_3(k)$ and their error

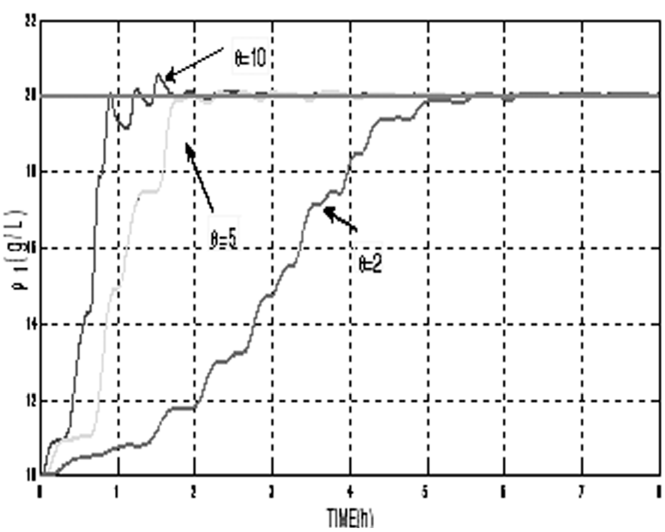

Figure 8: Evolution of the unknown parameter ρ_1, their estimate $\hat{\rho}_1(k)$ with θ = 2, θ = 5 and θ = 10

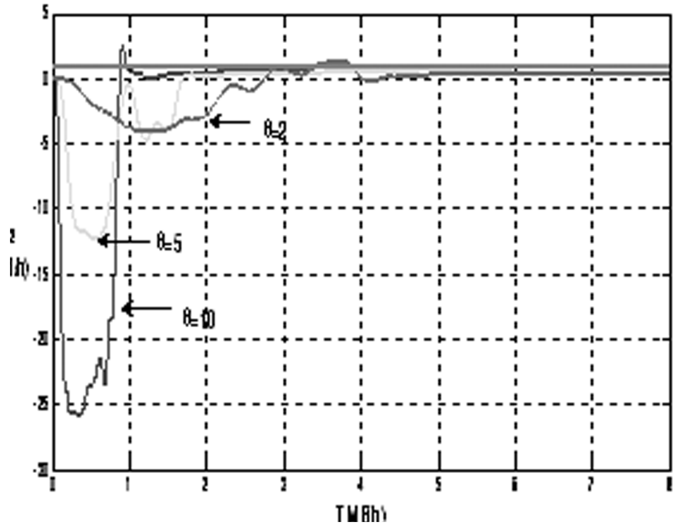

Figure 9: Evolution of the unknown parameter ρ_2, their estimate $\hat{\rho}_2(k)$ with θ = 2, θ = 5 and θ = 10

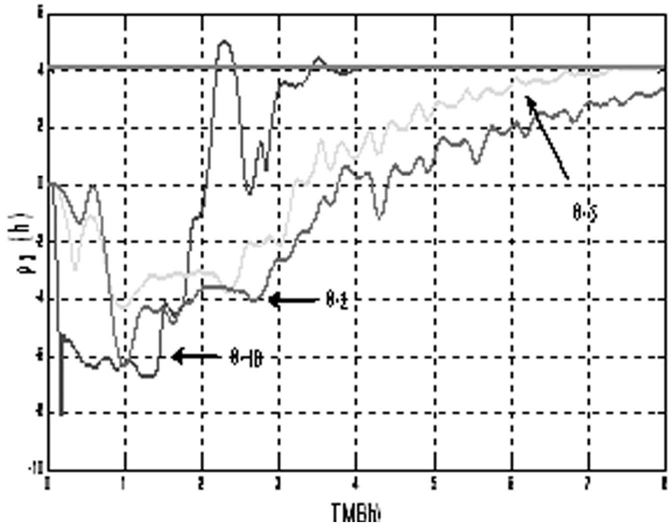

Figure10: Evolution of the unknown parameter ρ_3, their estimate $\hat{\rho}_3(k)$ with θ = 2, θ = 5 and θ = 10

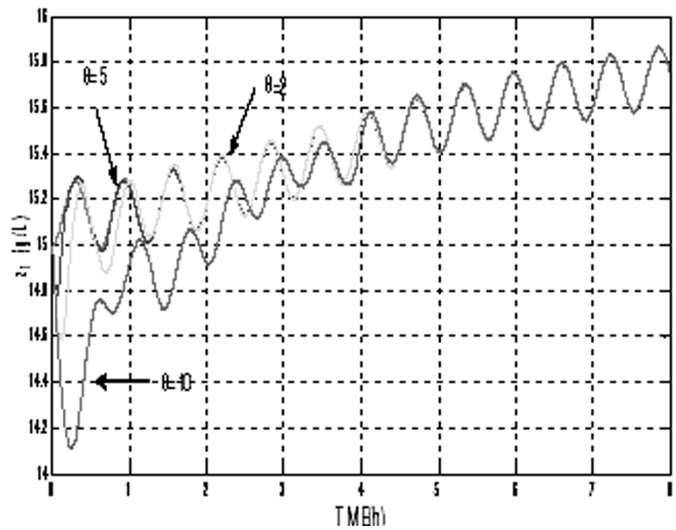

Figure 11: Evolution of the state variables $z_1(k)$, their estimate $\hat{z}_1(k)$ with θ = 2, θ = 5 and θ = 10

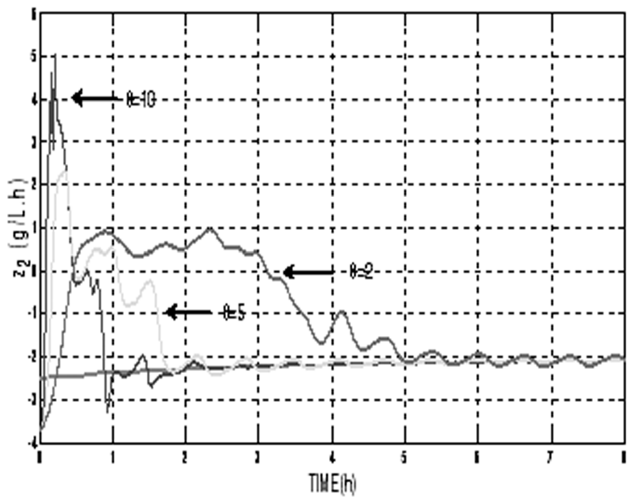

Figure 12: Evolution of the state variables $z_2(k)$ their estimate $\hat{z}_2(k)$ with θ = 2, θ = 5 and θ = 10

Figures 8, 9 and Figure 10 show the estimates of the three unknown parameters ρ_1, ρ_2 and ρ_3 for both different values of θ. The evolution of the state variable $z_1(k)$ is show in Figure 11. Similarly, the unknown trajectory of $z_2(k)$ is shown in Figure 12.

It easy to notice that for a low value of (θ = 2) allows to obtain a good estimation, but, these estimates vary slowly. In order to put forwards the strategy of choosing the design parameter θ, we have reported results with other value of θ, (θ = 5). The estimation of the state variables and the unknown parameters converge to their values as fast to the estimation with (θ = 2). Then, the proposed high gain observer (10) with a very important value of θ, (θ = 10), provides an accurate estimation of the state variables and the unknown parameters with a relatively important and quickly convergence.

6. CONCLUSION

In this paper, an adaptive high gain observer for a class of nonlinear discrete-time systems was addressed to estimate the states and the unknown parameters simultaneously. The formulation of the discrete observer design problem is realized after discretization using the Euler approximation of the continuous-time observer introduced by Farza *et al.* [26]. The performance of the proposed observer has been demonstrated and discussed in simulation through a typical bioreactor model. The satisfactory performance with smaller error and faster estimation can be obtained by the adequate choice of the design parameter of synthesis.

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