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Peristaltic Transport with Heat Transfer for Micropolar Fluid under the Effect of Hall Currents

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ABSTRACT

The effect of Hall current on the peristaltic transport with heat transfer for a micropolar fluid in a symmetric channel will be investigated. The equations of motion for micropolar fluids are introduced. The system of equations which describe this problem are transformed by using the transformations between a laboratory and a fixed frames. The equations of motion are solved analytically for low Reynolds number and long wavelength approximation.

The value of the microrotation at the boundary is taken into account, it linked with the rotation of the velocity by way of a coefficient that characterize the microrotation on the solid surfaces (varies in the range $0 \le \alpha \le 1$). The graphs of the longitudinal velocity, the microrotation velocity and temperature distribution are plotted and discussed for the physical parameters of the problem.

1. INTRODUCTION

The word peristalsis stems from the Greek word peristalikos. Peristalsis is defined as a wave of relaxation contraction (expansion) imparted by the walls of a flexible conduit, there by pumping the enclosed material, it is a nature's way of moving the content within hollow muscular structures by successive contraction of their muscular fibers [13, 14, 20, 22]. Peristalsis is now well-known to the physiologists to be one of the major mechanisms for fluid transport in many biological systems, it results physiologically from neuron muscular properties of the tubular smooth muscles [10, 15, 16, 28].

The peristaltic transport may be involved in many biological organs, e.g., swelling food through the esophagus, movement of chime in the gastrointestinal tract, urine transport from the kidney to the bladder through the ureter, transport of spermatozoa in the ducts efferents of male reproduction tract and in the cervical canal of the female, movement of ova in the fallopian tub, vasomotion of small blood vessels such as venules and capillaries as well as blood flow in arteries, and in many other glandular ducts [10, 11, 20, 22, 28]. There are also many industrial applications of the peristaltic transport like, blood pumps in heart lung machine, transport of corrosive fluid, where the contact of the fluid with the machinery parts is prohibited [22].

The theory of microfluid introduced by Eringen, deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and micro-motions of the fluid elements. A subclass of these fluids is the micropolar fluids, like blood, liquid crystals and polymers, can support couple stresses, body couples and exhibit the micro-rotational and micro-inertial effects, Eringen is the first one who put the theory to find a mathematical model of the micropolar fluids [12]. These equations are a generalization of the (Newtonian) Navier-Stokes equations and deal with three fields, velocity vector **V**, the pressure of the fluid P and microrotation vector **w**, together with some viscosity parameters and material constants to describe the behavior of the fluid [6, 8].

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Some researchers attempted study the peristaltic flow problems concerning the micropolar fluids without studying the Hall effect, [2, 9, 17, 18, 21, 23, 24, 27], and others studied the Hall and heat transfer effects with peristalsis or not for different fluids, [1, 3, 4, 5, 7, 19, 25, 26]. In the present study we investigated the effect of Hall current on peristaltic transport with heat transfer for a micropolar fluid in a symmetric channel taking the value of the microrotation at the wall into account.

2. BASIC EQUATIONS

The system of basic equations which describe the motion of a micropolar fluid with Hall current, heat and mass transfer can be written as

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \tag{1}$$

Conservation of liner momentum

$$(\lambda_v + 2\mu_v + k_v)\nabla(\nabla \cdot \mathbf{V}) - (\mu_v + k_v)\nabla \times \nabla \times \mathbf{V} + k_v \nabla \times \mathbf{w} - \nabla P + J \wedge B = \rho \dot{\mathbf{V}}, \tag{2}$$

Conservation of angular momentum

$$(\alpha_v + \beta_v + \gamma_v)\nabla\nabla \cdot \mathbf{w} - \gamma_v \nabla \times \nabla \times \mathbf{w} + k_v \nabla \times \mathbf{V} - 2k_v \mathbf{w} + \rho l = \rho j \dot{\mathbf{w}}, \tag{3}$$

Heat equation

$$k\nabla^2 T + \tau \cdot \nabla \mathbf{V} = \rho C_p \dot{T}, \tag{4}$$

with

$$(\dot{\mathbf{V}}) = \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)(\mathbf{V})\right],$$

$$J = \sigma\left[\mathbf{V} \wedge B - \frac{1}{n_e} J \wedge B\right],$$

$$\tau = (\tau_{ij}) = 2(\mu_v + k_v)e_{ij}, \qquad e_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right),$$

where $\mathbf{V}$ is the velocity vector of the fluid, J is the current density including the Hall effect, σ is the electric conductivity, e is the electric charge, n_e is the number density of electrons, $B = (0, 0, B_0)$ is the uniform magnetic field with magnetic flux density, $\mathbf{w} = (0, 0, w)$ is the microrotation vector, l is the body couple, ρ is the fluid density, j is the microinertia parameter, $\tau = (\tau_{ij})$ is the stress tensor, T is temperature, k is the thermal conductivity, and α_v, β_v, γ_v, λ_v, μ_v and k_v are the material parameters [12] (different viscosities that characterize the isotropic properties of the fluid), satisfy

$$2\mu_v + k_v \geq 0, \qquad k_v \geq 0, \qquad 3\alpha_v + \beta_v + \gamma_v \geq 0, \qquad \gamma_v \geq |\beta_v|. \tag{5}$$

3. MATHEMATICAL FORMULATION

Consider a cartesian coordinates in two-dimensions (X, Y) where X-axis is taken in motion direction while Y-axis is perpendicular on it and (U, V) are the velocity components in X and Y directions respectively, as shown in Fig. (1). Neglecting body couple, with solenoidal microrotation vector, then the equations of motion become

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\,, \tag{6}$$

$$(\mu_v+k_v)\,\nabla^2 U+k_v\,\frac{\partial w}{\partial Y}-\frac{\partial P}{\partial X}+\frac{\sigma B_0^2}{1+m^2}\,(mV-U)=\rho\left[\frac{\partial U}{\partial t}+U\,\frac{\partial U}{\partial X}+V\,\frac{\partial U}{\partial Y}\right], \tag{7}$$

$$(\mu_v+k_v)\,\nabla^2 V-k_v\,\frac{\partial w}{\partial X}-\frac{\partial P}{\partial Y}-\frac{\sigma B_0^2}{1+m^2}\,(mU+V)=\rho\left[\frac{\partial V}{\partial t}+U\,\frac{\partial V}{\partial X}+V\,\frac{\partial V}{\partial Y}\right], \tag{8}$$

$$\gamma_v\nabla^2 w+k_v\left(\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right)-2k_v w=\rho j\left[\frac{\partial w}{\partial t}+U\,\frac{\partial w}{\partial X}+V\,\frac{\partial w}{\partial Y}\right], \tag{9}$$

$$k\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)+(\mu_v+k_v)\left[\left(\frac{\partial U}{\partial X}\right)^2+\frac{1}{2}\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2+\left(\frac{\partial V}{\partial Y}\right)^2\right]=\rho c_p\left(\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right), \tag{10}$$

where $m=\frac{\sigma B_0}{e n_e}$ is the Hall parameter.

The geometry of the wall surface is defined as

$$\mathbf{Y}=\mathcal{H}=d+b\cos\frac{2\pi}{\lambda}\,(X-ct)\,, \tag{11}$$

where d is the half-width of the channel, b is the wave amplitude, λ is the wavelength, c is the velocity propagation, and t is the time.

The appropriate boundary conditions

$$\psi=0,\quad \frac{\partial^2\psi}{\partial Y^2}=0,\quad w=0,\quad T=T_0,\quad \text{at}\quad Y=0, \tag{12}$$

$$\psi=q,\quad w=\frac{-\alpha}{2}\left(\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}\right),\quad \frac{\partial\psi}{\partial Y}=0,\quad T=T_w,\quad \text{at}\quad Y=\mathcal{H}, \tag{13}$$

where ψ is the stream function ($U=\frac{\partial\psi}{\partial Y}$ and $V=-\frac{\partial\psi}{\partial X}$), q is the flux of flow, and according to [8] it is proposed to link the value of the microrotation value with the rotation of the velocity by way of a coefficient α,

$$\mathbf{w}\times\mathbf{n}=\frac{\alpha}{2}\,(\nabla\times\mathbf{V})\times\mathbf{n}\,, \tag{14}$$

where $\mathbf{n}$ is the normal unit vector on the boundary, ψ characterizes the microrotation on the solid surfaces and its value is evaluated by the relation

$$0\le\alpha\le\frac{\mu_v+k_v}{k_v}\,.$$

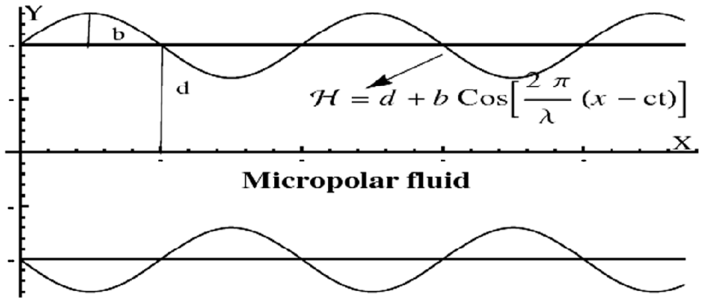

Figure 1: The Geometry of the Problem

Introducing a wave frame (x, y) moving with the velocity c away from the laboratory frame (X, Y), by the transformations

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad v = V \quad p(x) = P(X, t), \tag{15}$$

where u and v are the fluid velocity components and p is pressure in the wave frame of references.

Further, we introduce the following non-dimensional variables

$$\begin{gathered}\bar{x} = \frac{x}{\lambda}, \quad y = \frac{y}{d}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c\delta}, \quad h = \frac{\mathcal{H}}{d}, \quad \delta = \frac{d}{\lambda}, \\ \bar{\psi} = \frac{\psi}{cd}, \quad \bar{p} = \frac{d^2 p}{\mu\lambda c}, \quad R = \frac{\rho c d}{\mu}, \quad \bar{j} = \frac{j}{d^2}, \\ \bar{w} = \frac{wd}{c}, \quad M = \frac{\sigma B_0^2 d^2}{\mu}, \quad \Theta = \frac{T - T_0}{T_w - T_0}.\end{gathered} \tag{16}$$

After using transformation (15), dimensionless variables (16), after dropping bars and under the assumptions of long wave length ($\delta \ll 1$) and low Reynolds number, the equations (6-10) become

$$\left.\begin{aligned} \left(\frac{1}{1-N}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{N}{1-N}\right)\frac{\partial w}{\partial y} - \frac{\partial p}{\partial x} - \frac{M}{1+m^2}(u+1) = 0, \\ \frac{\partial p}{\partial y} = 0, \\ \left(\frac{2-N}{\beta^2}\right)\frac{\partial^2 w}{\partial y^2} - \frac{\partial u}{\partial y} - 2w = 0, \\ \frac{\partial^2 \Theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 = 0, \end{aligned}\right\} \tag{17}$$

where $N = \frac{k_v}{\mu_v + k_v}$ is the coupling number $(0 \le N \le 1)$, $\beta^2 = \frac{k_v d^2 (2\mu_v + k_v)}{\gamma_v(\mu_v + k_v)}$ is the micropolar parameter and $E_c = \frac{(\mu_v + k_v)c^2}{k(T_w - T_0)}$ is the Eckert number. By eliminating the pressure between the second and third equations in (17), the previous equations (in terms of stream function), where $H = \frac{M}{1+m^2}(1-N)$, will be

$$\psi_{yyyy} + N w_{yy} - H\psi_{yy} = 0, \tag{18}$$

$$\psi_{yy} - \left(\frac{2-N}{\beta^2}\right) w_{yy} + 2w = 0, \tag{19}$$

$$\frac{\partial^2 \Theta}{\partial y^2} + E_c(\psi_{yy})^2 = 0. \tag{20}$$

The channel wall equation will be

$$y = h = 1 + \phi \cos 2\pi x, \tag{21}$$

where $\phi = \frac{b}{d} < 1$ is the amplitude ratio.

Also, we can write down the non-dimensional boundary conditions in terms of the stream function as following

$$\psi = 0, \quad \psi_{yy} = 0, \quad w = 0, \quad \Theta = 0, \quad \text{at} \quad y = 0, \tag{22}$$

$$\psi = q, \quad w = \frac{-\alpha}{2}\psi_{yy}, \quad \psi_y = -1, \quad \Theta = 1, \quad \text{at} \quad y = h, \tag{23}$$

Introducing these conditions into equations (18, 19), the solution of the stream function and the microrotation velocity respectively are

$$\psi = A_0 \sinh(y\gamma_2) - \frac{A_1 \sinh(y\gamma_1) + A_2 y}{A_3}, \tag{24}$$

$$w = \frac{B_0 \sinh(y\gamma_2) - B_1 \sinh(y\gamma_1) + y(B_2 + B_3)}{N}, \tag{25}$$

$$\Theta = C_1(f_1(y) + f_2(y) + f_3(y) + f_4(y) + f_5(y)), \tag{26}$$

where

$$\gamma_1 = \frac{\sqrt{-\dfrac{\sqrt{8H(N-2)\beta^2 + (H(N-2)-(N+2)\beta^2)^2} - H(N-2) + (N+2)\beta^2}{N-2}}}{\sqrt{2}},$$

$$\gamma_2 = \frac{y\sqrt{\dfrac{\sqrt{8H(N-2)\beta^2 + (H(N-2)-(N+2)\beta^2)^2} - H(N-2) + (N+2)\beta^2}{N-2}}}{\sqrt{2}},$$

$A_i\,(i = 0, 1, 2, 3)$, $B_j\,(j = 0, 1, 2, 3)$, $C_k\,(k = 1, 2, ..., 12)$, $n_l\,(l = 1, 2, ..., 7)$ and $f_m(y)\,(m = 1, 2, ..., 5)$ are given in the appendix.

4. RESULTS AND DISCUSSION

In the present study, analytical solutions have been obtained for the problem of two dimensional peristaltic flow of a micropolar fluid in the presence of Hall effect, through a symmetric channel. The equations governing this motion have been solved under the conditions of low Reynolds number and long wave length, subject to a set of appropriate boundary conditions. The expressions of velocity, microrotation and temperature have been evaluated for different parameters and have been shown graphically through a set of figures.

The effects of coupling number N, micropolar parameter β and the Hall parameter m on the velocity profile are plotted and Shown in Figs (2-4). It is observed that the velocity decreases with increasing of N through a small distance behind the center and then this effect disappear and for all values of N as seen in Fig. (2) and the same behavior happen with the effect of β on the velocity profile as shown in Fig. (3).

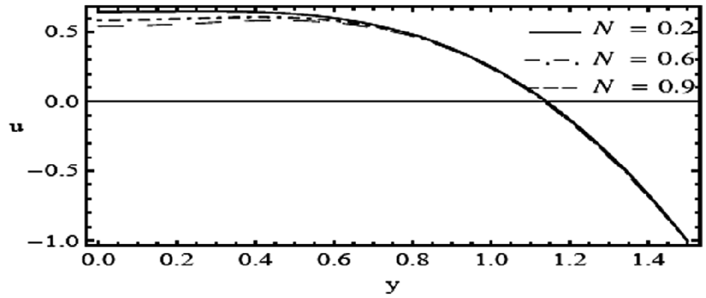

Figure 2: The Velocity Profile *u* Against *y* for Different Values of Coupling Number *N* when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $m = 1$, $M = 1.2$, $\beta = 3$, $q = 0.5$

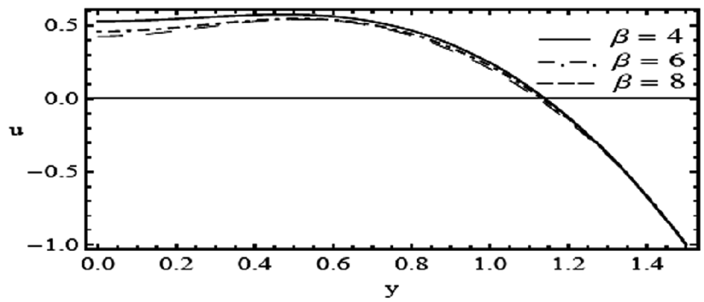

Figure 3: The Velocity Profile *u* Against *y* for Different Values of Micropolar Parameter β when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $m = 1$, $N = 0.5$, $M = 1.2$, $q = 0.5$

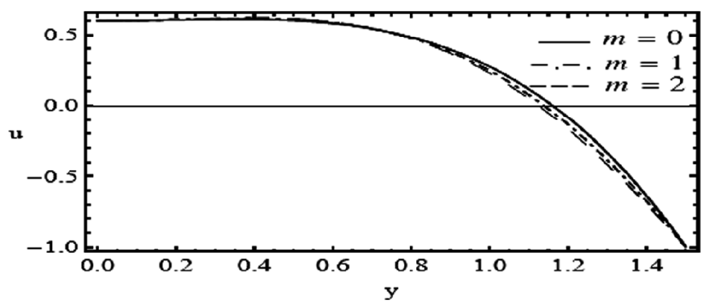

Figure 4: The Velocity Profile *u* Against *y* for Different Values of Hall Parameter *m* when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $N = 0.5$, $M = 1.2$, $\beta = 3$, $q = 0.5$

Figure (4) explained the effect of the Hall parameter *m* on the velocity profile, it is noticed that at the beginning the velocity takes one behavior for all values of *m* and then it is observed that the velocity decreased with the increasing of *m*.

Figures (5-7) are plotted to clear the effects of physical parameters *N*, β and *m* on the microtation velocity *w*, in Fig. (5) we observe that with increasing of *N* the microrotation velocity increasing and in

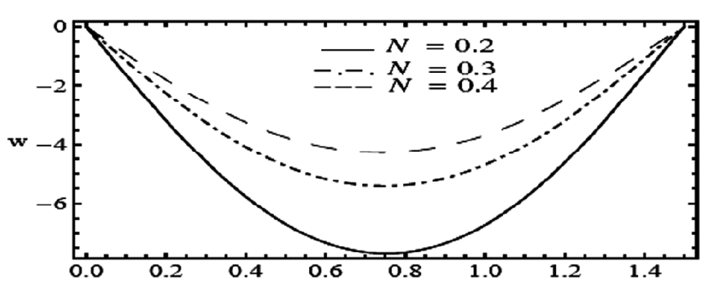

Figure 5: The Microrotation *w* Against *y* for Different Values of Coupling Number *N* when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $m = 1$, $M = 1.2$, $\beta = 3$, $q = 0.5$

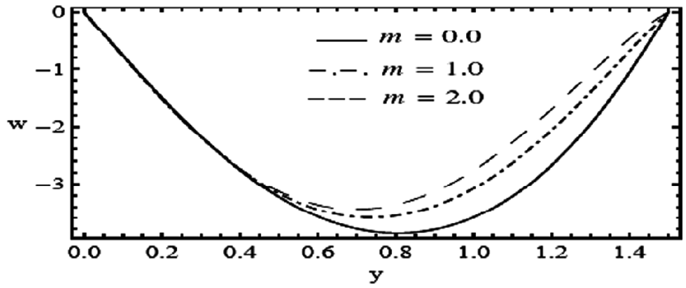

Figure 6: The Microrotation *w* Against *y* for Different Values of Micropolar Parameter β when: ϕ = 0.5, *x* = 1, α = 0, *m* = 1, *N* = 0.5, *M* = 1.2, *q* = 0.5

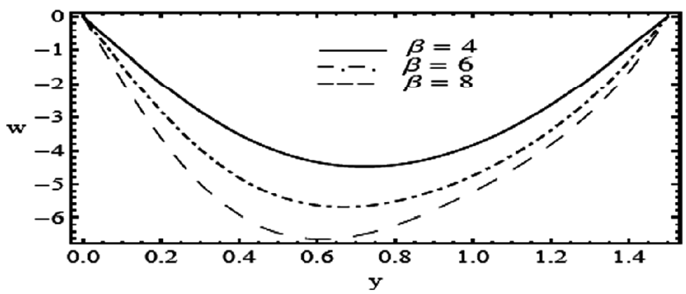

Figure 7: The Microrotation *w* Against *y* for Different Values of Hall Parameter *m* when: ϕ = 0.5, *x* = 1, α = 0, *N* = 0.5, *M* = 1.2, β = 3, *q* = 0.5

Fig. (6) near the center of the channel the microrotation velocity takes one behavior for all values of β and then it increasing with β, but in Fig. (7) it is observed that the microrotation velocity decreasing with increasing of *m*.

Figures (8-10) illustrated the behavior of the temperature distribution θ under the effects of physical parameters *N*, β and *m*, it is observed that Θ decreases with increasing of the values of *N* and β as shown in Fig. (8 and 9), but in Fig. (10) we observe that θ increases with increasing of *m*. The effects of the Eckert number E_c on the temperature distribution Θ is illustrated in Fig. (11), it is observed that the temperature increases with E_c.

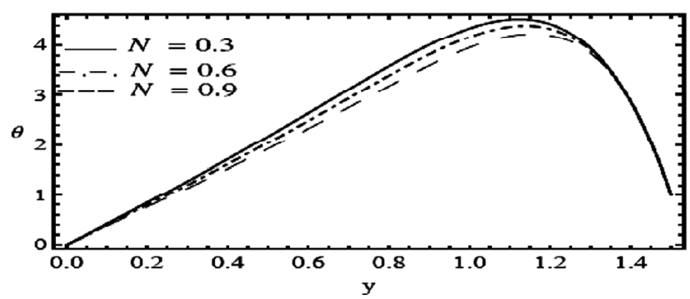

Figure 8: The Temperature Distribution θ Against *y* for Different Values of Coupling Number *N* when: ϕ = 0.5, *x* = 1, α = 1, *m* = 1, E_c = 5 *M* = 1.2, β = 3, *q* = 1

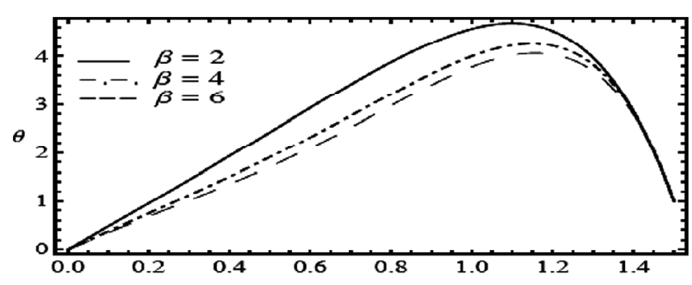

Figure 9: The Temperature Distribution θ Against *y* for Different Values of Micropolar Parameter β when: ϕ = 0.5, *x* = 1, α = 0, *H* = 3, *N* = 0.5, E_c = 5, *M* = 1.2, *q* = 1

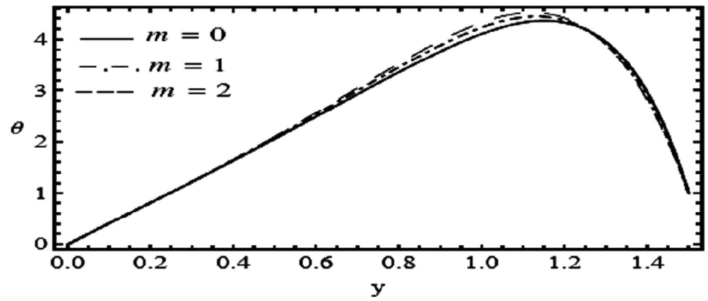

Figure 10: The Temperature Distribution θ Against *y* for Different Values of Hall Parameter *m* when: $N = 0.5$, $E_c = 5$, $\phi = 0.5$, $x = 1$, $\alpha = 1$, $M = 1.2$, $\beta = 3$, $q = 1$

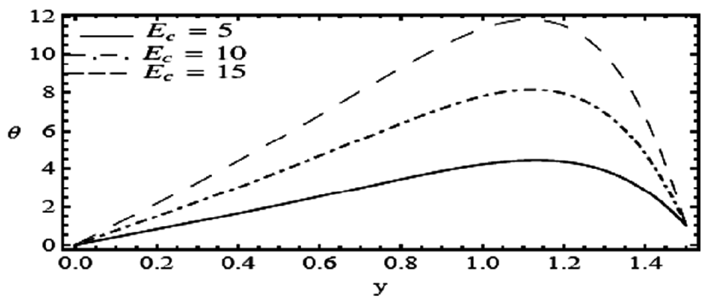

Figure 11: The Temperature Distribution θ Against *y* for Different Values of Eckert Number E_c when: $\phi = 0.5$, $x = 1$, $\alpha = 1$, $m = 1$, $N = 0.5$, $M = 1.2$, $\beta = 3$, $q = 1$

5. CONCLUSION

In the present study, the problem of two dimensional peristaltic flow of a miropolar fluid through a symmetric channel with the effect of Hall current is obtained. The equations governing the fluid flow, subjected to a set of appropriate boundary conditions, have been solved analytically under the conditions of low Reynolds number and long wave length. The solutions of these equations are obtained as functions of the physical parameters of the problem by using Mathematica Program, the effects of these parameters of the problem on these solutions have been shown graphically. It is observed that velocity profile and the temperature distribution decrease with increasing of the values of N and β and the opposite behavior happened for the microrotation velocity with theses parameters. Also, we find that velocity profile and microrotation velocity decrease with increasing of m and the temperature distribution increases with increasing of m and E_c.

6. APPENDIX

$$A_0 = -(n_1 \sinh(h\gamma_1) + n_2\gamma_1 \cosh(h\gamma_1) + n_3\gamma_1^2 \sinh(h\gamma_1))\,[1/(h\gamma_2 \sinh(h\gamma_1)\cosh(h\gamma_2)(2H(H+2) + \gamma_1^2(N-2)) + \gamma_1 \sinh(h\gamma_2)(n_6 - n_4\gamma_1) + n_5\gamma_2^2(\sinh(h\gamma_1) - h\gamma_1\cosh(h\gamma_1)))],$$

$$A_1 = n_1 \sinh(h\gamma_2) + n_2\gamma_2 \cosh(h\gamma_2) + n_3\gamma_2^2 \sinh(h\gamma_2),$$

$$A_2 = -q\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)\,(2H + \gamma_2^2(N-2)) + \gamma_2 \sinh(h\gamma_1)\,(2Hq\cosh(h\gamma_2) - n_5\gamma_2) + n_4\gamma_2^1(q\gamma_2\cosh(h\gamma_2) + \sinh(h\gamma_2)),$$

$$A_3 = h\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)\,(2H(H+2) + \gamma_2^2(N\alpha - 2)) + n_4\gamma_1^2(\sinh(h\gamma_2) - h\gamma_2\cosh(h\gamma_2)) + \gamma_2 \sinh(h\gamma_1)\,(n_7 - n_5\gamma_2),$$

$$B_0 = -((H-\gamma_2^2)(n_1 \sinh(h\gamma_1) + n_2\gamma_1\cosh(h\gamma_1) + n_3\gamma_1^2\sinh(h\gamma_1)))\,[1/(h\gamma_2\sinh(h\gamma_1)\cosh(h\gamma_2)(2H(H+2) + \gamma_1^2(N\alpha - 2)) + \gamma_1 \sinh(h\gamma_2)\,(\gamma_1(2 - N\alpha)\sinh(h\gamma_1) + n_6) + n_5\gamma_2^2(\sinh(h\gamma_1) - h\gamma_1\cosh(h\gamma_1)))],$$

$$B_1 = (H - \gamma_1^2)(n_1 \sinh(h\gamma_2) + n_2\gamma_2 \cosh(h\gamma_2) + n_3\gamma_2^2 \sinh(h\gamma_2))[1/(h\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)(2H(H+2) + \gamma_2^2(N\alpha - 2)) + \gamma_2 \sinh(h\gamma_1)(\gamma_2(2 - N\alpha)\sinh(h\gamma_2) + n_7) + n_4\gamma_1^2(\sinh(h\gamma_2) - h\gamma_2\cosh(h\gamma_2)))],$$

$$B_2 = H(-q\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)(2H + \gamma_2^2(N\gamma - 2)) + \gamma_2 \sinh(h\gamma_1)(2Hq\cosh(h\gamma_2) + \gamma_2(2 - N\alpha)\sinh(h\gamma_2)) + n_4\gamma_1{}^2(q\gamma_2\cosh(h\gamma_2) + \sinh(h\gamma_2)))[1/(h\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)(2H(H+2) + \gamma_2^2(N\alpha - 2)) + \gamma_2 \sinh(h\gamma_1)(\gamma_2(2 - N\alpha)\sinh(h\gamma_2) + n_7) + n_4\gamma_1^2(\sinh(h\gamma_2) - h\gamma_2\cosh(h\gamma_2)))],$$

$$B_3 = H_2(-q\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)(2H + \gamma_2^2(N\alpha - 2)) + \gamma_2 \sinh(h\gamma_1)(2Hq\cosh(h\gamma_2) + \gamma_2(2 - N\alpha)\sinh(h\gamma_2)) + n_4\gamma_1^2(q\gamma_2\cosh(h\gamma_2) + \sinh(h\gamma_2)))[1/(h\gamma_1 \sinh(h\gamma_2)\cosh(h\gamma_1)(2H(H+2) + \gamma_2^2(N\alpha - 2)) + \gamma_2 \sinh(h\gamma_1)(\gamma_2(2 - N\alpha)\sinh(h\gamma_2) + n_7) + n_4\gamma_1^2(\sinh(h\gamma_2) - h\gamma_2\cosh(h\gamma_2)))],$$

$$n_1 = 2H(h(H+2) + q), \quad n_2 = 2hH(H+1)q, \quad n_3 = (h+q)(N\alpha - 2), \quad n_4 = (N\alpha - 2)\sinh(h\gamma_1),$$

$$n_5 = (N\alpha - 2)\sinh(h\gamma_2), \quad n_6 = -2hH(H+2)\cosh(h\gamma_1), \quad n_7 = -2hH(H+2)\cosh(h\gamma_2),$$

$$C_1 = 1/(8hA_3^2(\gamma_1^2 - \gamma_2^2)^2), \quad C_2 = A_0A_1A_3, \quad C_3 = A_0^2A_3^2, \quad C_4 = -16C_2\sinh(\gamma_1 h)\sinh(\gamma_2 h),$$

$$C_6 = C_3 - C_3\cosh(2\gamma_2 h), \quad C_7 = \cosh(\gamma_1 h)\cosh(\gamma_2 h), \quad C_8 = C_4 - 2C_5 - C_6,$$

$$C_9 = C_4 + C_5 + 2C_6, \quad C_{10} = 2(A_1^2 + C_3), \quad C_{11} = -2A_1^2 + C_3, \quad C_{12} = A_1^2 - 2C_3,$$

$$f_1(y) = 2A_1^2 Ec\,\gamma_1^6(2y\gamma_2^2h(h-y) - h\sinh^2(y\gamma_1) + y\sinh^2(\gamma_1 h)),$$

$$f_2(y) = (2A_1^2 Ecy\gamma_1^8 h(y-h) + 32C_2 Ec\gamma_2^3\gamma_1^3(C_7 y - h\cosh(y\gamma_1)\cosh(y\gamma_2) + h - y)),$$

$$f_3(y) = \gamma_1^4(Ec\gamma_2^2(h(2A_1^2\cosh(2y\gamma_1) + 16C_2\sinh(y\gamma_1)\sinh(y\gamma_2) - C_3\cosh(2y\gamma_2) + C_{11}) + C_8 y) + 8A_3^2 y + C_{10}Ecy\gamma_2^4 h(y-h)),$$

$$f_4(y) = \gamma_1^2\gamma_2^2(Ec\gamma_2^2(h(-A_1^2\cosh(2y\gamma_1) + 16C_2\sinh(y\gamma_1)\sinh(y\gamma_2) + 2C_3\cosh(2y\gamma_2) + C_{12}) + C_9 y) - 16A\gamma_3^2 y - 4C_3 Ecy\gamma_2^4 h(y-h)),$$

$$f_5(y) = (2A_2^3\gamma_2^4(A_0^2 Ec\gamma_2^2(y\gamma_2^2 h(y-h) - h\sinh^2(y\gamma_2) + y\sinh^2(\gamma_2 h)) + 4y) + 2A_1^2 Ec\gamma_1^6(-2y\gamma_2^2 h(y-h) - h\sinh^2(y\gamma_1) + y\sinh^2(\gamma_1 h))).$$

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