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Supply Chain Inventory Model for Two Warehouses with Soft Computing Optimization

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ABSTRACT

Supply chain inventory optimization is a method of balancing investment to achieve the service-level goal. Here we consider two warehouse inventory models. One is of finite capacity treated as OW (Owned Warehouse) which is at market place and other warehouse is of infinite capacity treated as RW (Rented Warehouse) which is at some other place from the market Genetic algorithms and Simulated Annealing. Here the objective is to minimize the total cost. In this work Genetic algorithms and Simulated Annealing is used to optimize or to minimize the cost. The results produced from the Genetic algorithms and Simulated Annealing is compared with mathematical model and genetic algorithm. Results show that the Genetic algorithms and Simulated Annealing optimized the results more than traditional mathematical model and Genetic algorithms and Simulated Annealing.

Keywords: Supply chain Inventory, Production, two-warehouse, Genetic algorithms, Simulated Annealing, distributors and retailers.

1. INTRODUCTION

As discussed above the warehouse is an important place in the business entity and every businessman needed it during the business transaction of goods either finished or raw-materials. Now in the present market scenario and due to globalization of the market the business environment is highly competitive and no one wants to loss goodwill in the market and tries to fulfil the demand of their customer. For this, the distributors and retailers always stock the goods at their shop. In this cut-throat business environment suppliers offers some discount on bulk purchase during festival seasons and also they offers trade credit financing scheme to attract their retailers. To get benefited from these policies of the suppliers, retailers needed extra space to stock the products purchased in bulk during the offered period but due to small space in the busy market places, retailers faces problem of storage at their owned single warehouse and hence they required another storage space to stock their products purchased in excess. To resolve this problem they hire another storage space on rental basis for a short period. This rented warehouse becomes an additional storage space which is provided by private/public or government agencies and these spaces are used as secondary space for storing. The acquiring the space on rental basis for storing purposes brought the concept of two-warehouse in the Supply chain inventory modelling. In Supply chain inventory modelling, the two-warehouse concept was first time introduced by Hartley and thereafter many authors uses the concept of two ware-house considering one with limited capacity (Own ware-house) and other with unlimited capacity (Rented ware-house). In this concept often it is assumed that the carrying cost of items in rented ware-house is more than that of own ware-house due to better preservation facilities provided by owner of additional warehouse, therefore it is economical to consume first the goods kept in rented ware-house to reduce the holding cost incurred in rented warehouse. Latter we shall discussed the advantages and limitations of the two-warehouse modelling over the single warehouse modelling.

Genetic Algorithm

Genetic algorithms are very different from most of the traditional optimization methods. Genetic algorithms need design space to be converted into genetic space. So, genetic algorithms work with a coding of variables. The advantage of working with a coding of variable space is that coding discreteness the search space even though the function may be continuous. A more striking difference between genetic algorithms and most of the traditional optimization methods is that GA uses a population of points at one time in contrast to the single point approach by traditional optimization methods. This means that GA processes a number of designs at the same time. As we have seen earlier to improve the search direction in traditional optimization methods transition rules are used and they are deterministic in nature but GA uses randomized operators. Random operators improve the search space in an adaptive manner.

2. RELATED WORK

In this direction the concept of two warehouse modeling was introduced by Hartely (1976) which is later on carried by many other researchers. Hartely, in his research article does not consider the transportation cost incurred for transporting items from rented warehouse to own warehouse or retail shop/distribution center/retail outlet. The paper introduced by Hartely was extended by Srama (1987) introducing the transportation cost as one of the key factor affecting the inventory cost and infinite refilling rate. Further Murdeshwar and Sayhe (1985) extended the paper of Sarma with consideration of finite refilling rate. A research article was established by Dave (1988) introducing mentioning an additional case of bulk release pattern for each finite and infinite refilling rates of Murdeshwar paper and corrected the errors of this paper giving whole answer of the model given by Sarma. The concept of two ware house introduced by Hartely, further used by many researchers and still continuing with adding different business environment and cost affecting components of the inventory system. The above inventory model was analyzed by Bhunia and Maiti (1998) a sensitivity analysis is presented graphically on the optimal average cost. Zhou and Yang (2005) introduces a two-warehouse inventory model for items with stock-level dependent demand rate.

Kar et. al., (2001) developed a Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. Murdeshwar, Goswami and Chaudhuri (1992) developed an economic order quantity model for items with two levels of storage for a linear trend in demand. They derived the condition when to rent a warehouse and used a simple algorithm to find the maximum total profit per unit time. Ghosh and Chakrabarty (2009) introduced an order level inventory model under two level storage systems with time dependent demand.In all these models the cases of non-deteriorating items were established. Jiang et. al., (2015) developed "Joint optimization of preventive maintenance and inventory policies for multi-unit systems subject to deteriorating spare part inventory" (The deterioration of the inventory affects decision-making and increases losses. Block replacement and periodic review inventory policies were here used to evaluate a joint optimization problem for multi-unit systems in the presence of inventory deterioration). Wagner et. al., (1999) introducing "A genetic algorithm solution for one-dimensional bundled stock cutting" (The nature of this problem is such that the traditional approaches of linear programming with an integer round-up procedure or sequential heuristics are not effective. A good solution to this problem must consider trim loss, stock usage and ending inventory levels). Sadeghi et. al., (2015) extended "Two parameter tuned multi-objective evolutionary algorithms for a bi-objective vendor managed inventory model with trapezoidal fuzzy demand" (A bi-objective vendor managed inventory (BOVMI) model for a supply chain problem with a single vendor and multiple retailers, in which the demand is fuzzy and the vendor manages the retailers' inventory in a central warehouse). Diabat et. al., (2014) developed "Hybrid algorithm for a vendor managed inventory system in a two-echelon supply chain"(We try to find the optimal sales quantity by maximizing profit, given as a nonlinear and non-convex objective function. For such complicated combinatorial optimization problems, exact algorithms and optimization commercial software such as LINGO are inefficient, especially on practical-size problems). Guchhait et. al., (2013) extended "Two storage inventory model of a deteriorating item with variable demand under partial credit period" (The supplier also offers a partial permissible delay in payment even if the order quantity is less than the fixed ordered label. For display of goods, retailer has one warehouse of finite capacity at the heart of the market place and another warehouse of infinite capacity (that means capacity of second warehouse is sufficiently large) situated outside the market but near to first warehouse. Units are continuously transferred from second warehouse to first and sold from first warehouse. Combining the features of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) a hybrid heuristic (named Particle Swarm-Genetic Algorithm (PSGA)) is developed and used to find solution of the proposed model). Bera et. al., (2012) introducing "Inventory model with fuzzy lead-time and dynamic demand over finite time horizon using a multi-objective genetic algorithm" (A realistic inventory problem with an infinite rate of replenishment over a prescribed finite but imprecise time horizon is formulated considering time dependent ramp type demand, which increases with time. Lead time is also assumed as fuzzy in nature. Shortages are allowed and backlogged partially. Two models are considered depending upon the ordering policies of the decision maker). Wang et. al., (2011) developed "Location and allocation decisions in a two-echelon supply chain with stochastic demand – A genetic-algorithm based solution" (Decisions include locating a number of factories among a finite set of potential sites and allocating task assignment between factories and marketplaces to maximize profit). Kannan et. al., (2010) extended "A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling" (In order to overcome this issue, it is necessary to setup a logistics network for arising goods flow from end users to manufacturers. In this study, the optimum usage of secondary lead recovered from the spent lead–acid batteries for producing new battery is presented). Yun et. al., (2009) extended "Hybrid genetic algorithm with adaptive local search scheme for solving multistage-based supply chain problems" (The optimal design of supply chain (SC) is a difficult task, if it is composed of the complicated multistage structures with component plants, assembly plants, distribution centres, retail stores and so on. It is mainly because that the multistage-based SC with complicated routes may not be solved using conventional optimization methods). Farahani et. al., (2008) introducing "A genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain" (A bi-objective model is set up for the distribution network of a three-echelon supply chain, with two objective functions: minimizing costs, and minimizing the sum of backorders and surpluses of products in all periods). Nachiappan et. al., (2007) extended "A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain" (The operational parameters to the above model are: sales quantity and sales price that determine the channel profit of the supply chain, and contract price between the vendor and the buyer, which depends upon the understanding between the partners on their revenue sharing). Altiparmak et. al., (2006) developed "A genetic algorithm approach for multi-objective optimization of supply chain networks" (Supply chain network (SCN) design is to provide an optimal platform for efficient and effective supply chain management. It is an important and strategic operations management problem in supply chain management, and usually involves multiple and conflicting objectives such as cost, service level, resource utilization, etc). Maiti et. al., (2006) introducing "An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy" (This GA is based on Roulette wheel selection, whole arithmetic crossover and non-uniform mutation. Here, mutation is carried out for the fine-tuning capabilities of the system by non-uniform operator whose action depends on the age of the population. This methodology has been applied in solving multiple price break structure and implemented for multi-item deterministic inventory control system having two separate storage facilities (owned and rented warehouse) due to limited capacity of the existing storage (owned warehouse). Also, demand rate is a linear function of selling price, time and non-linearly on the frequency of advertisement. The model is formulated with infinite replenishment and shortages are not allowed. The stocks of rented warehouse (RW) are transported to the owned warehouse (OW) in bulk-release rule). Chan et. al., (2003) introducing "Solving the multi-buyer joint replenishment problem with a modified genetic algorithm" (The joint replenishment problem (JRP) is a multi-item inventory problem. The objective is to develop inventory policies that minimize the total costs (comprised of holding cost and setup cost) over the planning horizon). Mondal et. al., (2003) extended "Multi-item fuzzy EOQ models using genetic algorithm" (It uses genetic algorithms (GAs) with mutation and whole arithmetic crossover. Here, mutation is carried out along the weighted gradient direction using the random step lengths based on Erlang and Chi-square distributions. These methodologies have been applied to solve multi-item fuzzy EOQ models under fuzzy objective goal of cost minimization and imprecise constraints on warehouse space and number of production runs with crisp/imprecise inventory costs). Xie, et. al., (2002) extended "Heuristic genetic algorithms for general capacitated lot-sizing problems" (The lot-sizing problems address the issue of determining the production lot-sizes of various items appearing in consecutive production stages over a given finite planning horizon. In general capacitated lot-sizing problems, the product structure can be a general acyclic network, the capacity constraints can be very sophisticated, and all the known parameters can be time-varying). Disney et. al., (2000) developed "Genetic algorithm optimisation of a class of inventory control systems" (Benchmark performance characteristics. Five are considered herein and include inventory recovery to "shock" demands; in-built filtering capability; robustness to production lead-time variations; robustness to pipeline level information fidelity; and systems selectivity. A genetic algorithm for optimising system performance, via these five vectors is described).

3. ASSUMPTION AND NOTATIONS

The mathematical model of two warehouse inventory model for deteriorating items is based on the following notation and assumptions

Assumption:

1. Replenishment rate is infinite and lead time is negligible i.e. zero.
2. The time horizon of the inventory system is infinite.
3. Goods of OW are consumed only after the consumption of goods kept in RW due to the more holding cost in RW than in OW.
4. The OW has the limited capacity of storage and RW has unlimited capacity.
5. Demand vary with time and is linear function of time and given by $D(t) = \{(\delta + 1) + (\eta - 1)t \text{ if } t > 0\}$; where $(\delta + 1) > 0$ and $(\eta - 1) > 0$;
6. For deteriorating items a fraction of on hand inventory deteriorates per unit time in both the warehouse with different rate of deterioration.
7. Shortages are allowed and demand is partially backlogged at the beginning of next replenishment.
8. The unit inventory cost (Holding cost) in RW > OW.
9. We assume that the holding cost will be fixed till a definite time in RW and the will increased according to a fraction of ordering cycle length. So for holding cost (h_r), we have k a time moment before which holding cost is constant.

$$h_r = \{h_r \text{ if } t \leq (\lambda + 1)\}$$

Notations:

A	: Ordering cost per Order.
C_O	: Capacity of OW.
C_R	: Capacity of RW.
T	: The length of replenishment cycle.
Q_{max}	: Maximum Inventory level per cycle to be ordered.
t_1	: The time up to which inventory vanishes in RW.
t_2	: The time at which inventory level reaches to zero in OW and shortages begins.
$(\lambda + 1)$	: Definite time up to which holding cost is constant.
h_m	: The holding cost per unit time in OW.
h_r	: The holding cost per unit time in RW.

s_c : The shortages cost per unit per unit time.

L_c : The opportunity cost per unit per time.

$\Psi^r(t)$: The level of inventory in RW.

$\Psi^i(t)$: The level of inventory in OW where $i = 1, 2$.

$\Psi^s(t)$: Determine the inventory level at time t in which the product has shortages.

u : Deterioration rate in RW Such that $0 < u < 1$;

v : Deterioration rate in OW such that $0 < v < 1$;

C_p : Purchase cost per unit of items.

A_B : Maximum amount of inventory backlogged.

A_L : Amount of inventory lost.

C_P : Cost of purchase.

C_S : The present worth cost of shortages.

C_L : The present worth cost of lost sale

H_C : The present worth cost of holding inventory

P_R : Production rate which is taken as demand dependent i.e. $P_R = \Pi D(t)$

R_R : Retailer rate which is taken as demand dependent i.e. $R_R = \varepsilon D(t)$

D_R : Distribution rate which is taken as demand dependent i.e. $D_R = \delta D(t)$

TC_1 : Transportation Cost of Manufacturer to between warehouses

TC_2 : Transportation Cost of warehouses to between Distribution centres

$TC(t_1, T)$: The total relevant inventory cost per unit time of inventory system.

4. MATHEMATICAL FORMULATION OF MODEL AND ANALYSIS

(i) Manufacturer Model:

$$\begin{aligned} P_R &= \text{Production Rate} - \text{Demand} \\ &= \Pi\{(\delta + 1) + (\eta - 1)t\} - \{(\delta + 1) + (\eta - 1)t\} \end{aligned}$$

(ii) Two-warehouses Inventory Model: In the beginning of the cycle at $t = 0$ a lot size of Q_{max} units of inventory enters into the system in which backlogged $(Q_{max} - A_B)$ units are cleared and the remaining units M is kept into two storage as C_O units in OW and C_R units in RW.

During the time interval $[0, t_1]$ the inventory in RW decrease due to the demand and deterioration and is governed by the following differential equation:

$$\frac{d\Psi^r(t)}{dt} = \{(\delta + 1) + (\eta - 1)t\} - \{u\Psi^r(t)\};\ 0 \le t \le t_1 \qquad (1)$$

In the time interval $[0, t_2]$ the inventory level decreases in OW decreases due to deterioration only and is governed by differential equation

$$\frac{d\Psi^{1w}(t)}{dt} = -v\,\Psi^{1w}(t) \quad 0 \le t \le t_1 \tag{2}$$

During time interval $[t_1, t_2]$ the inventory level in OW is decreases due to demand and deterioration both and is governed by the following differential equation

$$\frac{d\Psi^{2w}(t)}{dt} = -\{(\delta + 1) + (\eta - 1)t\} - \{\beta\Psi^{r}(t)\} \quad t_1 \le t \le t_2 \tag{3}$$

Now at $t = t_2$ the inventory level vanishes and the shortages occur in the time interval $[t_2 T]$ a fraction f of the total shortages is backlogged and the shortages quantity supplied to the customers at the beginning of the next replenishment cycle. The shortages is governed by the differential equation

$$\frac{d\Psi^{s}(t)}{dt} = -f\{(\delta + 1) + (\eta - 1)t\} \quad t_3 \le t \le T \tag{4}$$

At the time $t = T$ replenishment cycle restarts. The objective of the model is to minimize the total inventory cost by the relevant cost as low as possible.

Now inventory level at different time intervals is given by solving the above differential equations (1) to (4) under boundary conditions

$$\Psi^{r}(t_1) = 0;\ \Psi^{1w}(0) = C_0;\ \Psi^{2w}(t_2) = 0;\ \Psi^{S}(t_2) = 0;$$

Therefore Differential eq. (1)

$$\Psi^{r}(t) = \left\{\frac{(\delta+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}\right\}\{(\alpha t_1 - 1)\}\,e^{\alpha(t_1 - t)} - \left\{\frac{(\delta+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}(\alpha t - 1)\right\} \tag{5}$$

$$\Psi^{1w}(t) = C_0 e^{-vt_1} \tag{6}$$

$$\Psi^{2w}(t) = \left\{\frac{(\delta+1)}{v} + \frac{(\eta-1)}{v^2}\right\}\{(vt_2 - 1)\}\,e^{v(t_2 - t)} - \left\{\frac{(\delta+1)}{v} + \frac{(\eta-1)}{v^2}(vt - 1)\right\} \tag{7}$$

$$\Psi^{S}(t) = f\left\{(\delta+1)(t_2 + t) + \frac{(\eta-1)}{2}(t_2^2 - t^2)\right\} \tag{8}$$

Now at $t = 0$, $\Psi^{r}(0) = C_R$ therefore equation (5) yield

$$C_R = \left\{\frac{(\eta-1)}{\alpha^2} - \frac{(\delta+1)}{\alpha}\right\} + \left\{\frac{(\delta+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}(\alpha t_1 - 1)\right\}e^{-\alpha t_1} \tag{9}$$

Maximum amount of inventory backlogged during shortages period (at $t = T$) is given by

$$A_B = -\Psi^{S}(T)$$

$$= f\left\{(\delta+1)(T - t) + \frac{(\eta-1)}{2}(T^2 - t^2)\right\}; \tag{10}$$

Amount of inventory lost during shortages period

$$A_L = (1 - A_B)$$

$$= \left(1 - f\left\{(\delta + 1)(T - t) + \frac{(\eta - 1)}{2}(T^2 - t^2)\right\}\right) \tag{11}$$

The maximum Inventory to be ordered is given as

$$Q_{max} = C_O + \Psi^r(0) + A_B$$

$$Q_{max} = C_O + \left\{\frac{(\eta - 1)}{\alpha^2} - \frac{(\delta + 1)}{\alpha}\right\} + \left\{\frac{(\delta + 1)}{\alpha} + \frac{(\eta - 1)}{\alpha^2}(\alpha t_1 - 1)\right\} e^{\alpha t_1}$$

$$+ f\left\{(\delta + 1)(T - t_2) + \frac{(\eta - 1)}{2}(T - t_2^2)\right\}; \tag{12}$$

Now continuity at $t = t_1$ shows that $I^{1w}(t_1) = I^{2w}(t_1)$ therefore from eq. (6) & (7) we have

$$(\eta - 1)v^2 t_2^2 - \alpha v^2 t_2 - (\alpha^2(C_O + Z) + (\eta - 1) - \alpha v) = 0 \tag{13}$$

where, $Z = \left\{\frac{(\delta + 1)}{v} + \frac{(\eta - 1)}{v^2}(vt_1 - 1)\right\} e^{-vt_1}$

Which is quadratic in t_2 and further can $(\eta - 1)e$ solved for t_2 in terms of t_1 i.e.

$$t_2 = \varphi(t_1) \tag{14}$$

where, $\varphi(t_1) = \frac{-(\delta + 1)^2 v^4 \pm \sqrt{D}}{2(\eta - 1)v^2}$ and $D = (\delta + 1)^2 v^4 + 4(\eta - 1)\, v^2((\eta - 1) - (\delta + 1)v)$

$$+ v^2\left(C_O + \left\{\frac{(\delta + 1)}{v} + \frac{(\eta - 1)}{v^2}(vt_1 - 1)\right\} e^{-vt_1}\right)$$

Next the total relevant inventory cost per cycle includes following parameters:

1. Ordering cost per cycle = A
2. Purchase cost per cycle = $P \times Q_{max}$
3. The present worth holding cost = H_C

Case: When $(\lambda + 1) > T$

$$\int_0^{t_1} h_r \Psi^r(t)\,dt + \int_0^{t_1} h_w \Psi^{1w}(t)\,dt + \int_{t_1}^{t_2} h_w \Psi^{2w}(t)\,dt$$

Holding cost for Case

$$H_C = h_r\left((\delta + 1)t_1^2 + (\eta - 1)t_1^3 + \frac{(\eta - 1)t_1^2}{\alpha} - \frac{(\eta - 1)t_1^2}{2\alpha}\right)$$

$$+ h_w\left(C_O t_1 + \frac{(\eta - 1)t_2^2}{v} - \frac{(\eta - 1)t_1 t_2}{v} + \frac{(\eta - 1)t_1^2}{2v} - \frac{(\eta - 1)t_2^2}{2v}\right) \tag{16}$$

The present worth of shortages cost

$$C_S = S_c f\left(\frac{(\hat{\delta}+1)T^2}{2} - \frac{(\hat{\delta}+1)t_2^2}{2} + \frac{(\eta-1)T^3}{6} - \frac{(\eta-1)t_2^3}{6} - (\hat{\delta}+1)t_1 T\right.$$
$$\left. + (\hat{\delta}+1)t_2^2 - \frac{(\eta-1)t_2^2 T}{2} + \frac{(\eta-1)t_2^3}{2}\right) \quad (17)$$

The present worth opportunity cost/Lost sale cost

$$CL = L_c\left(1 - \left(\frac{(\hat{\delta}+1)T^2}{2} - \frac{(\hat{\delta}+1)t_2^2}{2} + \frac{(\eta-1)T^3}{6} - \frac{(\eta-1)t_2^3}{6} - (\hat{\delta}+1)t_1 T\right.\right.$$
$$\left.\left. + (\hat{\delta}+1)t_2^2 - \frac{(\eta-1)t_2^2 T}{2} + \frac{(\eta-1)t_2^3}{2}\right)\right) \quad (18)$$

Present worth purchase cost

$$C_P = P_c\left(C_O + \left(\frac{(\eta-1)}{\alpha^2} - \frac{(\hat{\delta}+1)}{\alpha}\right) + \left\{\frac{(\hat{\delta}+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}((\hat{\delta}+1)t_1 - 1)e^{-\alpha t_1}\right\}\right.$$
$$\left. + f\left\{(\hat{\delta}+1)(T - t_2) + \frac{(\eta-1)}{2}(T^2 - t_2^2)\right\}\right) \quad (19)$$

Therefore Total relevant inventory cost per unit per unit of time is denoted and given

$$TC(t_1, T) = \frac{1}{T}\,[A + C_P + C_S + C_L + H_C]$$

$$= \frac{1}{T}\left[A + P_C\left(C_O + \left(\frac{(\eta-1)}{\alpha^2} - \frac{(\hat{\delta}+1)}{\alpha}\right) + \left\{\frac{(\hat{\delta}+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}((\hat{\delta}+1)t_1 - 1)e^{-\alpha t_1}\right\}\right.\right.$$
$$\left. + f\left[(\hat{\delta}+1)(T - t_2) + \frac{b}{2}(T^2 - t_2^2)\right]\right)$$
$$+ S_c f\left(\frac{(\hat{\delta}+1)T^2}{2} - \frac{(\hat{\delta}+1)t_2^2}{2} + \frac{(\eta-1)T^3}{6} - \frac{(\eta-1)t_2^3}{6} - (\hat{\delta}+1)t_1 T\right.$$
$$\left. + (\hat{\delta}+1)t_2^2 - \frac{(\eta-1)t_2^2 T}{2} + \frac{(\eta-1)t_2^3}{2}\right)$$
$$+ L_c\left(1 - \left(\frac{(\hat{\delta}+1)T^2}{2} - \frac{(\hat{\delta}+1)t_2^2}{2} + \frac{(\eta-1)T^3}{6} - \frac{(\eta-1)t_2^3}{6} - (\hat{\delta}+1)t_1 T\right.\right.$$
$$\left.\left. + (\hat{\delta}+1)t_2^2 - \frac{(\eta-1)t_2^2 T}{2} + \frac{(\eta-1)t_2^3}{2}\right)\right)$$

$$+ h_r\left((\delta+1)t_1^2 + (\eta-1)t_1^3 + \frac{(\eta-1)t_1^2}{\alpha} - \frac{(\eta-1)t_1^2}{2\alpha}\right)$$

$$+ h_w\left(C_O t_1 + \frac{(\eta-1)t_2^2}{v} - \frac{(\eta-1)t_1 t_2}{v} + \frac{(\eta-1)t_1^2}{2v} - \frac{(\eta-1)t_2^2}{2v}\right)\Bigg] \tag{20}$$

The total relevant inventory cost is minimum if

$$\frac{\partial TC}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial TC}{\partial T} = 0$$

and subject to the satisfaction of following:

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \frac{\partial^2 TC}{\partial t_1 \partial T} > 0$$

(iii) Transportation Cost: (Manufacturer to between Warehouses)

$$TC = T_1\{(\delta+1) + (\eta-1)t\}$$

(iv) Transportation Cost: (Warehouses to between Distribution Centres)

$$TC = T_2\{(\delta+1) + (\eta-1)t\}$$

(v) Distribution Centre:

$$D_R = \delta D(t)$$
$$= \delta\{(\delta+1) + (\eta-1)t\}$$

(vi) Retailer Cost

$$R_R = \varepsilon D(t)$$
$$= \varepsilon\{(\delta+1) + (\eta-1)t\}$$

Total Supply Chain inventory cost = Production Rate + Transportation Cost + Two-warehouses Inventory + Transportation Cost + Distribution Rate + Retailer Rate

$$TSCIC = P_R + TC_1 + TC(t_1, T) + TC_2 + D_R + R_R \tag{21}$$

$$TSCIC = [\Pi\{(\delta+1) + (\eta-1)t\} - \{(\delta+1) + (\eta-1)t\}] + T_1\{(\delta+1) + (\eta-1)t\}$$

$$+ \frac{1}{T}\left[A + P_C\left(C_O + \left(\frac{(\eta-1)}{\alpha^2} - \frac{(\delta+1)}{\alpha}\right) + \left\{\frac{(\delta+1)}{\alpha} + \frac{(\eta-1)}{\alpha^2}((\delta+1)t_1 - 1)e^{-\alpha t_1}\right\}\right.\right.$$

$$+ f\left\{(\delta+1)(T-t_2) + \frac{b}{2}(T^2 - t_2^2)\right\}$$

$$+ S_c f\left(\frac{(\delta+1)T^2}{2} - \frac{(\delta+1)t_2^2}{2} + \frac{(\eta-1)T^3}{6} - \frac{(\eta-1)t_2^3}{6} - (\delta+1)t_1 T\right.$$

$$+ (\delta + 1) t_2^2 - \frac{(\eta - 1) t_2^2 T}{2} + \frac{(\eta - 1) t_2^3}{2} \Bigg)$$

$$+ L_c \left(1 - \left(\frac{(\delta + 1) T^2}{2} - \frac{(\delta + 1) t_2^2}{2} + \frac{(\eta - 1) T^3}{6} - \frac{(\eta - 1) t_2^3}{6} - (\delta + 1) t_1 T \right.\right.$$

$$\left.\left. + (\delta + 1) t_2^2 - \frac{(\eta - 1) t_2^2 T}{2} + \frac{(\eta - 1) t_2^3}{2} \right)\right)$$

$$+ h_r \left((\delta + 1) t_1^2 + (\eta - 1) t_1^3 + \frac{(\eta - 1) t_1^2}{\alpha} - \frac{(\eta - 1) t_1^2}{2\alpha} \right)$$

$$+ h_w \left(C_O t_1 + \frac{(\eta - 1) t_2^2}{\nu} - \frac{(\eta - 1) t_1 t_2}{\nu} + \frac{(\eta - 1) t_1^2}{2\nu} - \frac{(\eta - 1) t_2^2}{2\nu} \right) \Bigg]$$

$$+ T_2\{(\delta + 1) + (\eta - 1)t\} + \delta\{(\delta + 1) + (\eta - 1)t\} + \varepsilon\{(\delta + 1) + (\eta - 1)t\} \qquad (22)$$

5. PROPOSED GENETIC ALGORITHM

Compute $t_n(k)$ for given u by Genetic Algorithm: For each given state *u* in order compute the optimal cost with respect to it, we must solve the following optimization problem

$$t_n(k) = \min_{b \geq k} T_k(k) - c_k \cdot k,$$

$$k \in \Pi_n, b \in \Xi_n$$

Now, we give a genetic algorithm procedure for solving the above optimization problem

Genetic Algorithm Procedure for optimal cost:

Step 1: Initialize pop size chromosomes randomly.

Step 2: Update the chromosomes by crossover and mutation operations.

Step 3: Calculate the objective values for all chromosomes.

Step 4: Compute the fitness of each chromosome according to the objective values.

Step 5: Select the chromosomes by spinning the roulette wheel.

Step 6: Repeat the second to fifth steps for a given number of cycles.

Step 7: Report the best chromosomes as the optimal cost for the given state *k*.

6. PROPOSED SIMULATED ANNEALING

Simulated Annealing (SA) is a technique for finding good solution to minimization problems. It simulates the physical annealing process of solidifying a metal to a uniform crystalline structure. In order to achieve this uniform crystalline structure the metal is first heated to a molten state and the gradually cooled down. The critical parameter of this process is the rate of cooling. If the cooling takes place too quickly energy gaps will be formed resulting in non-uniformity in the crystalline structure. On the other hand if the cooling takes place too slowly then time is wasted. The optimal cooling rate varies from metal to metal.

Let us assume that a minimization problem involving m design variables: $x_1, x_2, \ldots, x_m$ is to be solved the SA

The problem may be stated as follows:

$$\text{Minimize } P = J(X)$$

Subject to

$$X^{min} \leq X \leq X^{max}$$

Step 1: We assign a high initial temperature to molten metal say Y_0 select an initial solution X_0 at random and set a termination criterion ε to a small value and iteration number $y = 0$

Step 2: We calculate the temperature of $(y + 1)$-th iteration that is T_{y+1} as 60% of that of y-th iteration that is Y_y therefore $Y_{y+1} = 0.6 \times Y_y$ we generate a candidate solution for $(y + 1)$-th iteration that is X_{y+1} at random in the neighbourhood of X_y.

Step 3: If the change in energy $(\delta + 1)J = J(X_{t+1}) - J(X_t) < 0$ then we accept X_{y+1} as the next solution and set $y = y + 1$

Else we generate a random number r lying in the range of (0.0, 2.0) and if $r \leq \exp\left(\frac{-\delta J}{Y_{y+1}}\right)$ we accept X_{y+1} as the next solution and set $y = y + 1$

Else we reject X_{y+1} and set $y = y$ and go to step 2.

Step 4: If $|J(X_{y+1}) - J(X_y)| < \varepsilon$ and Y reaches a small value we terminate the program.

7. IMPLEMENTATION OF SYSTEM

In the proposed algorithm steps is applied on mathematical results obtained from the mathematical inventory model. It provides best cost solution and optimizes the results. MATLAB is used to implement the proposed work. MATLAB stands for matrix programming language which was developed by Math Works. It is a programming language that makes algebra programming simple. MATLAB is a fourth-generation high level programming language that provides an interactive environment for computation, visualization and programming. Functions of MATLAB that are used in proposed work are:

- Zeros: it will create an array of all zeros.
- Rand: It will generate array of random numbers.
- Sum: Sum of array elements.
- Ones: Create an array of all ones.
- Consume: It returns cumulative sum of all elements.
- Cost function: This function is used to optimize the results.
- Floor: Rounds the elements to the nearest integers less than or equal to value.
- Plot: To plot graphs.

8. NUMERICAL ANALYSIS

The following randomly chosen data in appropriate units has been used to find the optimal solution and validate the model of the three players the producer, the distributor and the retailer. The data are given as $(\delta + 1) = 610$, A = 2510, C = 2120, $(\eta - 1) = 0.45$, $h_w = 65$, $h_r = 45$, $P_C = 1510$, $u = 0.123$, $v = 0.124$, $S_C = 340$, $\lambda = 2.42$, $f = 0.15$ and $L_c = 110$.

A Binary genetic algorithm for lot Sizing Problem

The total 5 problem instances are run for the binary GA with holding cost of $(\delta + 1) + (\eta - 1)t = €8.32$, ordering cost of A = €91, shortages cost of $S_C = €7.81$ Production Rate $P_R = €7.50$, Transportation Cost $T_{1,2} = €6.50$, Distribution Rate $D_R = €7.60$ and Retailer Rate $R_R = €1.50$ in order to compare the results with those optimal by the Wagner-Whit in algorithm. For each problem instance, 5 replications are conducted. Minimum, maximum, average, and standard deviation are given together with the CPU times.

Simulate Annealing to solve this minimization Problem

The initial temperature of motal $Y_0 = 4210°K$, Initial solution selected at random $X_0 = \left(\frac{8.7}{8.7}\right)$

Termination criterion = 1.011, Let random number varying in the range of (2.0, 4.0) are as follows: 2.3, 2.8, 2.9, 2.8, 2.4, 2.7, 2.4, 2.6, 2.9, 2.7, and so on.

The values of decision variables are computed for the model for cases separately.

Table 4.1

Cost function	t_1^*	t_2^*	T^*	*Total relevant cost*
$TC(t_1^*, T)$	4.47	36.70	97.24	71351

Table 4.2
GA (Genetic Algorithm) Results

P	*WW*	*GA*			
	OPT	*BEST*	*MAX*	*AVG*	*STD*
1	32.50	32.50	31.75	38.00	3.25
2	32.00	32.00	31.75	38.75	2.69
3	30.00	30.00	35.00	31.00	4.00
4	30.00	30.00	33.00	39.00	6.00
5	30.25	30.25	30.50	32.70	4.58

Table 4.3
SA (Simulated Annealing) Results

P	*WW*	*SA*			
	OPT	*BEST*	*MAX*	*AVG*	*STD*
1	41.50	41.50	41.75	48.00	4.45
2	41.00	41.00	41.75	48.75	1.69
3	40.00	40.00	45.00	41.00	3.00
4	40.00	40.00	43.00	49.00	6.00
5	40.45	40.45	40.50	44.70	3.58

9. CONCLUSION

In this research paper, a new Genetic algorithms and Simulated Annealing has been proposed for the optimization of two warehouse inventory model with the objective of minimizing the total relevant cost. Two different cases have been considered to optimize the relevant cost. Furthermore the proposed Genetic algorithms and Simulated Annealing is very useful to optimize the cost. The Genetic algorithms and Simulated Annealing is implemented in MATLAB. The Genetic algorithms and Simulated Annealing is applied on mathematical model to optimize the cost. Consequently, it can be concluded that this Genetic algorithms and Simulated Annealing is a well-designed and promising method for optimization.

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