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### Transient Analysis on a Freewheel Differential For Baja Sae using Finite Element Method

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**Abstract:** A differential used to splits torque and power between the two wheels. Most vehicles use an open differential which uses a bevel gear arrangement. In a situation, where only one of the wheels has traction, an open differential would send only 50% of the torque and 0% of the power to this wheel. To overcome this problem, manufacturers generally provide a Limited Slip Differential. The Freewheel Differential improves the fuel economy of a vehicle, but will also handle torque distribution appropriately. When the inner wheel loses traction, and starts spinning faster, the outer wheel will receive the torque the moment both the wheels start spinning at the same speed. This paper deals with the development of a product and to realize the design of the product, thorough analysis of stresses and deformation has been conducted on each component to ensure the worthiness of the design.

**Keywords:** Transient analysis, finite element method, differential, sports car.

#### 1. INTRODUCTION

As in the case of all commercial vehicles, a differential is required to transmit the power and torque to the driving wheels through the drive shafts. For a Baja SAE car, the torque distribution via the differential is of huge importance as there can be cases where only one of the driving wheels is touching the ground or just one of the wheels has sufficient traction to push the car.

The freewheel differential triumphs over the standard open differential in most of these scenarios. It also doesn't complicate the overall assembly and would only have a marginal effect on the overall cost of the vehicle.

V. Moorthy et al. [8] mainly talk about the difference in fatigue life of similar gears with just one difference, that of the surface coating. Gears with different coatings were tested for 50 million cycles and their life was compared with each other and the gear with no surface coating. Santosh S. Patil et al. [5] focus on the impact

of introduction of coefficient of friction in the contact stress being experienced by a gear. The authors realize that the contact stresses obtained by Hertz Equation and the one obtained by including frictional coefficients is almost the same. However, this can throw some light on lack of lubrication of a gear set. Santosh S. Patila et al. [5] compare the contact stresses determined experimentally on a set of gears and the same obtained using FEM, including frictional coefficients. Niels L. Pedersen et al. [2] dwell into the optimization of the gear geometry in order to reduce bending stresses on the gear. The use of asymmetrical gears can be looked upon in this paper, however it needs the introduction of new standard cutting tools. As a result, implementing the findings of this paper on this paper would make it impractical. Friction losses are a major source of inefficiency in a gearbox. Determining these losses analytically would help us achieve a higher efficiency by reducing the frictional loss in the design phase itself. This paper can be extremely useful for a Baja SAE car is powered by a 10HP engine. Even the smallest of inefficiencies in the powertrain can make a huge impact on the overall performance of the vehicle. Huaiju Liu et al. [1] talk about the impact of lubrication on various parameters. It compares the situation of starved lubrication with the case where the gears are flooded with lubricant. It also talks about the minimum film thickness required and the coefficient of friction in the various lubrication scenarios. Yumei Hu et al. [10] talk about the impact of the modification coefficients and the helix angles on the real-time performance of the gears. The authors have also proposed that FEA techniques can be used to determine the impact. This paper would not be very helpful as spur gears have been chosen for this paper. Vivek Karaveer et al. [7] demonstrate a method to analyze for various stresses on pair of spur gears. This method can be deployed for the analysis of gears in this paper. Gears are designed following Indian Standards and the standard gear design procedure is followed. Followed by that, stress and strain simulations are performed on Ansys.

## **2. DESIGN APPROACH AND DETAILS**

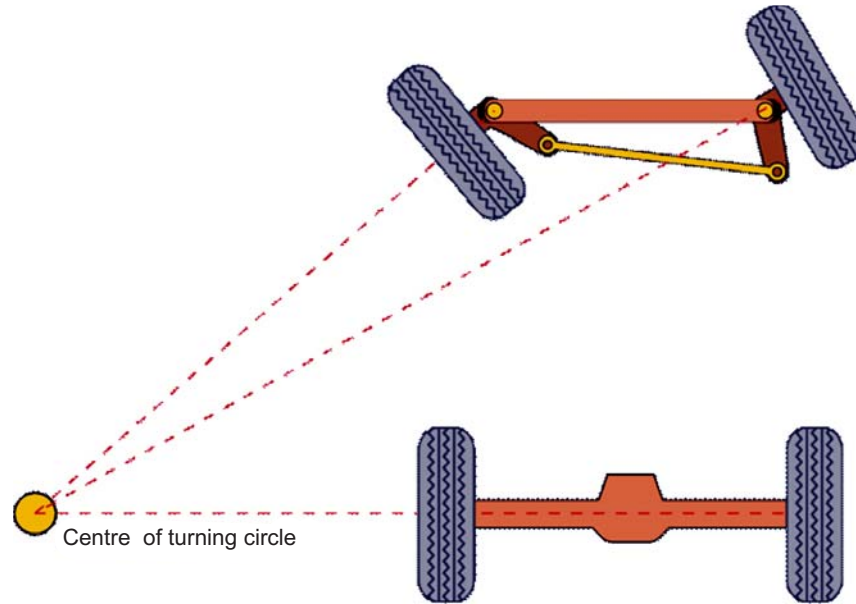
### **2.1. Conceptualization**

Any vehicle with more than 3 wheels, requires a differential to send the power from the gearbox to the wheels. A differential generally does the following tasks:

1. Multiplies the torque supplied from the gearbox
2. Distributes the torque and power between the two driven wheels of a car
3. Allows for the independence of rotation speed of the two driven wheels, even though they are driven from one engine.

The second and third tasks are the most vital and have the biggest impact on the performance of the vehicle. Distribution of power and torque is of utmost importance in unexpected situations, or while driving on surfaces with low traction. An open differential always splits the torque equally between the two wheels. However, this is not desirable in a situation where one of the powered wheels is jacked up due to huge undulations on the road or when one of the wheels has totally lost traction due to a slippery surface. A locked differential overcomes the problem of torque distribution but, it does not allow the two wheels to spin at independent RPM's. Limited Slip Differentials are a hybrid of locked differentials and open differentials. They act as a locked differential when the car is moving in a straight line and act as an open differential when the car turns. However, they do cause some wear in the tires and also are slightly more complicated.

A freewheel differential would overcome the problems posed by the present differentials. A freewheel allows transmission of power in one direction and free rotation in the other, much like the Ratchet Pawl Mechanism. Freewheels are most commonly used in bicycles. As in the case of a bicycle, the freewheel lets you transmit power to the wheels when you peddle forward but allows the wheel to rotate freely the moment you stop peddling.



**Figure 1: Ackerman Steering Geometry**

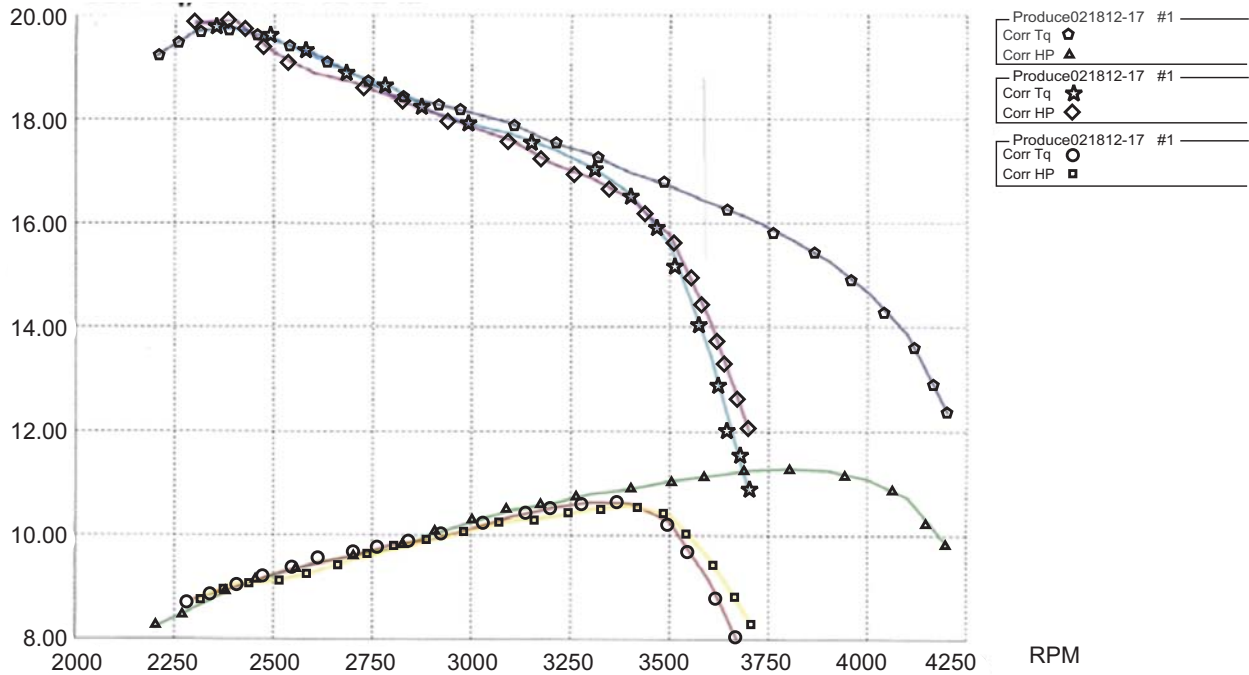
This differential aims to implement freewheels to make it work like a differential. The setup consists of one input shaft with a pinion meshed to a gear on the intermediate shaft (Figure 3). The intermediate shaft consists of another pinion which is then meshed with the output gear. The gear includes a hub, which houses two freewheels. Using two freewheels has a huge impact on the way the differential works. As seen from Figure 1, while taking a turn, all the four wheels of a car rotate at different RPM's as they move in circles of different diameters. Assuming the car to be a Rear Wheel Drive, and it performing a left turn, as shown in the figure, the rear left wheel will move slower than the rear right wheel. As a freewheel allows free rotation only when the wheel is moving faster than the input speed, the outer wheel will rotate freely and all of the torque will go to the inner wheel. If in case, the inner wheel loses traction due to excess torque, the torque will be sent to the outer wheel as and when the speed of both the wheels becomes the same. In a case where one of the wheels gets jacked up, all the torque would be supplied to the wheel with higher traction in accordance with Newton's Third Law of Motion.

The other advantage of such a differential would be a huge improvement in the fuel economy of the car. While riding a bicycle, we can stop peddling after attaining a good amount of speed, take some rest and still be moving without any problems, the engine of the car would be able to do the same with this kind of a differential being used. It will return to idling the moment the driver releases the throttle and the car would continue to move with a minimal loss in speed and a huge improvement in the fuel economy.

It should be noted that, in a case where the two wheels have to rotate at different RPM's, the wheel turning at a lower speed would be the one. These two freewheels are free to rotate separately. When the car has to move in a straight line, the two freewheels engage and they make both the tires rotate at the same speed.

## 2.2 Selection of a Gear Ratio

The engine produces 19.66Nm of torque at an RPM of 2400, as seen from the Run #4 data from the dyno test data given in **Figure 2**. Run #4 data is the most useful because the Baja SAE rulebook specifies that all the engines should be governed at an RPM of 3800 or lower. The engine is coupled with a CVT which has a ratio ranging from 3.9 to 0.9. Generally, the cars should have the capability of climbing a hill with a maximum gradient of 30°.



Test Summary and Comments for : PURDUE021812-19 #1			
Operator: ROBERT	1:56 pm 02/18/2012	PkTq: 19.76 @ 2400	2.56" Bore
Eng #:	Corr. To: 29.92/60 dry	PkHP: 11.28 @ 3800	2.436" Stroke
Customer: Peterson	Corr. Factor 1.041	12.54 cid 4 Cycle	1 Cylinders
PURDUE BAJA (2000-4200 gov) RUN #5			
Test Summary and Comments for : PURDUE021812-18 #1			
Operator: ROBERT	1:30 pm 02/18/2012	PkTq: 19.89 @ 2300	2.56" Bore
Eng #:	Corr. To: 29.92/60 dry	PkHP: 10.63 @ 3300	2.437" Stroke
Customer: Peterson	Corr. Factor 1.040	12.54 cid 4 Cycle	1 Cylinders
PURDUE BAJA (2000-3800 gov) RUN #4			
Test Summary and Comments for : PURDUE021812-17 #1			
Operator: ROBERT	1:27 pm 02/18/2012	PkTq: 19.90 @ 2400	2.56" Bore
Eng #:	Corr. To: 29.92/60 dry	PkHP: 10.56 @ 3400	2.437" Stroke
Customer: Peterson	Corr. Factor 1.040	12.54 cid 4 Cycle	1 Cylinders
PURDUE BAJA (2000-3800 gov) RUN #3			

Figure 2: Power and torque curve of 10HP Briggs & Stratton Engine

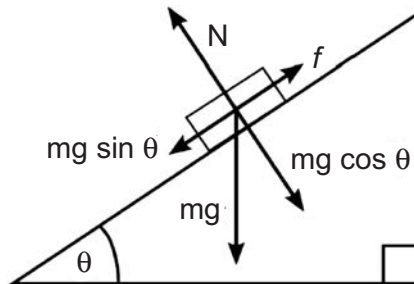


Figure 3: Free Body Diagram

Also, the diameter of the tires is set to 22". With these design constraints, it is found that the torque required at the wheels is 360Nm. Hence the minimum reduction from the gearbox will have to be 4.7. However, this reduction would give a top speed of 83 km/h, which is impossible to be achieved in a Mini Baja car because of the low power output from the engine and the huge amount of drag from the firewall. The theoretical top speed of a Mini Baja ATV is 55km/h. Hence the ratio needs to be adjusted to attain the theoretical top speed. With a reduction of 8.4066 in the gearbox, the top speed achieved can be limited to 53km/h which is extremely close to the theoretical top speed of the vehicle. This also increases the overall maximum torque to a value of 640Nm.

### 2.3. Selection of Freewheels

Plenty of options were available for the Freewheels. The two biggest manufacturers of industrial grade Freewheels are GMN and Renold. GMN is a German company and all their products are imported from Germany. However, all their products can be bought from the dealer present in Mumbai, with a lead time of about 2 weeks. Renold on the other hand has a lot more localized operation and specialize in various transmission components, from chains and sprockets to gears and freewheels. A thorough research was done on the viable options from both the brands. Freewheel model FND 473M by GMN was chosen because of its light weight and appropriate torque rating. The freewheels will have to press fit into the gear hub and hence the manufacturing of gears would require extra care.

### 2.4. Design of Components

For implementing the concept of a freewheel differential, there were two most viable options. They were:

1. Chain Drive
2. Gearbox

Both the options come with their own merits and shortcomings. While a chain drive can easily be modified later on, a gearbox would need extra effort in customization. A decision matrix was made, giving weightages to each of the factors and a decision was made based on the overall scores of the two systems.

**Table 1**  
**Chain Drive Vs Gear Drive**

<i>Aspect</i>	<i>Weightage</i>	<i>Chain Drive</i>	<i>Gear Drive</i>
Manufacturability	10	0	2
Ease of Assembly	10	1	1
Efficiency	10	1	2
Cost of Manufacturing	10	2	0
Total		40	50

Since the gear drive scored higher overall, it was decided to proceed with designing a gearbox.

#### 2.4.1. Design of Gears

A two stage reduction has been implemented to keep the assembly compact. Shigley's Mechanical design book [11] says that the most compact two stage assembly can be made by keeping the reductions at both the stages equal or as close to each other as possible. Hence a reduction of 2.933 in the first stage and 2.867 in the second stage. AISI 4340, oil quenched and tempered will be used to make the gears. The shafts will be made of EN19 as it satisfies the design constraints and also is easily available locally.

**There are two options in the type of gears:**

1. Helical gears
2. Spur gears

While the helical gears operate more silently, spur gears are easy to manufacture and don't generate axial loads. A decision matrix was made, assigning weightages to the different aspects.

**Table 2**  
**Spur Gears Vs Helical Gears**

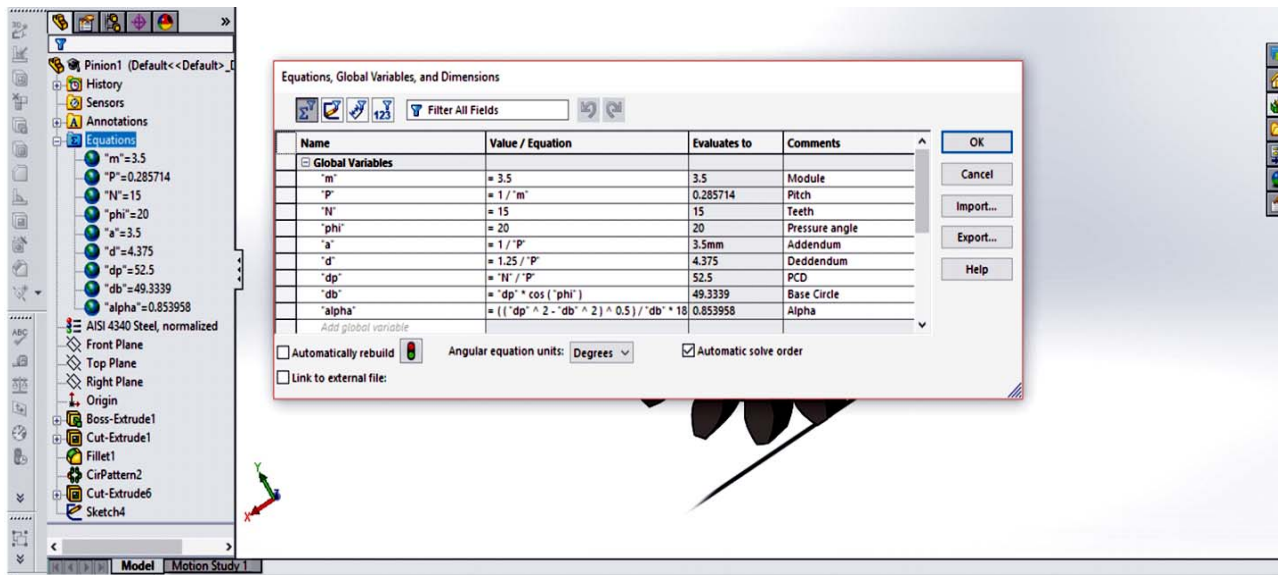
<i>Aspect</i>	<i>Weightage</i>	<i>Spur Gears</i>	<i>Helical Gears</i>
Manufacturability	10	1	0
Ease of Assembly	10	2	1
Amount of Raw Material	5	0	0
Cost of Manufacturing	10	2	1
Total		50	20

As seen from the matrix, for this particular application, spur gears come out with a higher score than helical gears due to various factors. As a result, spur gears were chosen for this paper.

Gears were design using the Lewis Bending method and designed for the Contact stresses predicted by the Hertziancontact stress equation.

For determining the loads on the gears, an averaged situation was used. Since the CVT can shift from a ratio of 3.9 to 0.9, an average shift to a ratio of 2.4 was assumed. The engine operates in the range of 1900-3800RPM. As a result, an average RPM of 2850RPM was assumed. At this RPM, the engine supplies a torque of 19.2Nm. As a result, an input torque for the first stage of gears was assumed to be 46Nm [13].

For modelling the gears, the dimensions were equation driven. It was noticed that the standard Solidworks toolbox gears don't offer much customization and also have a lot of backlash. Equation driven models on the other hand, are more accurate and don't have the inaccuracies of a default gear. They are also more customizable, which was essential for the second stage of gears in this paper.



**Figure 4: Equation Driven Dimensions for Modelling the gears**

The Figure 4 shows the different equations used to compute the dimensions of the gear. Out of all the variables, module and the number of teeth are the only independent variables, the other dimensions can be computed from these two. A parametric equation was used for making the involute curve. The two involute curves were drawn and the a cut extrude was made on the cylindrical surface. This feature was patterned all around the gear to replicate the number of teeth.

Analysis has been done on the first stage of gears for bending stresses. The result (Figure 5) shows a maximum bending stress of 72MPa, which is close to the Lewis bending value of 69MPa. The gears are made of EN24 material and will be heat treated to a hardness of HRC 45. The module of the gear has been chosen as per IS 2535.

**Table 3**  
**Design Loads on gears**

<i>Gear</i>	<i>Design Bending Stress(MPa)</i>	<i>Design Contact Stress(MPa)</i>
First Stage	250	828.125
Second Stage	205	1088.75

These actual bending loads have been obtained by using the following equations.

$$\frac{i + 1}{a * m * b * y} * [M]$$

Where,

*a* = Centre to Centredistance(mm)

*m* = module(mm)

*b* = face width(mm)

*y* = form factor

[M] = design torque(Nmm)

**Table 4**  
**Actual Loads on the Gears**

<i>Gear</i>	<i>Bending Load(MPa)</i>
First stage pinion	69
Second stage pinion	160

The specification of each of the gears designed are as follows:

**Table 5**  
**Geometry of the Gears**

	<i>Pinion 1</i>	<i>Gear 1</i>	<i>Pinion 2</i>	<i>Gear 2</i>
Module(mm)	3.5	3.5	4	4
Number of Teeth	15	44	15	43
PCD(mm)	52.5	154	60	172
Material	AISI4340	AISI4340	AISI4340	AISI4340
Rockwell Hardness	50	50	55	55
Face Width(mm)	31	31	30	30

### 2.4.2. Analysis of Gears

For conducting the analysis, the Solidworks assembly of the gears was saved in STEP format and imported to ANSYS Workbench. A no-separation [7] contact was created between the meshing teeth. The inner face of the gear was defined as a fixed support, whereas the pinion was defined as a Frictionless Support. The desired moment was applied to the inner face of the pinion. A body sizing of 2mm was used. The mesh was refined using a contact sizing of 0.7mm. This resulted in a total of 498708 elements.

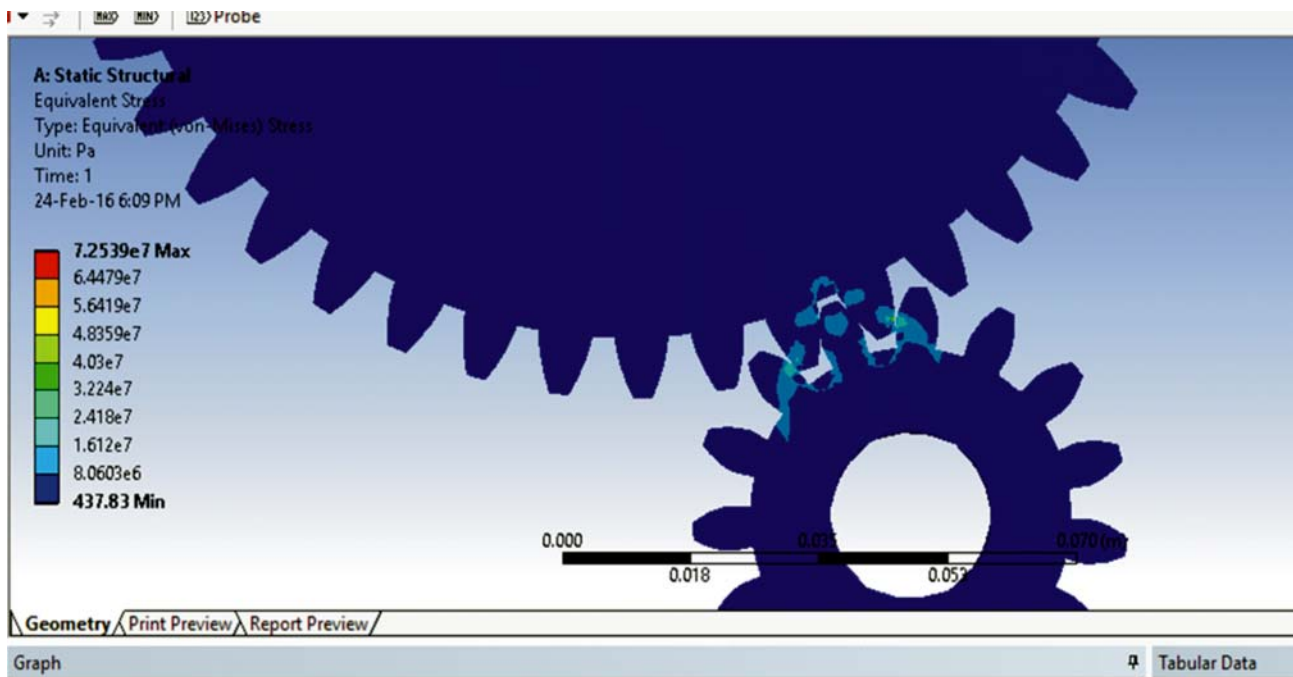


Figure 5: Bending stress on a sample gear set

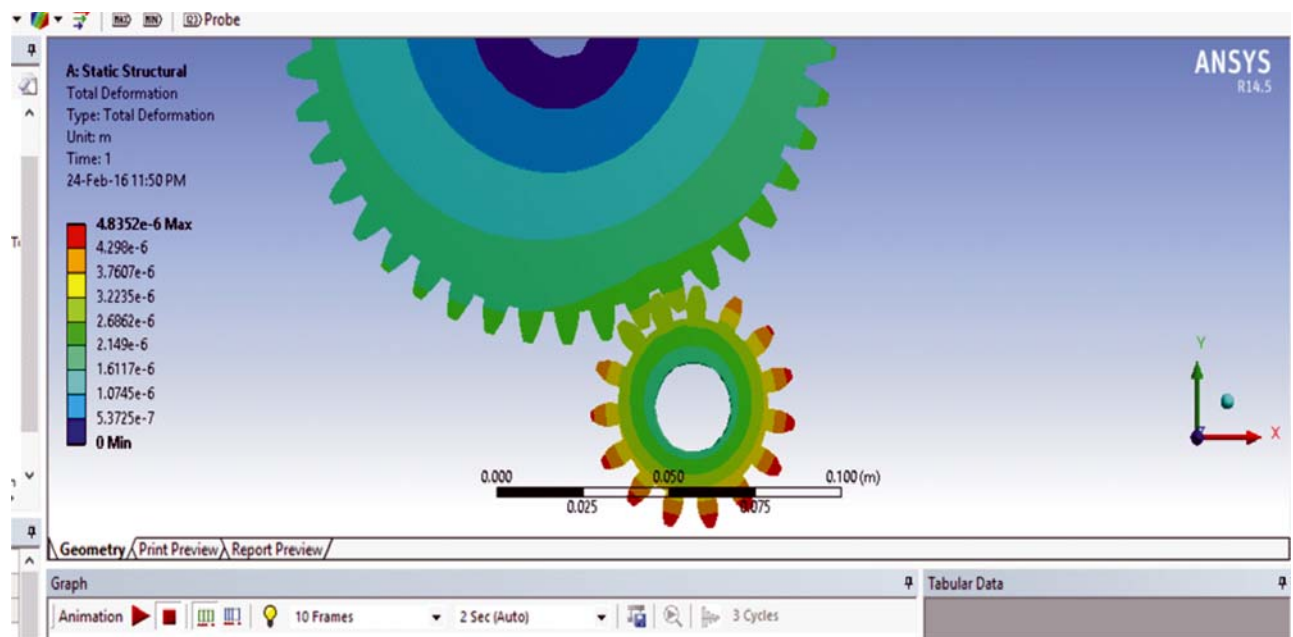


Figure 6: Deformation on a sample gear set



Spur gears have been chosen for this particular application as it can handle the load easily and is also cheaper to manufacture overall. Because the speed of the rotation is relatively low, the noise produced isn't an irritant. The stress computed (72MPa) is almost equal to the calculated stress of 69MPa. This shows that the results from the analysis are accurate.

**The gears have been analyzed for the following situations:**

1. Average torque with a fixed support at the gear
2. Average input torque with a frictionless support at the gear
3. Maximum possible input torque with fixed support at the gear
4. Maximum possible input torque with frictionless support at the gear

Having a fixed support on the gear emulates the situation of the car being stuck at a place and not being able to move. Whereas, the frictionless support at the gear emulates the situation of the car moving on the road [4].

The maximum torque has been computed using the power and torque curve shown in Figure 2, with the assumption that the CVT has not shifted out and hence is at a ratio of 3.9.

**Table 6**  
**Input torque on the first reduction**

Parameter	Value
Average Input Torque	46Nm
Maximum Input Torque	76.674Nm

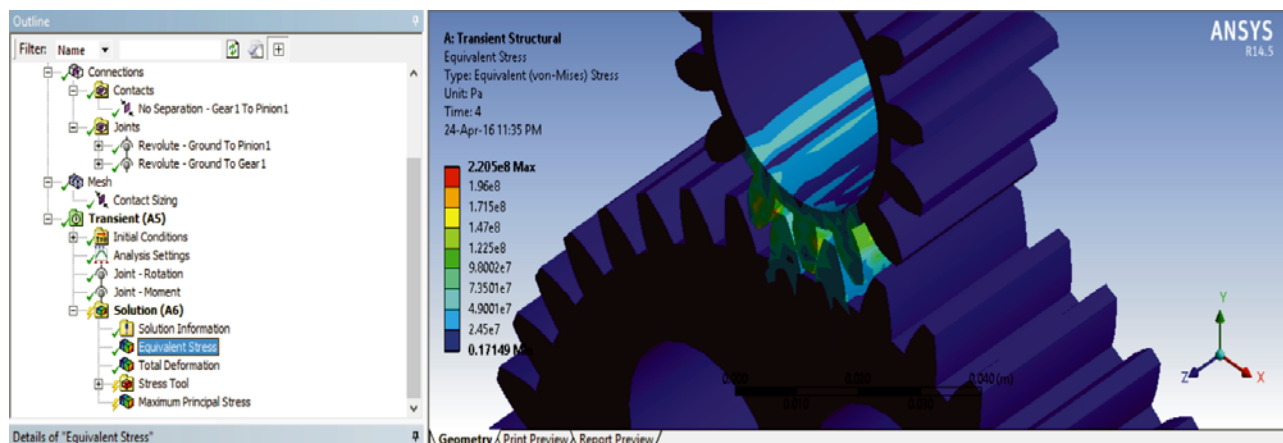
A transient analysis was also conducted on the gears to further examine the design.

**Transient Analysis:** The first stage of the gears was examined for stresses in transient conditions. The gears were analyzed for the following conditions:

**Table 7**  
**Transient loading conditions**

	Speed(RPM)	Torque(Nm)
Case 1	718	76.67
Case 2	4222	17.1

**Case 1:** Following were the results for the first case.



**Figure 7: Von Mises stresses on the first stage of gears in case 1 (Transient)**

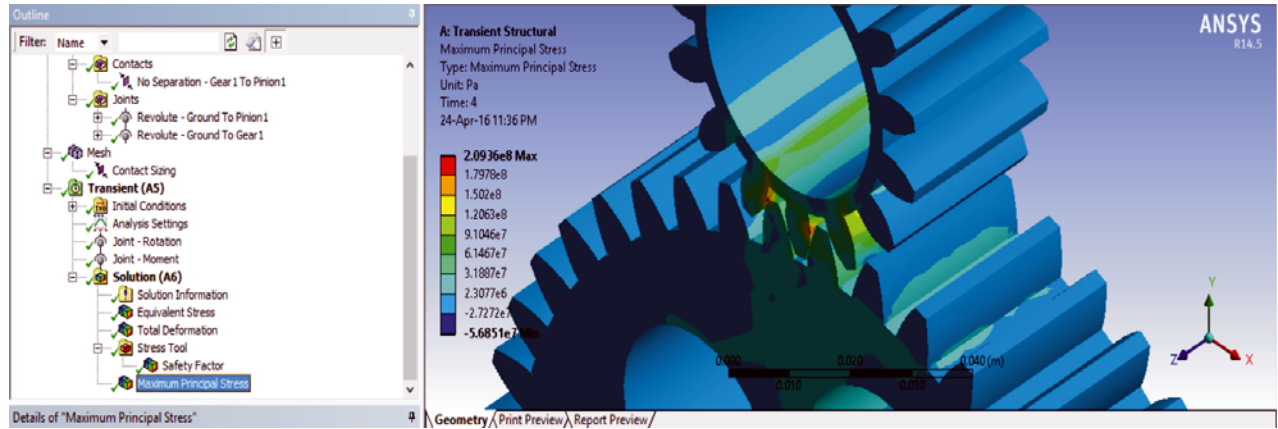


Figure 8: Maximum principal stresses on the first stage of gears in case 1 (Transient)

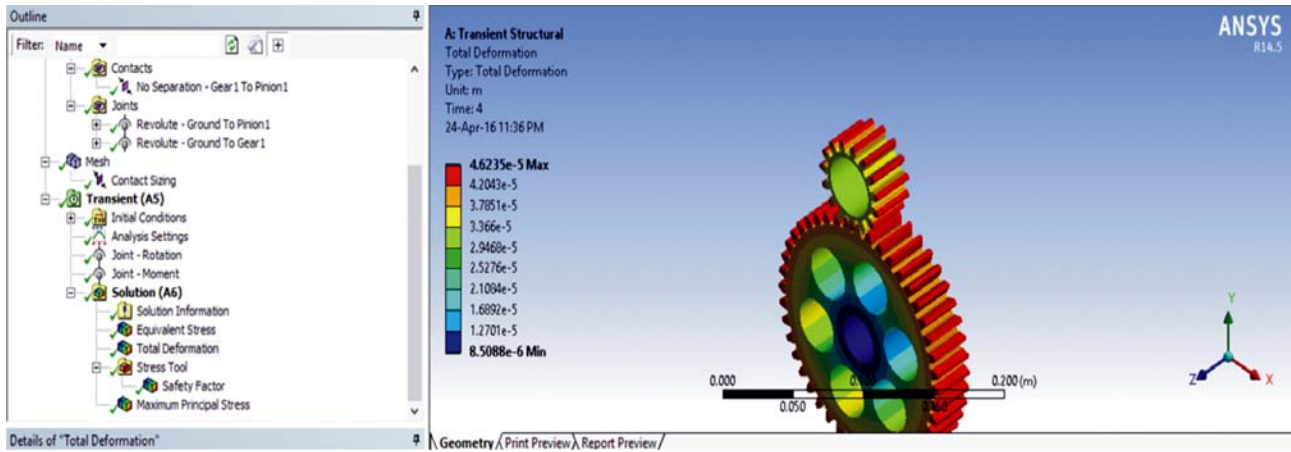


Figure 9: Total deformation of the first stage of gears in case 1 (Transient)

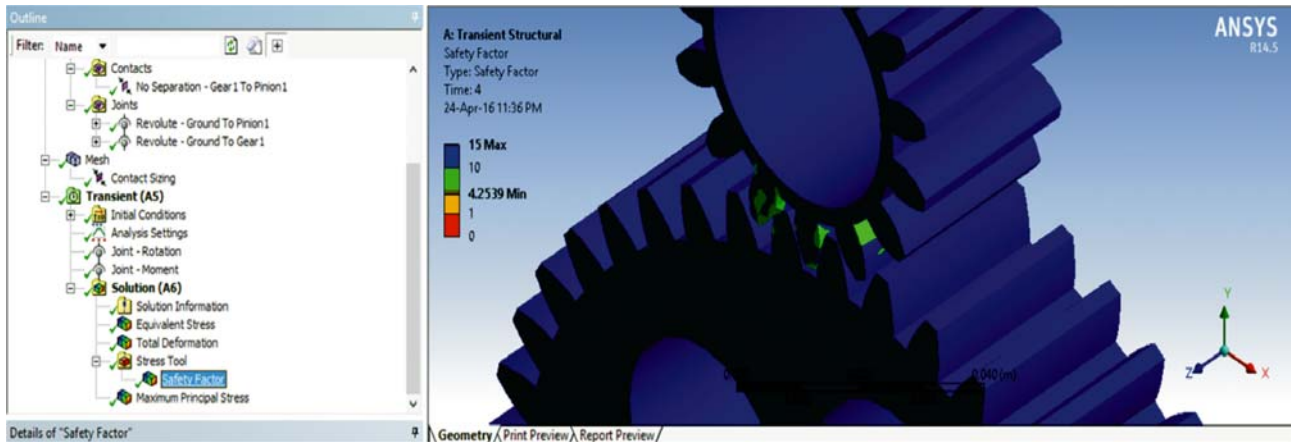


Figure 10: Factor of safety of the first stage of gears in case 1 (Transient)

As seen from the results, the FOS of the gears is 4.25, which is quite safe. This further validates the design for high torque scenarios. However, this had already been determined from the various cases of the static analysis.

**Case 2:** The primary aim of this analysis was to verify the robustness of this desing at a high speed and a low torque. The input RPM has been calculated assuming the Engine is governed at 3800RPM and the CVT has fully shifted out to a ratio of 0.9. The torque has been computed by extracting the value of the torque output from the dyno results of the engine.

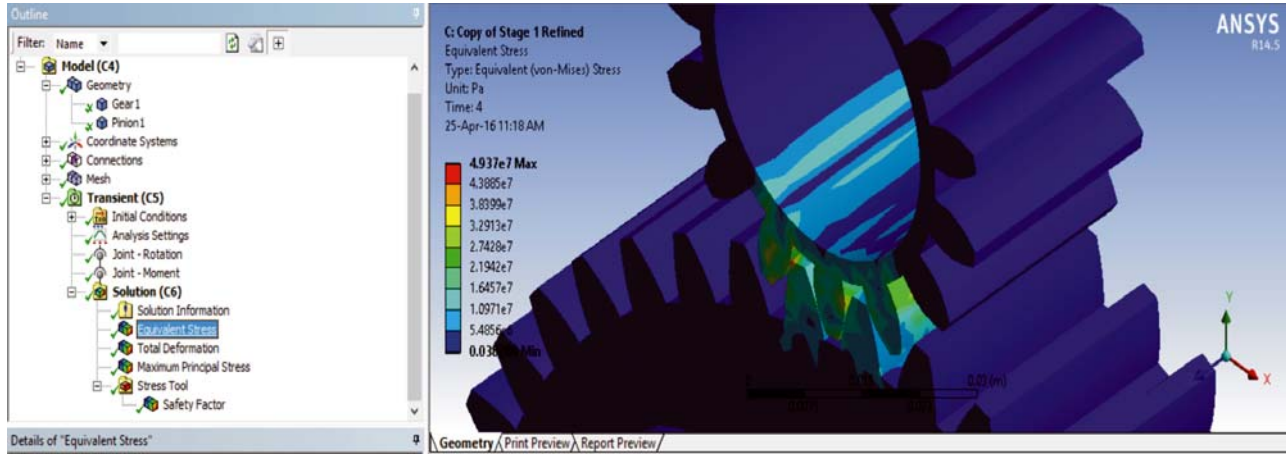


Figure 11: Von Mises stresses on the first stage of gears in case 2 (Transient)

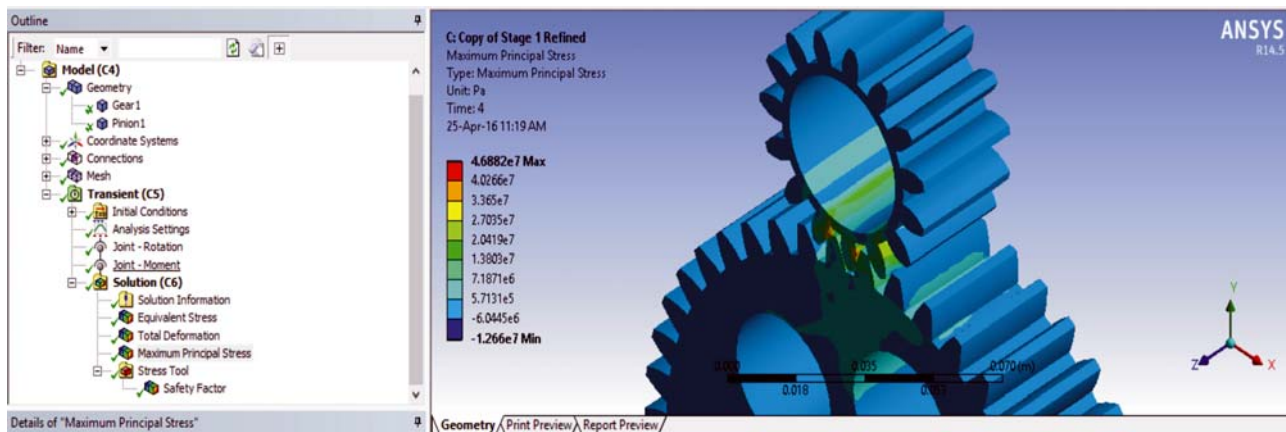


Figure 12: Maximum principal stresses on the first stage of gears in case 2 (Transient)

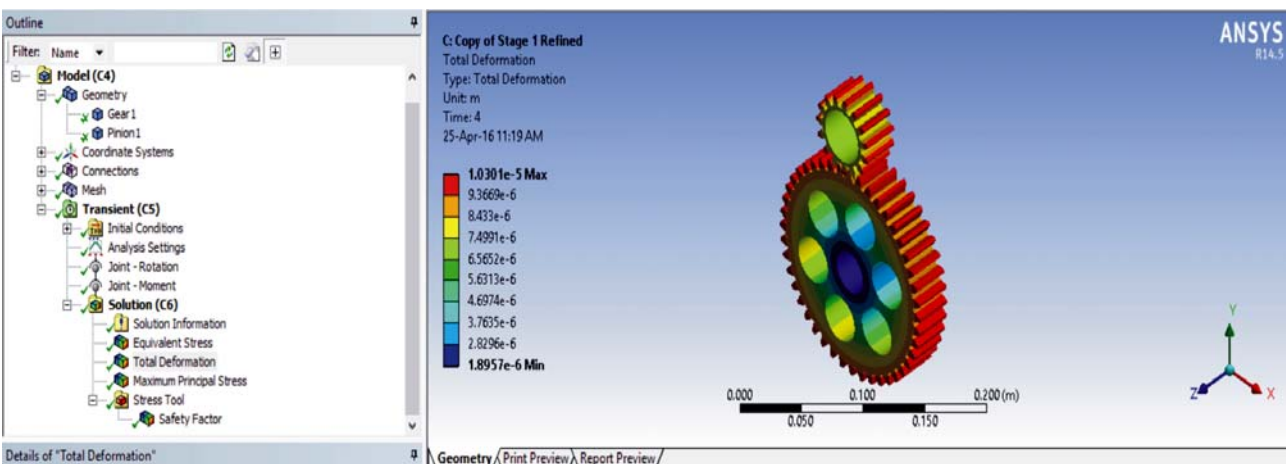
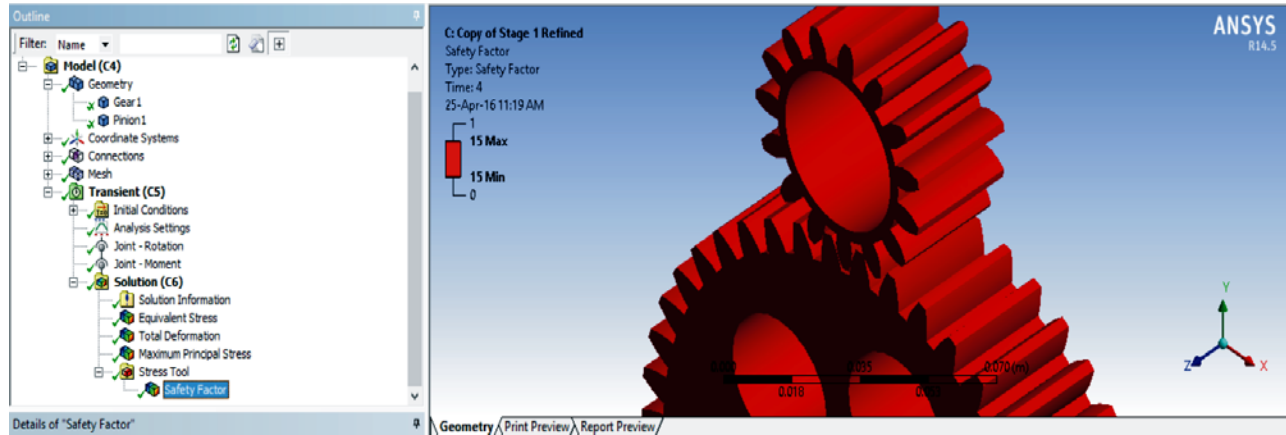


Figure 13: Total deformation on the first stage of gears in case 2 (Transient)



**Figure 14: Factor of safety of the first stage of gears in case 2 (Transient)**

As seen from the results, the minimum FOS is 15. Hence a high speed condition will do the gears no harm whatsoever.

For conducting a static analysis on the assembly, a bonded contact was set up between the shafts and the corresponding gears. Each of the gears had a No-Separation between them. The shafts were provided a cylindrical support at the bearing mounts. A moment on 46Nm was applied on the input shaft and the output shaft was fixed. A mesh was generated and then refined using contact sizing on 0.01m. A finer mesh was tried out, but it failed to give a result even after 24 hours, due to the lack of computational power.

**Table 9  
Technical Specifications**

<i>Operational Specifications</i>	<i>Value</i>
Max input torque	78Nm
<i>Performance Specifications</i>	
Gear Ratio	8.4066
Number of Stages	2
Weight	<14kg
Rated output speed	550RPM

### 3. CONCLUSION

A lot of work has been put in for the completion of this paper. However, no paper is perfect and this one is no exception. Many things were tried out during the course of this paper, however, not every task could be executed due to the lack of time and resources. A multiphase flow analysis of the lubricants inside the gearbox is a perfect example of one such task. The CFD problem was extremely difficult to set up and was giving many errors. Lack of reference material added to the problem. Due to the lack of time, this task couldn't be completed. Any future work can involve a CFD expert for this very reason. Apart from this, analysis on the effects of the vibration from the engine couldn't be accounted for, due to the lack of skills. It would've been ideal, if the entire design could be manufactured within this time period. For any future work, the researchers should focus on getting a small scale prototype ready and test that for vibrations and for analyzing the flow. This would give enough data on what refinements are to be made on the existing design in order to make the gearbox work better. Apart from this, there is also a lot of scope of weight reduction of the components, as discussed earlier. The present model has a total weight of about 13.5kg.

This product developed as a result of this paper, as a lot of potential to have a huge impact on the automotive industry. This concept is best suited for family vehicles, as they are built on a budget and demand a high fuel efficiency. This product can also revolutionize the differentials used in Baja SAE competitions. With the positive of having developed a really innovative product, this paper leaves a huge scope of further improvement.

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