International Review of Pure and Applied Mathematics
Volume 15 • Number 1 • Jan-June 2019

A COMPARATIVE STUDY ON OPTIMAL SOLUTION IN FUZZY TRANSPORTATION PROBLEM USING FUZZY NUMBERS

P.Priya & R.Priyanka

Abstract: This paper is concerned with the closed, bounded, and non empty feasible region of the transportation problem using fuzzy numbers which ensures the existence of an optimal solution to the balanced fuzzy transportation problem. The objective is minimize the transportation cost of some commodities through a complicated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Then the fuzzyfication of the cost of the transportation problem is discussed with the help of the numerical example. Finally we compared to the transportation cost.

Keywords: Intuitionistic triangular fuzzy number, Proposed Method, ABLC Method.

1. PRELIMINARIES

1.1 Fuzzy number

A fuzzy set A is defined on the set of real number R is said to be **Fuzzy number** if its

membership function $\mu_A: R \to [0,1]$ has the following properties.

a. A should be a common Fuzzy set.
b. α_A must be a closed interval for every $\in [0,1]$.
c. The support of A_1, O^+A should be bounded.

1.2 Intuitionistic Fuzzy Number

A intuitionistic fuzzy set $\tilde{A}'$ defined on the set of real number R is said to be intuitionistic fuzzy number

a. There exist an $A = \{\mu_{\tilde{A}'}(x), v_{\tilde{A}'}(x) : x \in X\}$.
   Where x is called the mean value of $\tilde{A}'$.
b. $\mu_{\tilde{A}'}$ is continuous mapping from R to the closed interval [0,1] and for all $x \in R$.

1.3 Intuitionistic Triangular Fuzzy Number

A triangular intuitionistic fuzzy number $\tilde{A}'$ is an intuitionistic fuzzy subset in R with the following membership function $\mu_{\tilde{A}'}(x)$ and non membership function $v_{\tilde{A}'}(x)$. Which is denoted by $\tilde{A}' = (p_1, p_2, p_3; p'_1, p_2, p'_3)$.

$$\mu_{\tilde{A}'}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, & p_1 \le x \le p_2 \\ \dfrac{p_3 - x}{p_3 - p_2}, & p_2 \le x \le p_3 \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{A}'}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p'_1}, & p_1 \le x \le p_2 \\ \dfrac{p_3 - x}{p'_3 - p_2}, & p_2 \le x \le p_3 \\ 0, & otherwise \end{cases}$$

where $p'_1 \le p_1 \le p_2 \le p_3 \le p'_3$ and $\mu_{\tilde{A}'}(x), v_{\tilde{A}'}(x) \le 0.5$ for $\mu_{\tilde{A}'}(x), = v_{\tilde{A}'}(x) \in R$.

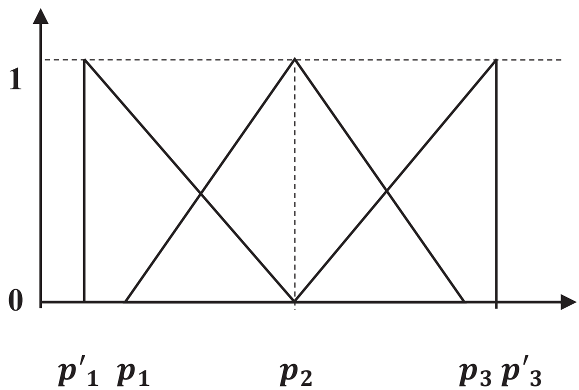


Figure 1. Intuitionistic Triangular Fuzzy Number

1.4 Definition

We define a accuracy function

Intuitionistic Triangular Fuzzy Number $a = \frac{(a_1+2a_2+a_3);(a'_1+2a_2+a'_3)}{8}$

2. METHODOLOGY

Even though there are many methods to find the basic feasible solution for a fuzzy transportation problem we have analysed a fuzzy verson of North west corner method, ABLC Method and Proposed method for solving fuzzy transportation problem.

Step:1 Construct the fuzzy transportation problem is balanced, if it is not.

Step:2

Proposed Method

(i) Choose the primary row (source) and verify that column (destination) unit has cost. Write the source under column 1 and related destination under column 2. Maintain this process for each source. However if any source has more than one same minimum value in different destination. Then write all these destination under column 2.

(ii) Choose those rows under colum1 which have unique destination. If destinations are not unique then go to step 3. Next delete that row or column where supply/demand exhausted.

(iii) If destination under column 2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all tjose sources where destinations are identical.

(iv) Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row or column where supply/demand exhausted.

(v) Repeat step3 and step4 for remaining sources and destinations till $(p + q - 1)$ cells are allocated.

(vi) Total cost is calculated as sum of product of cost and corresponding allocated value of Supply /demand.

$$\text{i.e., Total fuzzy cost} = \sum_{i=1}^{p} \sum_{j=1}^{q} C_{ij} X_{ij}$$

Step:3

Apex Base Least Cost Method

(i) Select the row or column corresponding to the least demand or supply.

(ii) Select the cell in an exceedingly marked row/ or column with minimum value. If tie happens, choose the cell with the smallest value average of it's fuzzy supply and demand.

(iii) Find the adjust row value and adjust column value for the chosen cell in the selected row/or column and select the adjust cell with minimum value. Allocate the corresponding minimum of its supply and demand.

(iv) After performing Step5 delete the row or column (where supply or demand becomes zero) for more computation.

(iv) Repeat Step4 to Step6 until all the demands are satisfied and all the supplies are exhausted.

(vi) This allotment yields a fuzzy initial basic feasible solution to the given fuzzy transportation problem.

$$\text{i.e., Total fuzzy cost} = \sum_{i=1}^{p}\sum_{j=1}^{q} C_{ij}X_{ij}$$

Step:3

Proposed Method

(i) Select the primary row (source) and verify that column (destination) unit has cost. Write the source under column 1 and related destination under column 2. Maintain this process for each source. However if any source has more than one same minimum value in different destination. Then write all these destination under column 2.

(ii) Select those rows under colum1 which have unique destination. If destinations are not unique then go to step 3. Next delete that row or column where supply/demand exhausted.

(iii) If destination under column 2 is not unique then select those sources where destinations are identical. Next find the difference between minimum and next minimum unit cost for all tjose sources where destinations are identical.

(iv) Check the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row or column where supply/demand exhausted.

(v) Repeat step3 and step4 for remaining sources and destinations till $(p + q - 1)$ cells are allocated.

(vi) Total cost is calculated as sum of product of cost and corresponding allocated value of Supply /demand.

$$\text{i.e., Total fuzzy cost} = \sum_{i=1}^{p}\sum_{j=1}^{q} C_{ij}X_{ij}$$

3. NUMERICAL EXAMPLE

A company has three sources S1,S2, and S3 and four destinations D1,D2,D3 and D4; the fuzzy transportation cost for unit quantity of the product from i^{th} source to the j^{th} destination is C_{ij}.

(-1,0,1;2,3,4)	(0,1,2;3,4,5)	(8,9,10;11,12,13)	(4,5,6;7,8,9)
(-2,-1,0;1,2,3)	(-3,-2,-1;0,1,2)	(2,4,5;6,7,8)	(-3,-1,0;1,2,4)
(2,3,4;5,6,7)	(3,6,7;8,9,10)	(11,12,14;15,16,17)	(5,6,8;9,10,11)

where $[c_{ij}]$ = and fuzzy availability of the product at sources are (1,3,5;6,7,8), (-2,-1,0;1,2,4),(5,6,7;10,12,13)

and the fuzzy demand of the product at destinations are (4,5,6;7,8,9),(1,2,3;5,6,7), (0,1,2;3,4,5),(-1,0,1;2,3,4) respectively,

4. SOLUTION

Step:1

Given fuzzy transportation problem is balanced one.

Table-1

Sources/ Destinations	D1	D2	D3	D4	Supply
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5)	(8,9,10; 11,12,13)	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S2	(-2,-1,0; 1,2,3)	(-3,-2,-1; 0,1,2)	(2,4,5; 6,7,8)	(-3,-1,0; 1,2,4)	(-2,-1,0; 1,2,4)
S3	(2,3,4; 5,6,7)	(3,6,7; 8,9,10)	(11,12,14; 15,16,17)	(5,6,8; 9,10,11)	(5,6,7; 10,12,13)
Demand	(4,5,6; 7,8,9)	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

Step:2

The minimum cost value for the corresponding sources S1,S2,S3 which represents the destinations D1,D2 respectively which is shown in Table-2

Table-2

Column 1	Column 2
S1	D1
S2	D2
S3	D1

Hence the destination D2 is unique for source S2. and allocate (S2,D2) in (-2,-1,0;1,2,4) (1,2,3;5,6,7) = (-2,-1,0;1,2,4).

Table-3

	D1	D2	D3	D4	Supply
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5)	(8,9,10; 11,12,13)	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S2	(-2,-1,0; 1,2,3)	(-3,-2,-1; 0,1,2) **(-2,-1,0;1,2,4)**	(2,4,5; 6,7,8)	(-3,-1,0; 1,2,4)	(-2,-1,0; 1,2,4)
S3	(2,3,4; 5,6,7)	(3,6,7; 8,9,10)	(11,12,14; 15,16,17)	(5,6,8; 9,10,11)	(5,6,7; 10,12,13)
Demand	(4,5,6; 7,8,9)	(1,2,3; 5,6,7) **(3,3,3;4,4,3)**	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

Delete row S2. And adjust demand {(1,2,3;5,6,7)- (-2, 1,0;1,2,4)}=(3,3,3;4,4,3).

Next the minimum cost value for the corresponding source S1,S3, and destination D1 respectively which shown in Table-4

Table-4

Column 1	Column 2
S1	D1
S3	D1

Hence the destinations are not unique. Because Sources S1,S3 have identical destination D1,D1.So we find the difference below minimum and next minimum unit cost for the sources S1,S3. The difference are (1,1,1;1,1,1) , (1,3,3;3,3,3) respectively for the sources S1,S3.

Hence the maximum difference is (1, 3, 3; 3, 3,3) represents S3. Now allocate (S3, D1)

Min{(5,6,7;10,12,13)(4,5,6;7,8,9)}=(4,5,6;7,8,9).

Proceeding like this..,

Table-5

Sources/ Destinations	D1	D2	D3	D4	Supply
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5) **(3,3,3;4,4,3)**	(8,9,10; 11,12,13) **(- 2,0,2;2,3, 5)**	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S2	(-2,-1-0; 1,2,3)	(-3,-2,-1; 0,1,2) **(-2,- 1,0;1,2,4)**	(2,4,5; 6,7,8)	(-3,-2,0; 1,2,4)	(-2,-1,0; 1,2,4)
S3	(2,3,4; 5,6,7) **(4,5,6;7,8, 9)**	(3,6,7; 8,9,10)	(11,12,14; 15,16,17) **(2,1,0;1,1, 0)**	(5,6,8; 9,10,11) **(-1,0,1;2,3,4)**	(5,6,7; 10,12,13)
Demand	(4,5,6; 7,8,9)	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

The total cost associated with these allocations is

X_{12} = (0,1,2;3,4,5)(3,3,3;4,4,3)

X_{13} = (8,9,10;11,12,13)(-2,0,2;2,3,5)

X_{22} = (-3, 2, 1;0,1, 2)(-2,- 1,0;1,2,4)

X_{31} = (2,3,4,;5,6,7)(4,5,6;7,8,9)

X_{33} = (11,12,14;15,16,17) (2,1,0;1,1,0)

X_{12} = (5,6,8;9,10,11)(-1,0,1;2,3,4)

Finally the total transportation cost is = (3,28,58;102,148,182)

=87.125

Step: 3

Using Apex Base Least Cost Method

Table-6

	D1	D2	D3	D4	S
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5)	(8,9,10; 11,12,13)	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S2	(-2,-1,0; 1,2,3) **(-2,-1,0;1,2,4)**	(-3,-2,-1; 0,1,2)	(2,4,5; 6,7,8)	(-3,-1,0; 1,2,4)	(-2,-1,0; 1,2,4) ←
S3	(2,3,4; 5,6,7)	(3,6,7; 8,9,10)	(11,12,14; 15,16,17)	(5,6,8; 9,10,11)	(5,6,7; 10,12,13)
Demand	(4,5,6; 7,8,9) **(6,6,6;6,6,5)**	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

Delete row S2. and adjust demand {(4,5,6;7,8,9)- (-2,-1,0;1,2,4)}=(6,6,6;6,6,5)

Table-7

	D1	D2	D3	D4	S
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5)	(8,9,10; 11,12,13)	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S3	(2,3,4; 5,6,7)	(3,6,7; 8,9,10)	(11,12,14; 15,16,17)	(5,6,8; 9,10,11) **(-1,0,1;2,3,4)**	(5,6,7; 10,12,13) **(6,6,6;8,9, 9)**
Deman d	(4,5,6; 7,8,9) **(6,6,6;6,6, 5)**	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

Delete column D4. and adjust supply {(5,6,7;10,12,13) - (-1,0,1;2,3,4)} = (6,6,6;8,9,9).

Table-8

	D1	D2	D3	Supply
S1	(-1,0,1; 2,3,4)	(0,1,2; 3,4,5) **(1,2,3;5,6,7)**	(8,9,10; 11,12,13)	(1,3,5; 6,7,8) **(0,1,2;1,1,1)**
S3	(2,3,4; 5,6,7)	(3,6,7; 8,9,10)	(11,12,14; 15,16,17)	(5,6,7; 10,12,13) **(6,6,6;8,9,9)**
Demand	(4,5,6; 7,8,9) **(6,6,6;6,6,5)**	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	

↑

Delete column D2.and adjust supply {(1,3,5;6,7,8)-(1,2,3;5,6,7)} = (0,1,2;1,1,1).

Proceeding like this..,

Table-9

Sources/ Destinations	D1	D2	D3	D4	S
S1	(-1,0,1; 2,3,4) **(0,1,2;1,1,1)**	(0,1,2; 3,4,5) **(1,2,3;5,6,7)**	(8,9,10; 11,12,13)	(4,5,6; 7,8,9)	(1,3,5; 6,7,8)
S2	(-2,-1,0; 1,2,3) **(-2,-1,0;1,2,4)**	(-3,-2,-1; 0,1,2)	(2,4,5; 6,7,8)	(-3,-1,0; 1,2,4)	(-2,-1,0; 1,2,4)
S3	(2,3,4; 5,6,7) **(6,5,4;5,5,4)**	(3,6,7; 8,9,10)	(11,12,14; 15,16,17) **(0,1,2;3,4,5)**	(5,6,8; 9,10,11) **(-1,0,1;2,3,4)**	(5,6,7; 10,12,13)
Demand	(4,5,6; 7,8,9)	(1,2,3; 5,6,7)	(0,1,2; 3,4,5)	(-1,0,1; 2,3,4)	

The total cost associated with these allocations is

$$X_{11} = (-1,0,1;2,3,4)\ (0,1,2;1,1,1)$$

X_{12} = (0,1,2;3,4,5)(1,2,3;5,6,7)

X_{21} = (-2,-1,0;1,2,3)(-2,-1,0;1,2,4)

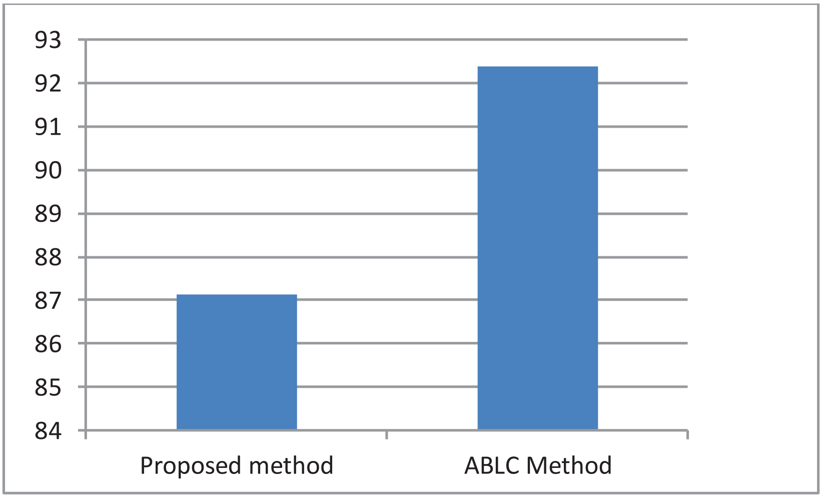
X_{31} = (2,3,4,;5,6,7)(6,5,4;5,5,4)

X_{33} = (11,12,14;15,16,17)(0,1,2;3,4,5)

X_{34} = (5,6,8;9,10,11)(1,0,1;2,3,4)

Finally the total optimal Cost is = (3,28,60;106,153,208)

= 92.375



5. COMPARATIVE STUDY

METHOD	OPTIMUM SOLUTION
Proposed Method	87.125
ABLC Method	92.375

6. CONCLUSION

In this paper, simple yet efficient parametric method was introduced to solve fuzzy transportation problem by using intuitionistic fuzzy numbers. The feasible region of the fuzzy transportation problem which ensures the existence of an optimal solution to the balanced fuzzy transportation problem. This method can be used for all kinds of fuzzy transportation problem, whether intuitionistic triangular fuzzy numbers with expected and unexpected data. The fuzzification of the cost of the fuzzy transportation problem is discussed with the help of numerical example.

Depends on the problem the reliability condition may be differing. These methods are easy to realize and apply. So it will be very helpful for decision makers who are production with logistic and supply chain problems.

REFERENCES

1. P.K. Gupta and Man Mohan, *Problems in Operation Research*, Sultan Chand and Sons, 2006
2. Dr. S. Murugesan and Mr.S. Srinivasan . *A new algorithm for solving fuzzy transportation problems with trapezoidal fuzzy numbers*. International journal of recent trends in engineering & research. ISSN 2455-1457, impact factor 3.344(2016)
3. Thangaraj Beaula, M.Priyadharshini, *Apex Base Least Cost Method for Fuzzy transportation Problem*, International Journal of Scientific & Engineering Research, Issue 2, february 2015. ISSN 2229-5518.
4. R. Jhon Paul Antony, S. Johnson Savarimuthu, T.Pathinathan. *Method for solving The transportation problem Using triangular intuitionistic fuzzy numbers.* International journal of Computing Algorithm.(IIR)(3 Feb 2014)
5. Md. Yasin Ali, Abeda sultana and A F M Khodadad khan. Comparision of fuzzy multiplication operation on triangular fuzzy number.IOSR Journal of mathematics.e-ISSN 2278-5728.p-ISSN:2319-765X.(July-Aug 2016).
6. K.Pramila and G.Uthra *Optimal solution of an intuitionistic Fuzzy Transportation Problem Annals of pure and applied Mathematics* Vol.8,No.2,2014, 67-73 ISSN:2279-087(P), 2279-0888, Publised on 17 December 2014.

P. Priya
Assistant Professor,
Department Of Mathematics,
St.Josep's College For Women

R. Priyanka
Assistant Professor,
Department Of Mathematics,
St.Josep's College For
Women, Tirupur, Tamil Nadu.