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Method for Estimating An Origin-Destination Matrix for Dynamic Assignment

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Abstract : The ability to control the real-time distribution of correspondences in the network and changes in traffic volumes allows us to predict congestions and to prevent undesirable effects of a sharp increase of the traffic flow in a particular section of the network. The operational efficiency of managerial decisions is directly dependent on the accuracy of predicted parameters of the traffic flow control. We propose a method for estimating dynamic changes in a correspondene matrix which applies a mesoscopic mathematical model of the network traffic flow distribution. Our model is based on the hypothesis of time distribution according to the generalized Erlang law. We developed a method of recursive extension of the Erlang parameters definition which employs Kalman filtering. When drawing an OD matrix based on the traffic counts data, it is the most difficult to form a link pro-portion matrix. We suggest an algotithm for estimating changes in a link proportion matrix in real time. A method of designing an OD matrix with the help of the known link proportion matrix has been developed and is described in this paper. In order to ensure the accuracy and reliability of estimation by Kalman Filtering, the estimated data are adjusted at each stage according to the data obtained by direct measurements.

Keywords : Real-Time Traffic, Mathematical Model, Origin-Destination Matrix, Link Pro-portion Matrix, Kalman Filtering.

1. INTRODUCTION

Nowadays the problem of predicting the network traffic flow distrribution in real time is very important for solving the problems of efficient traffic management. Having tools for on-line estimating the parametres of distribution of vehicles in an urban network we can calculate and predict delays for different traffic modes at approaching an intersection, prove the expediency of traffic control with or without traffic lights, determine the optimal parameters of traffic lights regulation with a short delay in response to the changes in traffic flow density in various directions, and select optimal routes for particular vehicles.

During the solving of the above mentioned problems very important is the information on the number of vehicles moving in the network from one point to another. This information is provided by the origin-destination matrix. Traditional methods of its estimation require large-scale selective studies which include selective surveys, number plates recording, etc. Moreover, these data are becoming outdated in the process of their collecting. That is why since 1970s there have been developed models that use data on traffic density in the network links to update the OD matrix [13].

Methods of drawing an OD matrix are currently divided into static and dynamic. Static approaches assume that fraffic flow density is time-independent or remains constant for a long period. Dynamic approaches suppose that traffic flow parameters are continuously changing. Due to the development of Intelligent Transport Systems during the last two decades the focus has been on dynamic origin-destination matrices. Particularly useful are methods which allow us to form an OD matrix using the data on traffic density since this value can be easily measured by sensors/detectors.

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For a static OD matrix, some reseacheres consider a transportation network as a sort of physical system. The idea of the gravity model was inspired by Newton's gravitational law according to which all bodies are gravitated with the force directly proportional to the product of their masses and inversely propotional to the squared distance between them. In a transportation system points generating / receiving traffic flows are similar to bodies, the total volume of OD flow is similar to the mass, and any costs of travel correspond to a physical distance. The gravity model was modified by adding extra conditions such as various balancing limitations and adjusted to the actual transportation network situation.The gravity model and its modifications (Gravity-Opportunity models) were employed for OD matrices by Robillard [12], Hogberg [4], Tamin and Willumsen [15].

Wilson [19], Van Zuylen [16], Van Zuylen and Branston [17] and some other researchers implemented the information minimization and entropy maximization methods for drawing OD matrices. The both methods are, in fact, identical. By analogy with the gravity model, the entropy model arose from physics, namely, from the second law of thermodynamics. It says that any closed system tends to approach a steady balance which is characterized by a maximum entropy of this system. A transportation system as a system of movememt of individuals in the urban network is characterized by a huge number of uncontrolled objects. With assupmption of permanent travel costs along the paths and the invariability of the network topology, a transportation system may be considered a closed one. The concept of entropy in traffic simulation was introduced by Wilson [19].

When drawing an OD matrix based on the traffic counts data, it is the most difficult to form a link proportion matrix which determins the traffic volume on each of the links for a certain origin-destination pair. To obtain an adequate model we must consider the cost of travel on links and in nods. In such cases the most widely applied theory is the transportation equlibrium method. Nguyen [10] was the first to apply this approach based on the equilibrium theory for estimating the OD matrix by mathematical programming. Yang, Sasaki, Iida, Meng and Bell [21, 22], Cascetta and Posterino [3] designed OD matrices by using various mathematical models and considering the transportation network loading. In this case non-linear programming is employed for obtaining OD matrices.

Path flow estimation-based models have been recently introduced. These determine the interrelationship between the OD matrix elememts and travel costs. The main idea of these models is in using a travel cost for each link in order to establish the user equilibrium (Sherali et al [14]; Nie and Lee [11]).

The main methods for estimating the dynamic origin-destinaytion matrix are the Kalman Filter [5] and a state-space model. By Kalman Filtering we can minimize the gap between the estimated results and the actual data. Kalman Filtering is an optimal linear state estimation process for a dynamic system which provides the minimum error variance by recursive estimation.

For dynamic updating of OD matrix Kalman Filtering was used by Ashok and Ben-Akiva [1, 2], Zhou, Qin and Mahmassani [20]. Nanne J. van der Zljpp and Rudi Hamerslag [6], having compared the Kalman Filtering with the least-square analysis and the constrained optimization, argued that the Kalman Filtering was more efficient. Unlike the reserches based on statistic methods of OD matrix estimation the reseaches employing dynamic OD matrix estimation are not numerous. They mainly study intersections, freeways and small closed networks.

In the present paper we suggest a method for estimation of dynamic changes in origin-destination matrices based on the mesooscopic matematical model of traffic flow distribution in the network developed by the author.

1. The Structure of Matrix $A_{STREETS}$ and Matrix $B_{INTERSECTION}$ Containing Information on the Network Traffic Flow Distribution

A detailed description of the developed mathematical model of traffic flow distribution which is suggested for drawing a dynamic origin-destination matrix is given in our earlier paper [9]. The model was later called TIMeR_Mod (**T**ransportation **I**ntelligent **Me**soscopic **R**eal-time **Mod**el). Initial data for solving transportation problems by TIMeR_Mod are the current scheme of urban network traffic and parameters of traffic flow distribution on lanes. The key hypothesis of our model is the assumption of time intervals between vehicles in lanes are distributed according to the Erlang law [18]. For this reason, to describe a traffic flow by TIMeR_Mod is to provide the generalized Erlang parameters. All the necessary data for this model are given in Matrix $A_{STREETS}$ and Matrix. Their structure is given below.

I. $A_{STREETS} = (S_1\ S_2\ S_3\ S_4$ Contr Pr Len Col AL... BL...)

1. No = the line number of matrix (it corresponds to the number of graph links joining Node I and Node II; the number of lines corresponds to the number of links);
2. S_1 and S_2 = the intersected freeways which form Node I;
3. S_3 и S_4 = the intersected freeways which form Node II;
4. Contr = the type of node (a signalized intersection or a simple one)
5. Pr = the priority (the main road or the minor road);
6. Len = the link length;
7. Col = the number of flows on the link;
8. AL = permissibility of turning to the left at Node I from Direction A;
9. AS = permissibility of through traffic at Node II from Direction A;
10. AR = permissibility of turning to the right at Node II from Direction A;
11. λA1, λA2 etc. = parameters λ in Direction A;
12. *k*A1, *k*A2 etc. = parameters *k* in Direction A;
13. BL = permissibility of turning to the left at Node I from Direction B;
14. BS = permissibility of through traffic at Node I from Direction B;
15. BR = permissibility of turning to the right at Node I from Direction B;
16. λB1, λB2 etc. = parameters λ in Direction B;
17. *k*B1, *k*B2 etc. = parameters *k* in Direction B.

II. $B_{INTERSECTION} = = \left(S_1\ S_2\ \lambda\ \text{Cline}\,1\ \ k\ \text{Cline}\,1 \ldots \lambda\ \text{Dline}\,1\ \ k\ \text{Dline}\,1 \ldots\right)$

1. The number of line coincides with the number of link between Node I and Node II in Matrix $A_{STREETS}$;
2. S1 and S2 = intersecting freeways which form Node I;
3. λCline1, λCline2 etc. = parameters λ of flows arriving at Node I in Direction C on the freeway crossing the given link at Node I;
4. *k*Cline1, *k*Cline2 etc = parameters *k* of flows arriving at Node I in Direction C on the freeway crossing the given link at Node I;
5. λDline1, λDline2 etc. = parameters λ of flows arriving at Node I in Direction D on the freeway crossing the given link at Node I;
6. *k*Dline1, *k*Dline2 etc. = parameters *k* of flows arriving at Node I in Direction D on the freeway crossing the given link at Node I;

The number of lines in the matrix is equal to the number of links in the graph. Parameters λ and *k* for each traffic direction are formed in the order of sequence of flows from right to left (from the wayside to the middle of the road). Matrix $A_{STREETS}$ already contains the data on all the network flows and movements at the intersections. Matrix $B_{INTERSECTION}$ was introduced for to make it more convinient to obtain all the necessary data for calculation and identification of the intersection in the process of modelling.

2. Method of Predicting and Adjusting the Erlang Distribution Parametrs

For predicting an OD matrix and the traffic density between particular OD pairs we use pevious observations. Unfortunately, errors are probable due to different measuring tools or simplification during mathematical simulation. Errors of both types are inevitable. However, Kalman Filter [5] enables to optimize predicting by minimizing the squared error variance.

For TIMeR_Mod we must measure by detectors time intervals between two successive vehicles in the traffic flow on the lane. With these measurements we obtain the following static point evaluations of parameters:

$\overline{x}_B$ is the sample mean of random value T (time interval between two successive vehicles on the lane), σ_B^2 is the sample variance of a random value.

Then we employ these evaluations to calculate the Erlang parameters necessary for all other calculations and solving transportation problems. (For more details, see: Naumova [7, 8]. It is important now that Erlang parameter λ_0 is obtained by:

$$\lambda_0 = S(k^*)\cdot\frac{1}{\overline{x}_B} = S(k^*)\cdot N \tag{2.1}$$

where $$k^* = \frac{\overline{x}_B^2}{\hat{s}^2};$$

N is the flow density on the lane.

Function $S(k^*)$ for different values of parameter $k = [k^*] + 1$ is dfferent.

The other Erlang parameters are : $\lambda_1 = x^{i+1}.\lambda_0,\ i \in \{1,..., k-1\}$;

$k = [k^*] + 1$ is an integer above k^*.

With changing of density, Erlang parameter λ_0 also changes. If we know the principle of density change, we can predict the value of parameter λ_0 and, therefore, solve the OD matrix prediction problems.

We design a linear regressive model for changing the density during a short period of time :

$N(t) = N_0 + nt$. Or, having split the time axis into intervals,

We obtain : $N_{k+1} = N_k + n.\Delta t.$

Denote : $(\lambda_0)_k$ is the value of parameter for time period k.

Then
$$\begin{aligned}(\lambda_0)_k &= S(k^*)\cdot N_k\\ (\lambda_0)_{k+1} &= S(k^*)\cdot N_{k+1} = S(k^*)\cdot(N_k + n\cdot\Delta t)\\ &= S(k^*)\cdot N_k + S(k^*)\cdot n\cdot\Delta t\end{aligned} \tag{2.2}$$

Since in the process of designing the regressive model of density some random errors ξ_k have naturally occurred the formula (2.2) must be transformed as follows:

$$(\lambda_0)_{k+1} = S(k^*)\cdot N_k + S(k^*)\cdot n\cdot\Delta t + \xi_\kappa \tag{2.3}$$

In this case the value $(\Lambda_0)_k$ which has been calculated on the basis of data obtained via the detector differs from its true value $(\lambda_0)_k$:

$$(\Lambda_0)_k = (\lambda_0)_k + \eta_k. \tag{2.4}$$

The model error ξ_k and the detector error η_k are random values and the laws of their distribution are not dependent on time (iteration number k). The mean values are: $E(\xi_k) = 0$, $E(\eta_k) = 0$. It is assumed that all random errors are independent from each other.

We must know the variance of random values ξ и η. The point evaluation of variance of random value ξ can be calculated off-line by direct mesurements, by expressing its observed values from model (2.3) and applying the corresponding formula of mathematical statistics [18].

Now, we apply the method of estimating the variance of randon value η:

$$\begin{aligned}\eta_k &= (\Lambda_0)_k - (\lambda_0)_k\\ &= S(k^*)\cdot\left(\frac{1}{\overline{x}_B} - \frac{1}{(\overline{x}_B)_{real}}\right)\end{aligned}$$

$$= S(k^*)\cdot\left(\frac{1}{\bar{x}_B} - N_{real}\right)$$

Then $D(\eta) = \left(S(k^*)\right)^2 \cdot D\left(\frac{1}{\bar{x}_B} - N_{real}\right)$ is the variance of random value η.

Our objective is to find the maximum approximation to the true value of parameter λ_0 at Step $(k+1)$. The idea of Kalman is to obtain the best approximation to the true value of $(\lambda_0)_{k+1}$. For this, it is necessary to choose the middle value between the detector readings and the predicted data. If we assign weight K to the detector reading, then the predicted value will have weight (1 – K):

$$\left(\lambda_0\right)_{k+1}^{opt} = K.\left(\Lambda_0\right)_{k+1} + (1-K)\times\left((\lambda_0)_k^{opt} + S(k^*)\cdot n\cdot\Delta t\right) \tag{2.5}$$

Number K is Kalman coefficient. It depends on the step of iteration. For the general case, in order to find the exact value of Kalman coefficient K, we must minimize the error:

$$\varepsilon_{k+1} = \left(\lambda_0\right)_{k+1} - \left(\lambda_0\right)_{k+1}^{opt}$$

$$\varepsilon_{k+1} = \left(\lambda_k + S(k^*)\cdot n\cdot\Delta t + \xi_k\right) - K\cdot\left((\lambda_0)_{k+1} + \eta_{k+1}\right) - (1-K)\cdot\left((\lambda_0)_k^{opt} + S(k^*)\cdot n\cdot\Delta t\right)$$

Having simplified, we obtain the following expression for the error :

$$\varepsilon_{k+1} = (1-K)\cdot(\varepsilon_k + \xi_k) - K\cdot\eta_{k+1} \tag{2.6}$$

The expected value of squared error :

$$E\left((\varepsilon_{k+1})^2\right) = (1-K)^2\cdot\left(E\left((\varepsilon_k)\right)^2 + D(\xi)\right) + K^2\cdot D\left((\eta)^2\right) \tag{2.7}$$

If we find the derivative by variable K and equate it to zero, then :

$$K_{k+1} = \frac{E\left((\varepsilon_k)^2\right) + D(\xi)}{E\left((\varepsilon_k)^2\right) + D(\xi) + D(\eta)} \tag{2.8}$$

K_{k+1} = Kalman coefficient for Step $(k+1)$.

Substitute (2.8) in (2.7), and obtain :

$$E\left((\varepsilon_{k+1})^2\right) = D(\eta)\cdot\frac{E\left((\varepsilon_k)^2\right) + D(\xi)}{E\left((\varepsilon_k)^2\right) + D(\xi) + D(\eta)} \tag{2.9}$$

Thus we predict the Erlang parameters values by using the data from the previous observations and the density prediction for the next step in the future.

Algorithm 1 for designing Matrix $A_{STREETS}$ ***and Matrix*** $B_{INTERSECTION}$ by computated solving of problems on efficient organization of vehicle traffic.

Initial data : Matrix and Matrix for the initial period; calculated off-line variations $D(\xi)$ and $D(\eta)$ for each link/lane. For the initial moment,

$$K_1 = 0{,}5,$$

$$\varepsilon_1 = \left(\lambda_0\right)_1 - \left(\lambda_0\right)_1^{opt},\ E\left((\varepsilon_1)^2\right) = (\varepsilon_1)^2$$

Step 1: For the current moment, calculating for each link/lane by empirical data : $\bar{x}_{B,}\ s_B^2$,

$$N_{k+1} = N_k + n.\Delta t.$$

Step 2: Calculating Kalman coefficient .

$$K_{k+1} = \frac{E\left((\varepsilon_k)^2\right) + D(\xi)}{E\left((\varepsilon_k)^2\right) + D(\xi) + D(\eta)}$$

Step 3: Calculating the adjusted Erlang parameters with Kalman coefficient:

$$(\lambda_0)_{k+1}^{opt} = K_{k+1}\cdot(\Lambda_0)_{k+1} + (1-K_{k+1})\cdot\left((\lambda_0)_k^{opt} + S(k^*)\cdot n\cdot\Delta t\right)$$

Step 4: Desinging Matrix $A_{STREETS}$ and Matrix $B_{INTERSECTION}$ for the next time interval.

Step 5: Calculating $E\left((\varepsilon_{k+1})^2\right) = D(\eta)\cdot\dfrac{E\left((\varepsilon_k)^2\right)+D(\xi)}{E\left((\varepsilon_k)^2\right)+D(\xi)+D(\eta)}$.

Step 6 : iterating points 1-5 for the next time interval.

The Erlang parameters are the basic element in calculating the travel cost function $G_p(x) \equiv G_p(x(N_p))$ by the network routes with the help of TIMeR_Mod [7, 8]. When predicting the elements of Matrix $A_{STREETS}$ and Matrix $B_{INTERSECTION}$ we get the possibility to predict the optimal network routes between any pair of points under the user equilibrium conditions.

3. Calculating a Dynamic Link Proportion Matrix

The OD matrix contains data on the number of vehicles which travel from origin to destination for time t:

$$OD = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \tag{3.1}$$

where $a_{j_1 j_2}$ = the number of vehicles which travel from origin j_1 to destination j_2; $a_{j_2 j_1}$ = the number of vehicles which travel in the opposite direction from j_2 to j_1. Denote the origine-destination pair as $j = \{j_1, j_2\}$.

A link proportion matrix which defines the traffic volume on each link for a particular OD pair is denoted as LP matrix. Traditionally, the LP matrix contains data on the share of vehicles moving on link i related to OD pair No j during time t (Zhou, Qin, Mahmassani in [20]). It must be designed for each link of the transportation network. Its dimention coinsides with that of the OD matrix. This paper suggests to keep two parameters in the LP matrix: the number N_{ij} of vehicles on the given link related to the given OD pair and the share LP_{ij} of these vehicles from the aggragate number of vehicles in the network.

OD matrices and LP matrices may be designed for particular OD pairs if necessary.

It is supposed that for the initial stage an OD matrix and an LP matrix have been designed off-line. Their further adjusting at each Step k is conducted according to the data on density of traffic flow on the lanes obtained from the detectors on-line. Besides, before the redistribution of correspondences at Step k we have already obtained Matrix $A_{STREETS}$ and Matrix $B_{INTERSECTION}$ with new values of Erlang parameters agjuusted by Kalman Filtering (see Point 2 of the present article).

Algorithm 2 for predicting the share of vehicles moving on each link related a particular OD pair:

Step 1: Choose OD pair j.

Step 2: Design for each OD pair j a set of optimal routes (as a set of links) and a number of vehicles using this routes for the next time period under the principle of user equilibrium.

Step 3 : Calculate the share of vehicles on link i related to OD pair j. It is equal to the ratio of density $(N_{ij})_k$, for OD pair j to the aggregate density flow $(N_i)_k$ on the link: $\left(LP_{ij}\right)_k = \dfrac{\left(N_{ij}\right)_k}{\left(N_i\right)_k}$ if the link belongs to the optimal route (if the link does not belong to the optimal route, then the share of vehicles is equal to zero).

Step 4 : Design an LP matrix for each of the links for the next time period taking into account the calculations of Step 3.

Step 5 : Adjust the density on the links in Matrix and Matrix by Kalman Filtering.

Step 6 : Go to Step 1.

Step 2 of Algorithm 2 requires separate explanation. Changes in traffic density occur in several OD pairs (more precisely, we control all pairs of the OD matrix). So, we suggest using Algorithm 2 developed by the author and presented in our previous paper which determins optimal routs for each pair at each stage. Calculations are made by analytical method using the model developed by the author (TIMeR_Mod). Applying the analytical method we can make calculations faster.

Since the time axis is divided into small intervals, we can assume that the optimal route calculated at at Step 2 of Algorithm 2 for a particular demand for the given OD pair remains constant for all $\Delta(N_{ij})_k$ demands. So, the number of correspondences for OD pair j at each step only changes due to the changes in one route which is optimal for the given step.

For Step 3 we must take into account the following fact: if the link does not belong to the optimal route, then the share of vehicles is equal to zero. So we do not have to recaclulate it. Recalculations are only needed for the share of those links where we observe nonzero changes of density $\Delta(N_{ij})_k$ for the given OD pair.

4. Designing an Origin-Destination Matrix by the Known Link Proportion Matrix for Dynamic Assignment

When we know the LP matrix for the current and previous time periods, we can obtain the percentage of changes of number of vehicles related to each OD pair j for each link i :

$$\left(\Delta_{ij}\right)_k = \left(\mathrm{LP}_{ij}\right)_k - \left(\mathrm{LP}_{ij}\right)_{k-1} \tag{4.1}$$

Then the mean percentage change $\left(\bar{\Delta}_j\right)_k$ for OD pair j at Step k is a point evaluation of the real change of number of vehicles related to each OD pair j at the current moment:

$$\left(\bar{\Delta}_j\right)_k = \frac{\sum_i \left(\Delta_{ij}\right)_k}{m} \tag{4.2}$$

Here m is the number of links on which the optimal route for each OD pair j has been drawn at the given step (we neglect those links where the percentage change in the LP matrix is zero). Then the estimated value for the number of correspondences for the next step is:

$$\mathrm{D}_{j,k+1} = \mathrm{D}_{j,k} \times (1 + \left(\bar{\Delta}_j\right)_k) \tag{4.3}$$

With elements $\mathrm{D}_{j,k+1}$ we design an OD matrix for the next step.

For an LP matrix for Step (k + 1) we calculate :

- The change of density for OD pair j on link i : $\Delta\left(\mathrm{N}_{ij}\right)_{k+1} = \mathrm{D}_{j,k} \times \left(\bar{\Delta}_j\right)_k$
- The density for OD pair j on link i : $\left(\mathrm{N}_{ij}\right)_{k+1} = \left(\mathrm{N}_{ij}\right)_k + \Delta\left(\mathrm{N}_{ij}\right)_{k+1}$
- The share of vehicles moving on link i related to OD pair j : $\left(\mathrm{LP}_{ij}\right)_{k+1} = \dfrac{\left(\mathrm{N}_{ij}\right)_{k+1}}{\left(\mathrm{N}_i\right)_{k+1}}$

5. Adjusting a Link Proportion Matrix by Kalman Filtering in Case of Known Origin-Destination Matrix for All the Origin-Destination Pairs

When predicting an OD matrix and an LP matrix we inevitably come across with errors in calculations. So, it is recommended to adjust the matrices at each stage by comparing the data calculated via the model with empirical values.

If the OD matrix was designed for all possible OD pairs j then the following equality is true :

$$\sum_j \left(\mathrm{N}_{ij}\right)_k = \left(\mathrm{N}_i\right)_k \tag{5.1}$$

That is for each link at each step the sum of densities related to the density for OD pair j on link i is equal to the density on link i. Density $(\mathrm{N}_i)_k$ at each step is mesured by detectors. Here measurement errors are probable:

$$\left(\tilde{\mathrm{N}}_i\right)_k = \left(\mathrm{N}_i\right)_k + \eta_k, \tag{5.2}$$

$\left(\tilde{\mathrm{N}}_i\right)_{kk}$ where is the measured value of density on the link, $(\mathrm{N}_i)_k$ is the true value, η_k is the measument error.

$$\left(\mathrm{N}_{ij}\right)_{k+1} = \left(\mathrm{N}_{ij}\right)_k + \mathrm{D}_{j,k} \times \left(\bar{\Delta}_j\right)_k + \xi_{j,k} \tag{5.3}$$

where is the model error.

Summarize (5.3) by j subject to (5.1):

$$\left(\mathrm{N}_i\right)_{k+1} = \left(\mathrm{N}_i\right)_k + \sum_j \mathrm{D}_{j,k} \times \left(\bar{\Delta}_j\right)_k + \sum_j \xi_{j,k} \tag{5.4}$$

Denote $$u_k = \sum_j \mathrm{D}_{j,k} \times \left(\bar{\Delta}_j\right)_k$$

$$\xi_k = \sum_j \xi_{j,k}$$. Then:

$$\left(\mathrm{N}_i\right)_{k+1} = \left(\mathrm{N}_i\right)_k + u_k + \xi_k \tag{5.5}$$

Model error ξ_k and detector error η_k are random values. The laws of their distribution are not dependent on the time (on iteration number k). Mean values $\mathrm{E}(\xi_k) = 0$, $\mathrm{E}(\eta_k) = 0$. Variance of random values $\xi_k = \left(\mathrm{N}_i\right)_{k+1}^{real} - \left(\left(\mathrm{N}_i\right)_k + u_k\right)$ and $\eta_k = \left(\tilde{\mathrm{N}}_i\right)_k - \left(\mathrm{N}_i\right)_k = \frac{1}{\bar{x}_B} - \mathrm{N}_i^{real}$ must be estimated by calculating directly before the on-line process.

It is assumed that all random errors are independent from each other: there is no any relashionship between the error at the moment of time k and the error at another moment of time k'.

$$\left(\mathrm{N}_i\right)_{k+1}^{opt} = \mathrm{K} \cdot \left(\tilde{\mathrm{N}}_i\right)_{k+1} + (1-\mathrm{K}) \cdot \left(\left(\mathrm{N}_i\right)_k^{opt} + u_k\right) \tag{5.6}$$

To find the value of Kalman coefficient K, we mustminimize the error :

$$\varepsilon_{k+1} = \left(\mathrm{N}_i\right)_{k+1} - \left(\mathrm{N}_i\right)_{k+1}^{opt}$$

$$\varepsilon_{k+1} = \left(\mathrm{N}_i\right)_k + u_k + \xi_k - \mathrm{K}.\left(\tilde{\mathrm{N}}_i\right)_{k+1} - (1-\mathrm{K}) \cdot \left(\left(\mathrm{N}_i\right)_k^{opt} + u_k\right)$$

After simplifying we obtain :

$$\varepsilon_{k+1} = \left(1-\mathrm{K}\right) \cdot \left(e_k + \xi_k\right) - \mathrm{K} \cdot \eta_{k+1} \tag{5.7}$$

Mathematical expectation of the square of error :

$$\mathrm{E}\left((\varepsilon_{k+1})^2\right) = (1-\mathrm{K})^2 \cdot \left(\mathrm{E}\left((\varepsilon_k)\right)^2 + \mathrm{D}(\xi)\right) + \mathrm{K}^2 \cdot \mathrm{D}\left((\eta)^2\right) \tag{5.8}$$

Similarly to Point 2 of the present paper, having differentiated (5.8) by variable K and having equated the obtained equation to zero, we will have the Kalman coefficient at Step $(k+1)$:

$$\mathrm{K}_{k+1} = \frac{\mathrm{E}\left((\varepsilon_k)^2\right) + \mathrm{D}(\xi)}{\mathrm{E}\left((\varepsilon_k)^2\right) + \mathrm{D}(\xi) + \mathrm{D}(\eta)}. \tag{5.9}$$

Therefore,
$$E\left((\varepsilon_{k+1})^2\right) = D(\eta)\cdot\frac{E\left((\varepsilon_k)^2\right)+D(\xi)}{E\left((\varepsilon_k)^2\right)+D(\xi)+D(\eta)}$$

Then, we adjust the LP matrix elements by the recursive formula:

$$\left(N_{ij}\right)_{k+1}^{opt} = K_{k+1}\cdot\left(\tilde{N}_{ij}\right)_{k+1} + (1-K_{k+1})\cdot\left(\left(N_{ij}\right)_k^{opt} + D_{j.k}\times\left(\bar{\Delta}_j\right)_k\right) \quad (5.10)$$

For the initial stage (as per Point 2 of the present paper) we assume: $K_1 = 0,5$, $\varepsilon_1 = \left(N_i\right)_1 - \left(N_i\right)_1^{opt}$, $E\left((\varepsilon_1)^2\right) = (\varepsilon_1)^2$.

6. Adjusting an Origin-Destination Matrix in Case of Need to Control the Number of Correspondences Only between Particular Origin-Destination Pairs

If the OD matrix was not designed for all possible OD pairs, then equation (5.1) is not true and, therefore, it is impossible to control the estimated values by measuring density as it is described in the previous point.

However, if we are able to control the number of correspondences leaving each point given in the OD matrix, then we are able to adjust the estimated values by Kalman Filtering.

$$\tilde{D}_{j,k} = D_{j,k} + \eta_k \quad (6.1)$$

where $\tilde{D}_{j,k}$ is the measured value of the number of correspondences at Step *k*, related to OD pair *j*; $\tilde{D}_{j,k}$ is the true value, η_k is the measument error.

$$D_{j,k+1} = D_{j,k}\times(1+\left(\bar{\Delta}_j\right)_k) + \xi_k \quad (6.2)$$

$$D_{j,k+1} = D_{j,k} + D_{j,k}\times\left(\bar{\Delta}_j\right)_k + \xi_k$$

where ξ_k is the model error.

$$\left(D_{j.k+1}\right)^{opt} = K_{k+1}\cdot\tilde{D}_{j,k+1} + (1-K_{k+1})\cdot\left(D_{j,k}^{opt} + D_{j,k}\times\left(\bar{\Delta}_j\right)_k\right) \quad (6.3)$$

Similarly, obtain the recursive formulas :

$K_{k+1} = \frac{E\left((\varepsilon_k)^2\right)+D(\xi)}{E\left((\varepsilon_k)^2\right)+D(\xi)+D(\eta)}$ is the Kalman coefficient at Step (*k* + 1), where value $E\left((\varepsilon_{k+1})^2\right)$ is calculated as follows:

$$E\left((\varepsilon_{k+1})^2\right) = D(\eta)\cdot\frac{E\left((\varepsilon_k)^2\right)+D(\xi)}{E\left((\varepsilon_k)^2\right)+D(\xi)+D(\eta)}$$

Variance $D(\xi)$ and variance $D(\eta)$ must be calculated off-line beforehand.

Formula (6.3) is the estimated value of the OD matrix elements for the next step.

2. CONCLUSION

Estimating and predicting an OD matrix in dynamic study is one of the critical problems in transportation. The methodology suggested in the present paper takes into account both regressive dependence in changes of traffic flow density distribution and random factors which influence these values. In order to adjust the estimated data, Kalman Filtering is used since it is the best linear filter. Calculations are based on the eqilibrium flow distribution in the network and are adjusted at each iteration. All these factors allow us to improve the quality of estimated parameters.

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