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ENHANCEMENT OF BUCKLING LOADS OF PIEZOELECTRIC SMART STRUCTURES

Bohua Sun*

ABSTRACT

In this paper, in the light of Budiansky's work on elastic buckling formulation, we present the dynamic buckling of piezoelectric smart structures in a general way. Using the virtual work principle of smart structures, the governing equations for pre-buckling, buckling and post-buckling states have been formulated. The critical load and the coefficients of post-buckling path have been given explicitly.

Keywords: Buckling, piezoelectric, smart materials

1. INTRODUCTION

1.1. Buckling [1-14]

As we known, for thin walled structures the membrane stiffness is generally much greater than the bending stiffness. A thin wall structure can absorb a great deal of membrane strain energy without excessive deformation taking places. Whereas, it deforms much more in order to absorb an equivalent amount of bending strain energy. When the structure is loaded in such a way that most of its strain energy is in the form of membrane compression, should there be a way that this stored-up membrane energy be converted into bending energy, the shell may fail rather dramatically in a process called "buckling", i.e. as it exchanges its membrane energy for bending energy. Very large deflections are generally required to convert a given amount of membrane energy into bending energy.

The way in which buckling occurs depends on how the structure is loaded and on its geometrical and material properties. The prebuckling process is often nonlinear if there is a reasonably large percentage of bending energy being stored in the structure throughout the loading history.

According to the percentage of bending energy, the two basic ways in which a conservative elastic system may loose its stability are: nonlinear collapse (snap-through, or over-the-lump) and bifurcation buckling. Nonlinear collapse is predicted by means of a nonlinear analysis. The stiffness of the structure or the slope of the load-deflection curve, decrease with increasing load. At the collapse load the load-deflection curve has zero slope, and if the load is maintained as the structure deforms, failure of the structure is usually dramatic and almost instantaneous. This type of instability failure is often called "snap-through", a nomenclature was derived from the many early tests and theoretical models of shallow arches, caps and cones. These nonlinear systems initially deform slowly with increasing load. As the load approaches the maximum value, the rate of deformation increases until, reaching a status of neutral equilibrium in which the average curvature is almost zero, these shallow structures subsequently "snap-through" to a post-buckled state which resembles the original structure in an inverted form. The term "bifurcation buckling" refers to a different kind of failure, the onset of which is predicted by means of an eigenvalue analysis. At the buckling load, or bifurcation point on the load-deflection path, the deformation begins to grow in a new pattern which is quite different from the prebuckling pattern. Failure or unbounded growth of this new deflection mode occurs if the post-bifurcation load-deflection curve has a negative slope and the applied load is independent of the deformation amplitude.

* Centre for Mechanics, Smart Structures and Microsystems, Department of Mechanical Engineering, Faculty of Engineering, Cape Peninsula University of Technology, P. O. Box 1906, Bellville 7535, South Africa, *E-mail: sunb@cput.ac.za*

1.2. Smart Structures [15-48]

In the past, material systems and structures were designed based on their passive response to external loads. In recent years, the construction and operation of large or precision space structures, which are extremely flexible and inherently low in natural damping, has sparked a keen interests in "intelligent" or "smart" structures which provide an active response to external loads. Such smart structures have a network of sensors and actuators which monitor and control the response of the structures, respectively. Smart structures are those structures made of smart materials, in other words, are those which incorporate actuators and sensors that are integrated into the structure and have structural functionality, as well as integrated control logic, signal conditioning and power amplification electronics. Such actuating, sensing and signal processing elements are incorporated into a structure for the purpose of influencing its states or characteristics, be they mechanical, thermal, optical, chemical, electrical or magnetic. For example, a mechanically intelligent structure is capable of altering both mechanical states, i.e. its position or velocity, or its mechanical characteristic, i.e. its stiffness or damping. An optically intelligent structure could, for example, change colour to match its background.

According to Crawley (1994) [22], there are three historical trends which have combined to establish the potential feasibility of smart structures. The first is the transition to laminated materials. In the past, structures were manufactured from large pieces of monolithic material which were machined, forged, or formed to a final structural shape, making it difficult to imagine the incorporation of active elements. The exploitation of the off-diagonal terms in material constitutive relations is a second trend which enables intelligent structures at this time. These off-diagonal terms of full constitutive relations, include characteristics of its mechanical, optical, electromagnetic, chemical, physical and thermal properties The third and perhaps most obvious advance comes from the electrical engineering and computer science disciplines. With their advances in microelectronics, bus architectures, switching circuitry, fiber optic technology and artificial intelligence and control disciplines.

In the next decades, it is expected that there will be widespread applications of the technology under development, in its current and evolutionary forms. The breath of application of this technology is expected not only to span the aerospace industry but become widespread in the construction, automotive, and machine tool industries as well.

1.3. Buckling of Piezoelectric Smart Structures

Actuating materials currently under use include shape memory alloys, magneto-strictive materials, piezoceramics and electricstrictive materials. Piezoelectric materials which exhibit mechanical deformation when an electric field is applied and, conversely, generate a charge in response to mechanical deformation can be used as actuators and sensors, respectively. The axial forces and the corresponding moments generated by the actuator can be used to resist the forces acting on the structures. Piezoelectric smart structures have been investigated by several researchers [15-48].

The majority of the work has been concentrated in the area of active vibration control of structures, and the dynamic buckling of a structure due to an axial load has received scant attention from the scientific community, which is not the case of "Everybody loves a buckling problem" [10] for the conventional structures. Buckling of smart structures belongs to dynamic buckling, which is generally meant to describe the buckling of structures under transient loads. It is obvious that all loads are transient.

Baz and Tampe (1989)[44] devised an active buckling control system to enhance the elastic stability characteristics of long slender beams. They used an external shape memory alloy (SMA) actuator in the form of a helical spring to counterbalance the applied compressive load. Baz, *et al*. (1991)[45], achieved buckling control of a fiberglass composite by activating shape memory alloy wires embedded along the neutral axis. Their results showed that the critical buckling loads could be increased three times when compared to that of an uncontrolled beam. Mollenhauer, *et al*. (1992)[46], analyzed the buckling of a stiffened aluminum panel with SMA actuators using a commercial software package. Murali Krisha and Mei (1992)[47] presented a FEM formulation based on the von Karman large deflection plate theory for composite plates with piezoelectric actuator layers, and calculated the critical buckling voltage of the structures. Similar to Lee (1990)[30], Chan and Bhatia (1993)[20] used the first-order shear deformation plate theory (FSDT), to study the dynamic buckling behavior. Their numerical results have demonstrated the effectiveness of the closed-loop system, incorporating the sensors and actuators, in enhancing the critical buckling load of the plate.

Beside the study of the above specific cases, there is a little general discussion on the dynamic buckling of the smart structures. In this paper, to fill the gap, we present the dynamic buckling of smart structures in a general way. Using virtual work principle of smart structures, the governing equations for pre-buckling, buckling and post-buckling states have been formulated. The critical load and the coefficients of post-buckling path have been given explicitly.

The following remarkable results have been obtained for the problem satisfied by these simplifications:

- The split of the critical load, in other words, the critical load can be decomposed into non-piezoelectric and piezoelectric parts, which is the mathematics-physical foundation of closed-loop control.
- The post-buckling coefficients λ_1 and λ_2 depend explicitly on the mechanical field; but post-buckling mode u_2 depends on both mechanical and electric fields.
- The generalized load-"shortening" is only depended on the mechanical field, and piezoelectric has no any contribution!
- The control mechanism of smart structures includes two aspects: one is control of pre-buckling motion and another is control of electric energy in the buckling state.

2. GENERAL FORMULATION OF SMART STRUCTURES

In order to serve to later sections but not for completeness, some general formulations of the structures will be highlighted as follows:

2.1. The Electric-mechanical Lagrangian Function

$$L = \frac{1}{2}\rho\dot{u}\dot{u} - H(\varepsilon, E),, \tag{1}$$

where, $H = H(\varepsilon, E)$ is electric enthalpy density, ε is strain tensor and E is electric displacement and $\dot{u}$ is velocity.

For linear materials, $H(\varepsilon, E) = \frac{1}{2}c\varepsilon\varepsilon - eE\varepsilon - \frac{1}{2}kEE$, where c, e and k are called the elastic, piezoelectric, and dielectric permittivety constants, respectively.

2.2. Constitutive Equations

In general case, we have $\sigma = \frac{\partial H}{\partial \varepsilon}$ and $D = -\frac{\partial H}{\partial E}$.

For linear case, we have

$$\sigma = c\varepsilon - eE, D = -e\varepsilon + kE, \tag{2}$$

With these relations (2), the electric entropy H can be rewritten as

$$H(\varepsilon, E) = \frac{1}{2}\sigma\varepsilon - \frac{1}{2}DE,, \tag{3}$$

For laminated smart structures, the above constitutive equations must be transformed into arbitrary coordinates. These operations would not change the constitutive equations in form. In this paper, for clarity of interpretation and also to avoid loss of generality, we omit specific transformation, and assume our constitutive equation will always take the right forms of your problems.

2.3. Electric Potential

For electric field, if we denote its potential as φ, the electric displacement is given by

$$E = -\nabla\varphi, \tag{4}$$

2.4. Hamilton's Principle

From Budiansky (1974)[11] and Sun (1992)[8], we can always write Hamilton's principle in a very simple form

$$\sigma\delta\varepsilon - D\delta E - \lambda d\Delta[u,\,\varphi] = \rho\dot{u}\delta\dot{u},\,, \tag{5}$$

Since $\delta\varepsilon = \varepsilon'\,[u]\delta u$, and $\delta E = E'[\varphi]\delta\varphi$, the above Hamilton principle can be rewritten as

$$\sigma\varepsilon'[u]\delta u - DE'[\varphi]\delta\varphi - \lambda d\Delta[u,\varphi] = \rho\dot{u}\delta\dot{u}, \tag{6}$$

In the above relations, we have endowed the symbol σ have three meanings: it is the stress state σ; it is a linear operator on stains, making $\sigma\delta\varepsilon$ the total work of the stresses acting through $\delta\varepsilon$; and it is a function $\sigma[\varepsilon]$ of strain. Same interpretation is also valid for both $D(E)$ and E. This understanding of the symbol σ is very useful in the deriving of buckling and post-buckling equations, and the buckling load of the practical problems.

2.5. Some Simplifications

The results found are very general, for more so than would usually be required, and in various particular problems one or more of the following simplifications can be invoked.

(i) linear stress-strain relation, $\sigma^{(n)} = 0$ for $n> = 2$;

(ii) quadratic strain-displacement relation, $\varepsilon^{(n)} = 0$ for $n> = 3$;

(iii) linear shortening-displacement relation, $\Delta^{(n)} = 0$, for $n> = 2$.

In many problems all three of these conditions are met. Limitation to elastic strains makes (i) valid in all but rubberlike or hyperelastic materials. A quadratic strain-displacement (ii) relation is, of course, obtained when the Lagrangian strain tensor, or a simplified variant thereof, is employed. The linearity of $\Delta[u]$ occurs in dead-loading situations(though not in hydrostatic loading) or for conservative system.

Another simplifying assumption, independent of (i)-(iii) above, is that of a (iv) linear fundamental state.

$$\begin{gathered}\sigma[k\varepsilon_0] = k\sigma\,[\varepsilon_0]\;\; \varepsilon[ku_0] = k\varepsilon[u_0]\;\; \Delta[k\,u_0] = k\Delta[u_0] \\ D[kE_0] = kD[E_0],\; E[k\varphi_0] = kE[\varphi_0]\end{gathered} \tag{7}$$

This assumption provides the common situation associated with buckling problems in which the prebuckling displacement, stresses, and strains vary linearly with λ, and the load λ appears linearly in the bifurcation eigenvalue problem. We will define assumption (iv) to mean a special kind of linearity associated with the function $\sigma[\varepsilon]$, $\varepsilon[u]$ and $\Delta[u]$, and the fundamental solutions u_0, σ_0, and ε_0, in which not only do the identities hold for any constant k but, in addition, the operators $\sigma_0^{(n)}$, $\varepsilon_0^{(n)}$, $\Delta_0^{(n)}$ are all independent of σ for $n>–1$. This guarantees that the fundamental solution of the field equations will be linear in λ, and it also makes (6.2) a linear eigenvalue problem. If there is no special noting, the above simplification will be used to all following formulations.

3. BIFURCATION ANALYSIS

In order to discover conditions for bifurcation buckling, we assume first that there exist a fundamental solution $u_0(\lambda)$, $\varepsilon_0(\lambda)$, $\sigma_0(\lambda)$, $D_0(\lambda)$, $E_0(\lambda)$ that varies smoothly with l as the load increases from zero. The above variational equation requires that

$$\sigma_0\varepsilon'[u_0(\lambda)] - D_0E'[\varphi_0(\lambda)] - \lambda d\Delta[u_0(\lambda),\varphi_0(\lambda)] = \rho\dot{u}_0\delta\dot{u},\,, \tag{8}$$

for all δu and $\delta\varphi$. Now suppose that , for some range of λ, there is another solution

$$\begin{gathered}u = u_0(\lambda) + v(\lambda),\quad \sigma = \sigma_0(\lambda) + s(\lambda),\quad \varepsilon = \varepsilon_0(\lambda) + \tau(\lambda), \\ \dot{u} = \dot{u}_0(\lambda) + \dot{v}(\lambda),\quad \varphi = \varphi_0(\lambda) + \psi(\lambda),\quad D = D_0(\lambda) + d(\lambda)'\end{gathered} \tag{9}$$

that intersects the fundamental one at generalized buckling load lc, in the sense that

$$\lim_{\lambda\geq\lambda_c}[v(\lambda),\; s(\lambda),\; \tau(\lambda),\; \psi(\lambda),\; d(\lambda)] = 0.\,, \tag{10}$$

It will be further assumed that $u_0(\lambda)$, $\varepsilon_0(\lambda)$, $\sigma_0(\lambda)$, $D_0(\lambda)$, $E_0(\lambda)$ exist for l greater than lc, so that a true bifurcation, rather than a limited point, is implied by (9) and (10).

The bifurcation buckling mode will be defined as

$$u_1 = \lim_{\lambda \geq \lambda c} \frac{v}{|v|}, \quad \dot{u}_1 = \lim_{\lambda \geq \lambda c} \frac{\dot{v}}{|v|}, \tag{11}$$

and also define the associated stress and strain modes by

$$\sigma_1 = \lim_{\lambda \geq \lambda c} \frac{s}{|v|}, \ \varepsilon_1 = \lim_{\lambda \geq \lambda c} \frac{\tau}{|v|}, \ \varphi_1 = \lim_{\lambda \geq \lambda c} \frac{\psi}{|v|}, \ d_1 = \lim_{\lambda \geq \lambda c} \frac{d}{|v|}, \tag{12}$$

where $|\ |$ represents a suitable norm; note $|u_1| = 1$.

Since the u, ε, σ, ψ given by (9) must satisfy equilibrium

$$\begin{aligned} &[\sigma_0(\lambda) + s(\lambda)]\varepsilon'[u_0(\lambda) + v(\lambda)] - [D_0(\lambda) + d(\lambda)]E'[\varphi_0(\lambda) + \psi(\lambda)] - \\ &\qquad \lambda d\Delta[u_0(\lambda) + v(\lambda), \varphi_0(\lambda) + \psi(\lambda)] = \rho\dot{u}\delta\dot{u}' \end{aligned} \tag{13}$$

and, under the assumption that ε, σ, Δ and G are analytical in the vicinity of $u_0(\lambda)$, a Taylor-series expansion gives

$$\begin{aligned} &\sigma = \sigma_0 + \sigma_0' s + \tfrac{1}{2}\sigma_0'' s^2 + \cdots; \ \varepsilon = \varepsilon_0 + \varepsilon_0' v + \tfrac{1}{2}\varepsilon_0'' s^2 + \cdots; \ E = E_0 + E_0'\varphi + \tfrac{1}{2}E_0''\varphi^2 + \cdots \\ &D = D_0 + D_0' d + \tfrac{1}{2}D_0'' d^2 + \cdots; \ \Delta = \Delta_0 + \Delta_{0u}' v + \Delta_{0\varphi}'\psi + \tfrac{1}{2}[\Delta_0'' v^2 + \Delta_{0u\varphi}'' v\psi + \Delta_0''\psi^2] + \cdots; \end{aligned} \tag{14}$$

for sufficiently small $|\lambda - \lambda_c|$. Substitute (14) into (13) and notice (8). Dividing (13) by $|v|$, and letting $\lambda \to \lambda_c$, gives

$$\sigma_1\varepsilon_c'\delta u + \sigma_c\varepsilon_c'' u_1\delta u - [D_1 E_c'\delta\varphi + D_c' E_c''\ \varphi_1\delta\varphi] - \lambda_c[\Delta_{0u}'' u_1\delta u + \Delta_{0\varphi}''\varphi_1\delta\varphi] = \rho\dot{u}_1\delta\dot{u}, \tag{15}$$

Applying the simplifications we have $\sigma_c = \lambda_c \dfrac{d\sigma(\lambda_c)}{d\lambda}$, $\Delta''[\cdots]$, the above buckling equation becomes

$$\sigma_1\varepsilon_c'\delta u + \lambda_c \frac{d\sigma(\lambda_c)}{d\lambda}\varepsilon_e'' u_1\delta u - D_1 E_c'\delta\varphi = \rho\dot{u}_1\delta\dot{u}, \tag{16}$$

as the variational equations governing the dynamical buckling mode u_1 and the critical load λ_c.

If we let $\delta u = u_1$, and $\delta\varphi = \varphi_1$ in Eq. (16), we have the dynamic buckling loads as follows:

$$\lambda_c = -\frac{\rho u_1^2 - \sigma_1\varepsilon_c' u_1}{\Omega} - \frac{D_1 E_c'\varphi_1}{\Omega}, \quad \text{where } \Omega = \frac{d\sigma(\lambda_c)}{d\lambda}\varepsilon_c'' u_1^2, \tag{17}$$

From this remarkable Eq. (17), we can easily see that the dynamic buckling loads of smart structures can be decomposed into two terms, one is no-piezoelectric part and another is with piezoelectric term, which is very important for the close loop feedback control in terms of $\dfrac{D_1 E_c'\varphi_1}{\Omega}$, which is clearly indicate that feedback control can only applied to the first derivative of piezoelectric energy with respect to the first buckling mode u_1. That is the mathematics-physical foundation of active control of smart structures.

4. POST-BUCKLING ANALYSIS

In order to determine v in (9) for $\lambda \geq \lambda_c$, introduce the scalar parameter x defined by

$$\xi = \langle v, u_1 \rangle, \tag{18}$$

where the bracket symbol represents any bilinear inner product, the only restriction on this inner product is that $\langle u_1, u_1 \rangle \neq 0$. Indeed, it is particularly convenient to choose the norm $|v| = \langle v,v \rangle^{1/2}$, for then $\langle u_1, u_1 \rangle = 1$, and this choice will be assumed henceforth.

Following the similar way given by Budiansky, for $\lambda = \lambda_c + \lambda_1\xi + \lambda_2\xi^2 + ...$, we have the following results:

$$\lambda_1 = -\frac{3}{2\Xi}\sigma_1\varepsilon_c'' u^2, , \tag{19}$$

If $\lambda_1 = 0$, we have

$$\lambda_2 = -\frac{2\sigma_1\varepsilon_c'' u_1 u_2 + \sigma_2\varepsilon_c' u_1^2}{\Xi}, \tag{20}$$

where $\Xi = \frac{\sigma_c \varepsilon'' u_1^2}{\lambda_c}$. In the case of $\lambda_1 = 0$, we have the governing equation of u_2,

$$\sigma_2\varepsilon_c'\delta u + \sigma_c\varepsilon_c'' u_2\delta u + \sigma_c\varepsilon_c'' u_1\delta u - D_2 E_c'\delta\varphi = 0, , \tag{21}$$

This is the governing equation of post-buckling state, and it is a non-homogeneous equation. There is an apparent lack of uniqueness in the solution. Once u_2 is found, λ_2 as given by (20) can be computed. In practice calculation of u_i (i = 1, 2,...) can be found by perturbation procedure.

It is seen that the post-buckling coefficients depend only on the buckling mode u_1, but the calculation of λ_2 would generally require the determination of u_2 as well, and so on, and the coefficients λ_1 and λ_2 are only explicitly depended on the mechanical field; actually post-buckling mode u_2 depends on both mechanical and electric fields.

An anti-symmetric bifurcation corresponds to $\lambda_1 \neq 0$, and in symmetric bifurcation the load rises or falls during buckling according to the sign of λ_2. The results for λ_i are, of course, not valid if $\Xi = 0$.

5. GENERALIZED LOAD- "SHORTENING" RELATION

For some problems, the load-"shortening" relation may be interesting. The load- "shortening" relation for a perfect structure has been obtained by using the normal energy approach (Budiansky, 1974). In this paper, the generalized load-"shortening" relation will be obtained by the variational principle.

For small ξ, then $v \approx \xi u_1$ and we have

$$\Delta - \Delta_0 \approx -\frac{1}{2}\xi^2\Xi, , \tag{22}$$

This relation is quite general including cases in which the prebuckling variation of shortening with λ is not linear. The generalized load-"shortening" is only depended on the mechanical field, and piezoelectric field has no any contribution on the shortening!

In the case of symmetric bifurcation, it is useful to define the initial postbuckling stiffness of the perfect structure as $K = \frac{d\lambda}{d\Delta}$ at $\lambda = \lambda_c$, and compare it to the corresponding prebuckling stiffness K_0 at $\lambda = \lambda_c$ on the fundamental path. we find $\frac{1}{K} = \frac{1}{K_0} + \frac{\Xi}{2\lambda_2}$, where $\Xi = \frac{\sigma_c\varepsilon'' u_1^2}{\lambda_c}$.

6. CONCLUSION

The majority of the work has concentrated in the area of active vibration control of structures, and the dynamic buckling of a structure due to an axial load has received scant attention from the scientific community. Some investigations for specific problems have shown that the critical buckling load of a structure could be increased few times more when compared to that of an uncontrolled one. In this paper, to fill the gap, we presented the dynamic buckling of the smart structures in a general way. Using the virtual work principle of smart structures, the governing equations for pre-buckling, buckling and post-buckling states have been formulated. The critical load and the coefficients of post-buckling path have been given explicitly.

We believe that following remarkable results have been obtained for the problem satisfied by these simplifications:

- The split of the critical load, in other words, the critical load can be decomposed into non-piezoelectric and piezoelectric parts, which is the mathematics-physical foundation of closed-loop control.
- The post-buckling coefficients λ_1 and λ_2 depend explicitly on the mechanical field; but post-buckling mode u_2 depends on both mechanical and electric fields.
- The generalized load- "shortening" depends only on the mechanical field.
- The control mechanism of smart structures includes two aspects: one is control of pre-buckling motion and another is control of electric energy in the buckling state.

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