

FPGA Implemenatrion of 9 Tab 2D Daubechies Wavelet Filter Using Algebraic Integer

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ABSTRACT

In this paper a novel Algebraic Integer (AI) based DAUB 9 wavelet filter is designed. The main objective of the proposed design is increase the level of image accuracy and reduces the error level. The AI technique is used to mapping the irrational number into rational one, so the computation complexity can be reduced. The Tab unit is designed using the Wallace tree multiplier and Carry Save Adder in order to increase the speed of the operation. The Final Reconstruction Step (FRS) appears only once in the design so the quantization noise is reduced. The PSNR, MSE factors are analyzed for the DAUB4, DAUB6 and DAUB9 for the proposed scheme. The proposed design is simulated in Modelsim 6.6c and implemented in Altera DE2 kit.

Key Terms: Image processing, Algebraic Integer, Reconstruction step, DWT, Daubechies Wavelet filter.

1. INTRODUCTION

Image Processing is the one of the important technique to process the image in the field of Discrete Wavelet Transform. Image processing technique is taken the input is image such as Photo or video [3] and the input image is processed to produce the output such as an image, image related information or parameters. Image processing is refer to process of image in 2D that is $f(x, y)$ Where x, y are the co-ordinate system. The image processing can be divides into three sections that is Image enhancement, Image restoration, Image compression. In the field of imaging the Discrete Wavelet Transform plays an important role. DWT is also used in the applications of medical imaging, pattern recognition, video processing [8] etc. The Discrete Wavelet Transform (DWT) representation provides the multi resolution analysis of signal with localization in time and frequency so it can provide a efficient way of image coding [4]. The image information can be analyzed by using the concept of multi resolution analysis. This multi resolution decomposition is the process of decompose the image into different frequency of components. The computational complexities of image processing can be reduced by using the DWT concept [8]. The discrete wavelet transform can be divided into following two method that is 1D DWT and 2D DWT which is based on the architecture. The architecture for 1D-DWT can be extended to architectures of 2D-DWT by the repeated use of 1D DWT. The 2D-DWT can be computed by row-column method that is computing 1D-DWT in rows followed by 1D-DWT in columns. The minimizing the latency and storage size is necessary in most applications, so mapping 1D-DWT architectures to 2D-DWT architectures is important. This Discrete wavelet transform can be uses the following transform techniques to analyze the image i.e. Haar transform, Fourier transform and Daubechies wavelet transform. Each and every transform technique has a specific way to coding the images and one transform operation can be various from other transform. The image can be computed in the DWT as uses the lifting and convolution based approach techniques. In the lifting scheme requires fewer arithmetic operation but in the convolution based approach require more arithmetic operation and

also the adder, multiplier delay of the lifting based approach can be longer than the convolution based method. The Daubechies wavelet transform is most commonly used Discrete Wavelet Transform and this can be formulated by the mathematician Ingrid Daubechies. Compared than Haar wavelet transform Daubechies wavelet can be developed with many variations and this can be formulated by the use of recurrence relation in order to generate the discrete samples of the image. In the Daubechies wavelets transform are chosen to have the highest number of vanishing moments that is refer as A , the tab used in the transform can be calculated as $N = 2A$. Here DN as used to find the length or number of taps, and $dB A$ as refer as the number of vanishing moments. In this proposed paper 9 tab wavelet filter are designed so $N = 9$ or $D9$ and $dB9/2$ are referred as the same wavelet transform. This Daubechies Wavelet Transform can be various from $DAUB2$ to $DAUB20$ that is highly localized to highly smooth variation and $DAUB 2$ have 2 coefficient like that $DAUB 9$ in the proposed design have 9 coefficients [1][8]. The Daubechies Wavelet Transform as designed using filter bank or lifting scheme means the hardware complexity is reduced towards the multiplier less implementation design [2]. In this Daubechies wavelet transform is generate the filter coefficients as the form of irrational numbers so this irrational number can introduce the approximation error in the process. This approximation error as in the entire process can degrade the image quality or loss of information in the reconstruction side. The Algebraic Integer (AI) based concept used to eliminate the errors by mapping the irrational coefficients into rational one form or array of integer form [8]. The Reconstruction step can be used at the final stage of design to reduce the amount of truncation operation so the Quantization error can be reduces dramatically.

2. 2D DISCRETE WAVELET TRANSFORM

The DWT is known as the worldwide accepting technique for image processing field. Basically the 2D DWT can achieved by the repeated use 1D DWT method. The 2D DWT architecture can be perform the following two stages to process the image that is decomposition and reconstruction level. In previous design the Daubechies wavelet filter is designed by Algebraic Integer (AI) technique. The output of the Daubechies wavelet filter is in the form of irrational numbers so the computational error level can be increased during the truncation or rounding operation perform over the filter bank associated with the Daubechies Wavelet. The Algebraic Integer is mainly used to convert those irrational coefficients into rational or array of integer value. So the AI technique can be used to avoid the quantization error during the filtering process. In the proposed design Daub9 wavelet filter is designed based on Algebraic integer concept. In the Existing deign is based on Daub4 and Daub9 wavelet filters using single Final Reconstruction Step (FRS) technique. The intermediate reconstruction step between the numbers of stages can introduce the computational error at the entire filtering process so the error level is increased. In the proposed design also FRS used only once in the entire operation that is the final level of the design is having a FRS block. The DWT have the following advantages that is it have a excellent energy compaction, low bit error rate compared than DCT [13] and well coding design for better image compression techniques.

3. FILTER BANKS AND SUB BAND CODING OF DAUBECHIES WAVELET

In the image processing, a 2D analysis and synthesis of the filter banks are performed in the wavelet transform. In the 2D filtering concept, the 1D concept filter bank is first applied to the columns of the image and then applied to the rows. The image has $R1$ rows and $R2$ columns i.e. $256*256$ size, now apply the 1D analysis filter bank to each column to obtain a two subband images, each having $R1/2$ rows and

$R2$ columns. And then apply the 1D analysis filter bank to each row and to obtain the four subband images and each having $R1/2$ rows and $R2/2$ columns. This can be illustrated in the diagram below.

The input image I is taken for perform the filtering operation. The image pass through the low pass and high pass blocks in rows and perform the down sampling operation by 2, that is produce a two sub

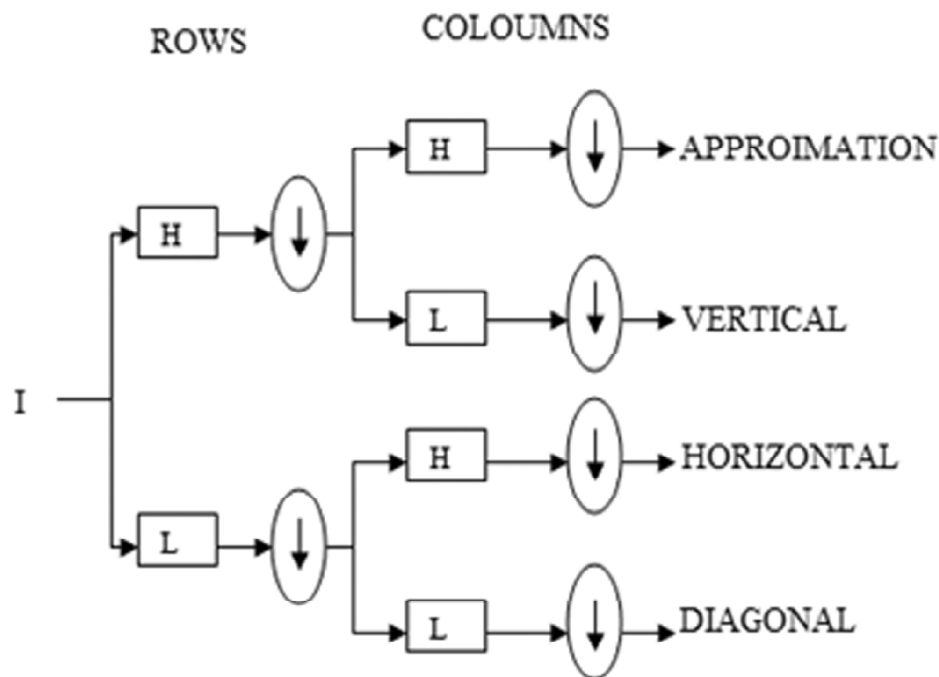


Figure 1: 2D analysis of Filter Bank

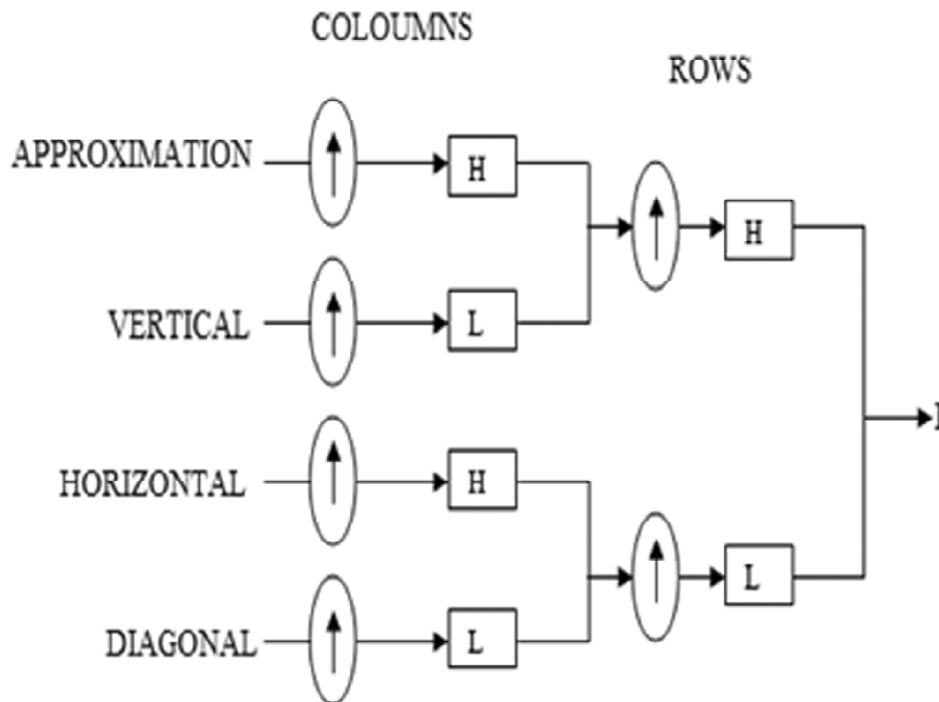


Figure 2: 2D synthesis of Filter Bank

band image. These two images perform the columns wise operation and down sampling by 2, that is producing a four sub band image as output. This four sub band image is named as the coarse approximation details, horizontal details, vertical details and diagonal details. Apart from the coarse approximation detail the remaining three details are discarded or threshold in image compression applications. The wavelet decomposition can be achieved by using the subband coding concept. The term subband coding is defined as the band of information that can be split into number of information and that can be encoded individually and independently. These subband coding is used in the image and signal processing decomposition stages.

4. ALGEBRIAC INTEGER (AI) CONCEPT

In the image processing field lot of transforms are used those are Haar, Symlet, Gaussian etc. In the proposed design is uses the Daubechies Wavelet Transform, the Daubechies Wavelet can be various from DAUB2-DAUB20 i.e. various from highly localized to highly smooth in the image and video processing applications [4][2]. The AI technique is mainly used to reduce the computational error by mapping the irrational coefficients into array of integer. An algebraic integer is a complex number that is a root of some monic polynomial with coefficients in \mathbb{Z} , where \mathbb{Z} = set of integers. The following steps are needed to derive the coefficients [10] of DAUB4, DAUB6 and DAUB9 wavelet filter.

The polynomial equation is given by,

$$P(y) = \sum_{K=0}^{N-1} \binom{2N-1}{K} y^K (1-y)^{N-1-K}$$

Where, $P(y)$ = polynomial variable

N = Number of vanishing moments.

The polynomial $F(z)$ can be obtaining from the equation of $T(z)$ is shown in below.

$$T(Z) = \alpha Z^{-(N-1)} \prod_{K=1}^{2N-2} (Z - ZK)$$

The $F(Z)$ can be derived from $T(Z)$ and that can be converted into $F(e^{j\omega})$ in order to obtain the final coefficients value,

$$T(Z) = |F(Z)|^2 = F(Z).F(Z)^{-1}$$

Sub, $Z = e^{j\omega}$ in the $F(Z)$ equation to obtain the result,

$$F(e^{j\omega}) = S(\omega)$$

The final equation is,

$$H(\omega) = \left(\frac{1+e-j\omega}{2} \right) S(\omega)$$

For DAUB4 Wavelet filter using above equation to derive the four coefficients shown in below,

$$h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$$

$$h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$$

$$h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}} \text{ and}$$

$$h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$$

Here h_0, h_1, h_2, h_3 are DAUB4 filtering coefficients.

The same way can be used to derive a DAUB 6 also,

$$h_0 = \frac{1 + \sqrt{10} + \sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}}$$

$$h_1 = \frac{5 + \sqrt{10} + 3\sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}}$$

$$h_2 = \frac{10 - 2\sqrt{10} + 2\sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}}$$

$$h_3 = \frac{10 - 2\sqrt{10} - 2\sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}}$$

$$h_4 = \frac{5 + \sqrt{10} - 3\sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}} \text{ and}$$

$$h_5 = \frac{1 + \sqrt{10} - \sqrt{5 + 2\sqrt{10}}}{16\sqrt{2}}$$

The above coefficients are in the irrational form so the source of computational error is increased [7]. The substitution integer for the irrational values in equation means the amount computational error can be decreased dramatically. Using this AI based technique means the DWT architecture can be implemented efficiently and it uses less amount of hardware components, so the hardware complexities can be reduced.

5. 2D DAUBECHIES 9 TAB DESIGN

In this proposed design, the DAUB 9 design is based on the AI concept. In this 9 tab Daubechies wavelet filter, the each tab units are designed by using the Wallace tree multiplier and carry save adder. In the existing tab design is based on the Array multiplier and Ripple carry adder, so in the proposed design operation speed can be increased. This 2D Daubechies wavelet filter can be designed by the repeated use of 1D filter and the amount of processing time is based on how much bits used in the input, in the proposed design 8 bit input as taken from the 256*256 size image. First the input image can be converted into hexadecimal format and it can be read both row & columns wise using the address decoder unit. These 0 to 65535 bits can be stored using BRAM (Block Random Access Memory) unit present in the Altera Kit. The filter coefficients values are stores in the LUT (Logic Look Up Table). The pixel value from the BRAM and the filter coefficient values are multiplied and then added using the tab unit. This operation can be performed entire the design by two times and final pixel values are reconstructed using single FRS at the final stage design. Using the FRS only once in the design means the error level can be reduced greatly because the FRS is the only source introduce the error into the design.

6. SIMULATION AND RESULT DISCUSSION

The entire proposed design has been designed using the Verilog coding and simulated by using the Modelsim 6.6c Altera software. The DAUB4, DAUB6 and DAUB9 wavelet filter simulation are obtained. The final simulation can produce the reconstructed hexadecimal text document and this document can be read by using the MATLAB R2011b software. Using the MATLAB code the final output image can be obtained from the MATLAB simulation. The DAUB4, DAUB6 and DAUB9 output image compared with the input

image is shown in below figure 3. The following two factors are mainly used to calculate the image accuracy level and Error level. Those factors are Peak signal to Noise Ratio (PSNR) and Mean Square Error (MSE). The MSE is defined as the square of the difference between the noise less image to noise image, it can be increased one means the error level also increased otherwise it will be decrease [12]. The PSNR is most commonly used to measure the quality of reconstruction of image and it can be increased one for better image accuracy. These two factors can be calculated by using the following equation,

$$MSE = \frac{1}{M.N} \sum_{i=1}^{m-1} (I(i, j) - K(i, j))^2$$

Where, I = Noise free Image

K = Reconstructed Image

$$PSNR = 10 \log_{10}(MAX^2/MSE)$$

The PSNR calculation for Daubechies wavelet filter is shown in below table 1.

The speed of the operation can be calculated in terms speed factor as shown in below table 2.

Table 1
PSNR and MSE calculated values for different DAUB.

<i>Factors</i>	<i>DAUB 4</i>	<i>DAUB 6</i>	<i>DAUB 9</i>
Mean Square Error in %	2.17	1.85	1.30
Peak Signal to Noise Ratio in dB	58.17	59.36	60.52

Table 2
DAUB 9 existing and proposed design

<i>Factor</i>	<i>DAUB 9 Unit Array Multiplier and Ripple Carry Adder</i>	<i>DAUB 9 Unit Wallace Tree Multiplier and Carry Save Adder</i>
Frequency in MHz	36.83	74.94

7. CONCLUSION

In this proposed paper DAUB 9 wavelet filter is designed using Algebraic Integer concept. The AI technique has a significant advantage of low computation error and reduced implementation complexity. The main objective of the proposed design is achieve good image accuracy, reduces the error level and increases the

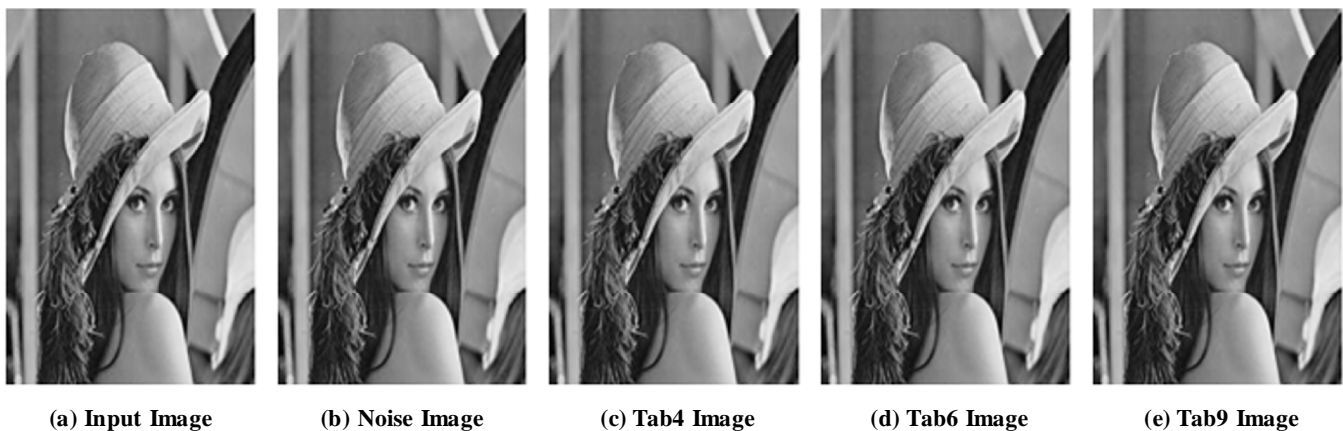


Figure 3: MATLAB simulation output images

speed of operation. The DAUB 9 design can achieve a PSNR in the range of 60.52. In the Daubechies Wavelet filter design, the number of taps increased means the level of image accuracy also increased. The introduced design can be simulated using the Modelsim and implemented in Altera DE2 kit.

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