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Modified Kudryashov Method for Finding Exact Solitary Wave Solutions of Variable Coefficient KdV Burger Equation and Modified KdV Equation

M. H. M. Moussa[*1], Rehab M. El-Shiekh[*2] & A. Fayza. Elsebaee[**]

ABSTRACT

In this paper, we use the modified Kudryashov method or the rational Exp-function method to construct the solitary traveling wave solutions of variable coefficient KdV Burger and modified KdV equations. These equations play a very important role in the mathematical physics and engineering sciences.

Keywords: Modified Kudryashov method, Rational exp-function method, Soliton, Traveling wave solution.

1. INTRODUCTION

As the mathematical model of complex physics phenomena, nonlinear partial differential equations (PDEs) are involved in many fields from physics to biology, chemistry and engineering etc. in the past decades, great efforts have been made to search for powerful methods to obtain exact solutions. There exist some methods such as inverse scattering method [1], Hirota's method [2], homogeneous balance method [3], Jacobi elliptic function method [4], extended tanh-function method [5], Bäcklund transformation method [6], algebra method [7], sine-cosine method [8], Homotopy perturbation method (HPM), Variational iterative method, Homotopy analysis method (HAM), Homotopy padē method (HpadēM) [9-10], F-expansion method was proposed to construct periodic wave solutions of nonlinear PDEs [11-12] and so on.

In this study, we use the rational Exp-function method to obtain the exact solitary wave solutions of the variable coefficient KdV Burger equation and modified KdV equation. The matter of this method is the modification of the approach by Kudryashov therefore we can entitle it as the modified Kudryashov method. The variable coefficient KdV Burger equation and modified KdV equation can be shown in the form of [13-14]

$$u_\tau + \mu_1 u u_\zeta - \mu_2 u_{\zeta\zeta} + \mu_3 u_{\zeta\zeta\zeta} - \mu_4 h(\tau) u_\zeta = 0, \tag{1}$$

$$u_\tau + \mu_4 u^2 u_\zeta + \mu_2 u_{\zeta\zeta\zeta} + \mu_3 h_2(\tau) u_\zeta = 0. \tag{2}$$

2. THE MODIFIED KUDRYASHOV METHOD

To illustrate the basic idea of the modified Kudryashov method, we first consider a general form of the nonlinear equation

$$F(u, u_\tau, u_\zeta, u_{\zeta\zeta}, u_{\zeta\zeta\zeta}) = 0. \tag{3}$$

* Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Hiliopolis, Cairo, Egypt, *E-mails: M_h_m_moussa@yahoo.com*[1] *& Rehab_el_shiekh@yahoo.com*[2]

** Department of Mathematics, Sciences Faculty, Helwan University, Helwan, Cairo, Egypt, *E-mail: Fayza_elsebaee@yahoo.com*

Where, P is a polynomial function with respect to the indicated variables. Making use of the travelling wave transformation

$$u = u(x), \qquad x = p(\tau)\zeta + q(\tau). \tag{4}$$

Where $p(\tau)$ and $q(\tau)$ are arbitrary functions of τ to be determined later, then Eq. (1) reduces to a nonlinear ordinary differential equation (ODE).

In this section, we shall seek a rational function type of solution for a given partial differential equation, in terms of $\exp(x)$, of the following form

$$u(x) = \sum_{k=0}^{m} \frac{a_k(\tau)}{(1+\exp(x))^k}. \tag{5}$$

Where, $a_0(\tau), a_1(\tau), \cdots, a_m(\tau)$ functions of τ to be determined. We can determine m by balance the linear term of the highest order in (1, 2) with the highest order nonlinear term. Differentiating (5) with respect to x, introducing the result into Eq.'s (1, 2), and setting the coefficients of the same power of e^x equal to zero, we obtain algebraic equations. The rational function solution of the Eq.'s (1, 2) can be solved by obtaining $a_0(\tau), a_1(\tau), \cdots, a_m(\tau)$ from this equation.

3. EXACT SOLUTIONS OF EQ. (1) BY USING MODIFIED KUDRYASHOV METHOD

For time-dependent h, Using the balancing procedure we get $m = 2$. This suggests the choice of $u(x)$ as

$$u(x) = a_0(\tau) + \frac{a_1(\tau)}{1+\exp(x)} + \frac{a_2(\tau)}{(1+\exp(x))^2}. \tag{6}$$

Substituting (6) and the ansatz (4) in to Eq. (1) and setting the coefficients of the same powers of e^x equal to zero, we obtain the following algebraic equations

$$\frac{da_0(\tau)}{d\tau} = 0,$$

$$\frac{da_0(\tau)}{d\tau} + \frac{da_1(\tau)}{d\tau} + \frac{da_2(\tau)}{d\tau} = 0,$$

$$\mu_4(\tau)h(\tau)p(\tau)a_1(\tau) + 5\frac{da_0(\tau)}{d\tau} + \frac{da_1(\tau)}{d\tau} - \mu_2 a_1(\tau)p^2(\tau) - \mu_1 p(\tau)a_0(\tau)a_1(\tau) - a_1(\tau)\frac{dq(\tau)}{d\tau}\zeta - \mu_3 p^3(\tau)a_1(\tau) = 0,$$

$$\begin{aligned}
-a_1(\tau)\frac{dq(\tau)}{d\tau} &- 3\mu_1 p(\tau)a_1(\tau)a_2(\tau) + \mu_4 h(\tau)p(\tau)a_1(\tau) + 2\mu_4 h(\tau)p(\tau)a_2(\tau) - 2a_2(\tau)\frac{dq(\tau)}{d\tau} \\
&+ \mu_2 a_1(\tau)p^2(\tau) + 2\mu_2 a_2(\tau)p^2(\tau) - 2\mu_1 p(\tau)a_0(\tau)a_2(\tau) + 3\frac{da_2(\tau)}{d\tau} + 4\frac{da_1(\tau)}{d\tau} \\
&- \mu_3 a_1(\tau)p^3(\tau) - \mu_1 p(\tau)a_1^2(\tau) - 2\mu_3 p^3(\tau)a_2(\tau) - \mu_1 p(\tau)a_0(\tau)a_1(\tau) \\
&- 2a_2\frac{dp(\tau)}{d\tau}\zeta - 2a_1\frac{dp(\tau)}{d\tau}\zeta - \mu_1 p(\tau)a_1^2(\tau) + 5\frac{da_0(\tau)}{d\tau} = 0,
\end{aligned}$$

$$\begin{aligned}
-4a_2(\tau)\frac{dq(\tau)}{d\tau} &+ 3\frac{da_2(\tau)}{d\tau} - 2\mu_2 p^2(\tau)a_2(\tau) - 3a_1(\tau)\frac{dq(\tau)}{d\tau} + 6\frac{da_1(\tau)}{d\tau} - 4\mu_1 p(\tau)a_0(\tau)a_2(\tau) \\
&- 2\mu_1 p(\tau)a_1^2(\tau) + 4\mu_4 h(\tau)p(\tau)a_2(\tau) - 3\mu_1 p(\tau)a_0(\tau)a_1(\tau) + 3\mu_4 h(\tau)p(\tau)a_1(\tau) \\
&- 3\mu_1 p(\tau)a_2(\tau)a_1(\tau) - 3a_1(\tau)\frac{dp(\tau)}{d\tau}\zeta - 4a_2(\tau)\frac{dp(\tau)}{d\tau}\zeta + 14\mu_3 p^3(\tau)a_2(\tau) \\
&+ \mu_2 p^2(\tau)a_1(\tau) + 10\frac{da_0(\tau)}{d\tau} + 3\mu_3 p^3(\tau)a_1(\tau) = 0,
\end{aligned}$$

$$\frac{da_2(\tau)}{d\tau} - 2a_2(\tau)\frac{dp(\tau)}{d\tau}\zeta - 3\mu_1 p(\tau)a_0(\tau)a_1(\tau) + 2\mu_4 h(\tau)p(\tau)a_2(\tau) - 3a_1(\tau)\frac{dp(\tau)}{d\tau}\zeta$$
$$+ 4\frac{da_1(\tau)}{d\tau} - 2a_2(\tau)\frac{dq(\tau)}{d\tau} - 4\mu_2 p^2(\tau)a_2(\tau) + 3\mu_3 p^3(\tau)a_1(\tau) - 8\mu_3 p^3(\tau)a_2(\tau)$$
$$- \mu_2 p^2(\tau)a_1(\tau) + 10\frac{da_0(\tau)}{d\tau} - \mu_1 p(\tau)a_1^2(\tau) - 3a_1(\tau)\frac{dq(\tau)}{d\tau} + 3\mu_4 h(\tau)p(\tau)a_1(\tau)$$
$$- 2\mu_1 p(\tau)a_0(\tau)a_2(\tau) = 0. \quad (7)$$

Solving the system of algebraic equations (7) with the aid of Maple, we obtain the following result

$$a_0(\tau) = c, \qquad a_1(\tau) = \frac{24\mu_2^2}{25\mu_1\mu_3},$$
$$a_2(\tau) = -\frac{12\mu_2^2}{25\mu_1\mu_3}, \qquad p(\tau) = \frac{\mu_2}{5\mu_3},$$
$$q(\tau) = \frac{-\mu_2}{125\mu_3^2}\int\left[6\mu_2^2 + 25\mu_1\mu_3 c - 25\mu_3\mu_4 h(\tau)\right]d\tau + c_1.$$

Finally the solution $u(x)$ of Eq. (1) becomes

$$u(x) = c + \frac{24\mu_2^2}{25\mu_1\mu_3(1+e^x)} - \frac{12\mu_2^2}{25\mu_1\mu_3(1+e^x)^2}. \quad (8)$$

Where x as in anstaz (4)

$$x = \frac{\mu_2}{5\mu_3}\zeta - \frac{\mu_2}{125\mu_3^2}\int\left[6\mu_2^2 + 25\mu_1\mu_3 c - 25\mu_3\mu_4 h(\tau)\right]d\tau + c_1.$$

As an example to illustrate the properties of the solution we plot the first solution in Fig. (1), if we take the constant integration = 0 and

$$\mu_4 = -1, \qquad c = 1, \qquad \mu_2 = \mu_3 = \mu_1 = 1 \quad \text{and} \quad h(\tau) = \tau$$

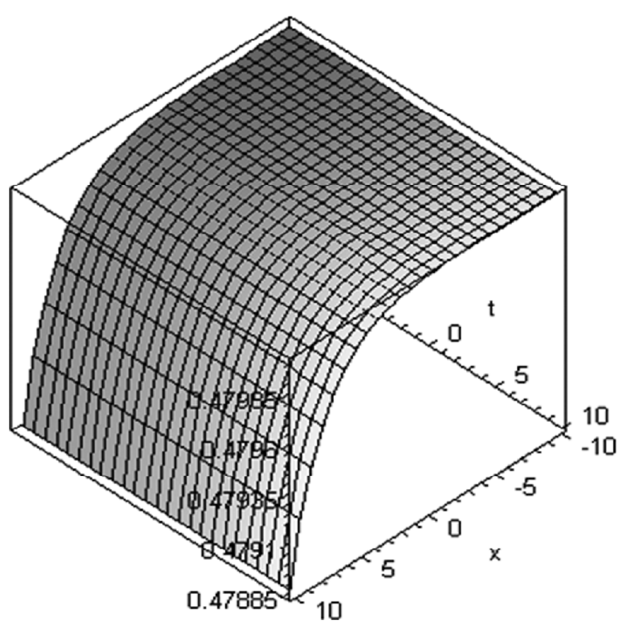

Figure 1: Which is a Solitary Wave Solution of Eq. (1)

4. EXACT SOLUTIONS OF EQ. (2) BY USING MODIFIED KUDRYASHOV METHOD

For time-dependent $h_2(\tau)$, Using the balancing procedure we get $m = 1$. This suggests the choice of $u(x)$ as

$$u(x) = a_0(\tau) + \frac{a_1(\tau)}{1+\exp(x)}. \tag{9}$$

Substituting (9) and the ansatz (4) in to Eq. (2) and setting the coefficients of the same powers of e^x equal to zero, we obtain the following algebraic equations

$$-\frac{da_0(\tau)}{d\tau} = 0,$$

$$-\frac{da_0(\tau)}{d\tau} - \frac{da_1(\tau)}{d\tau} = 0,$$

$$2\mu_4(\tau)a_0^2(\tau)p(\tau)a_1(\tau) + 2a_1\frac{dp(\tau)}{d\tau}\zeta + 2\mu_4 p(\tau)a_1^2(\tau)a_0(\tau) - 3\frac{da_1(\tau)}{d\tau} + 2a_1(\tau)\frac{dq(\tau)}{d\tau} - 6\frac{da_0(\tau)}{d\tau} + 2\mu_3 h_2(\tau)p(\tau)a_1(\tau) - 4\mu_2 a_1(\tau)p^3(\tau) = 0,$$

$$-\frac{da_1(\tau)}{d\tau} - 4\frac{da_0(\tau)}{d\tau} + \mu_3 h_2(\tau)a_1(\tau)p(\tau) + \mu_4 a_1(\tau)a_0^2(\tau)p(\tau) + \mu_2 a_1(\tau)p^3(\tau) + a_1(\tau)\frac{dq(\tau)}{d\tau} + a_1\frac{dp(\tau)}{d\tau}\zeta = 0,$$

$$\mu_4 a_1(\tau)a_0^2(\tau)p(\tau) + a_1\frac{dp(\tau)}{d\tau}\zeta + \mu_4 a_1^3(\tau)p(\tau)\mu_2 a_1(\tau)p^3(\tau) - 4\frac{da_0(\tau)}{d\tau} - 3\frac{da_1(\tau)}{d\tau} + 2\mu_4 a_0(\tau)a_1^2(\tau)p(\tau) + \mu_3 h_2(\tau)a_1(\tau)p(\tau) + a_1(\tau)\frac{dq(\tau)}{d\tau} = 0. \tag{10}$$

Solving the system of algebraic equations (10) with the aid of Maple, we obtain the following result

$$a_0(\tau) = c, \qquad a_1(\tau) = -2c, \qquad p(\tau) = \frac{\sqrt{-6\mu_2\mu_4}\,c}{3\mu_2},$$

$$q(\tau) = \int\left[\left(\frac{-\mu_4 c^2}{3} - h_2(\tau)\mu_3\right)\left(\frac{\sqrt{-6\mu_2\mu_4}\,c}{3\mu_2}\right)\right]d\tau + c_1.$$

Finally the solution $u(x)$ of Eq. (2) becomes

$$u(x) = c + \frac{-2c}{(1+e^x)}. \tag{11}$$

Where x as in anstaz (4)

$$x = \frac{\sqrt{-6\mu_2\mu_4}\,c}{3\mu_2}\zeta + \int\left[\left(\frac{-\mu_4 c^2}{3} - h_2(\tau)\mu_3\right)\left(\frac{\sqrt{-6\mu_2\mu_4}\,c}{3\mu_2}\right)\right]d\tau + c_1.$$

As an example to illustrate the properties of the solution we plot the first solution in Fig. (2), if we take the constant integration = 0 and

$$\mu_4 = -1, \qquad c = 1, \qquad \mu_2 = \mu_3 = 1 \quad \text{and} \quad h(\tau) = \tau$$

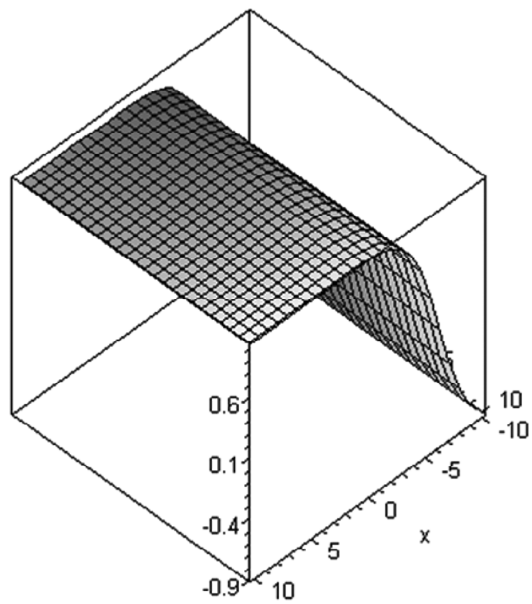

Figure 2: Which is a Solitary Wave Solution of Eq. (2)

5. CONCLUSIONS

In this paper, we have applied the modified Kudryashov method or the rational Exp-function method on the variable coefficient K-dV Burger equation and modified K-dV equation. New solitary wave solutions are obtained for both equations.

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