

# Synchronization of Chaotic Systems using NAC and Its Application to Secure Communication

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**Abstract:** This paper aims towards synchronization of Bhalekar-Gejji chaotic system and its application to secure communication. Synchronization is achieved using suitable nonlinear active control (NAC) scheme. Proposed synchronization is used to achieve secure communication using NAC. For secure communication, the information signal is masked with one of the state of chaotic system at transmitter end and retrieved at receiver end. Synchronization and convergence of error dynamics are achieved by using required Lyapunov stability condition. MATLAB simulation results validate synchronization of Bhalekar-Gejji chaotic system and its application to secure communication. Simulation results reveal that synchronization and its application using NAC is achieved successfully. Further, performance of NAC is given based on performance criteria terms as average synchronization error for both synchronization and its application to secure communication.

## 1. INTRODUCTION

Chaotic systems are being potentially used for secure communication and signal processing because of their potential mechanism for signal design and generation. Chaotic signal can be used in various ways for masking information bearing waveforms, because they are typically noise like and broadband, therefore difficult to predict [1]. Another important property of any chaotic system is aperiodic long time behavior, arises from deterministic dynamical system which is sensitivity to initial condition [2]. This important property leads to interesting and common applications in the field of secure communication [3, 4].

Synchronization of two identical chaotic systems with different initial condition was illustrated by Pecora & Carroll in 1990 [5] states that two chaotic systems can be synchronized from different initial conditions. During last three decades, chaos synchronization has attracted a great attention from various fields [2]. There are two methods for synchronization of chaotic system as drive-response scheme and coupling scheme. Drive-response also called master-slave system which is widely used. In this scheme output of the master (drive) system is use to control the slave system. Synchronization is achieved when zero error between the states of master and slave system. Various control schemes have been developed for synchronization of chaotic system in the last two decade such as nonlinear active control method [6], adaptive control method [7], backstepping control method [8], sampled data feedback method [9], time delay feedback method [10], sliding mode control method [11], passive control [12], optimal control [13], fuzzy control [14], PID control [15], etc.

Motivated with above discussion and problem, this paper put forward the synchronization and its application to secure communication using nonlinear active control. NAC is used here for synchronization

of two identical Bhalekar-Gejji [16, 17] chaotic systems. NAC is used because of its design simplicity, fast response [6]. Synchronization is achieved and applied to secure communication between master and slave systems using single driving chaotic signal. For illustration purpose message signal is transmitted at transmitter end in the form of saw tooth and square wave and same message is received at the receiver end.

Rest of paper is organized as follows: Section 2 deals with description of Bhalekar-Gejji chaotic system and its synchronization using NAC. In Section 3, design of NAC to achieve secure communication using Bhalekar-Gejji chaotic system is given. In Section 4, simulation results are shown for validation and verification of proposed scheme. Finally, conclusions and future scopes are addressed in Section 5.

## 2. SYNCHRONIZATION OF CHAOTIC SYSTEM USING NAC

In this section, description of Bhalekar-Gejji chaotic system and its synchronization using NAC is discussed.

### 2.1 System Description and its Synchronization

In 2011, S. B. Bhalekar et al. [16] proposed the Bhalekar-Gejji dynamical system. Further S. B. Bhalekar [17] explored the parameters and presented some interesting properties of BG chaotic system in 2012. The 3D Bhalekar-Gejji chaotic system is described by

$$\begin{cases} \dot{x}_1 = wx_1 - x_2^2 \\ \dot{x}_2 = \mu(x_3 - x_2) \\ \dot{x}_3 = \alpha x_2 - \beta x_3 + x_1 x_2 \end{cases} \quad (1)$$

where  $x_1, x_2, x_3$  are the states and  $w < 0, \mu, \alpha, \beta > 0$  are parameters of system (1). BG system exhibit chaotic behavior for  $w = -2.667, \mu = 10, a = 27.3, b = 1$ . The phase portrait is shown in Fig. 1.

$$\begin{cases} \dot{y}_1 = wy_1 - y_2^2 + u_1 \\ \dot{y}_2 = \mu(y_3 - y_2) \\ \dot{y}_3 = \alpha y_2 - \beta y_3 + y_1 y_2 + u_2 \end{cases} \quad (2)$$

where  $y_1, y_2, y_3$  are states and  $u_1, u_2$  are the control inputs added in slave system (2). Synchronization between master and slave system is achieved for any initial conditions  $x(0)$  and  $y(0)$ , if:

$$\lim_{t \rightarrow \infty} \|e(t)\| = \lim_{t \rightarrow \infty} \|y - x\| = 0 \quad (3)$$

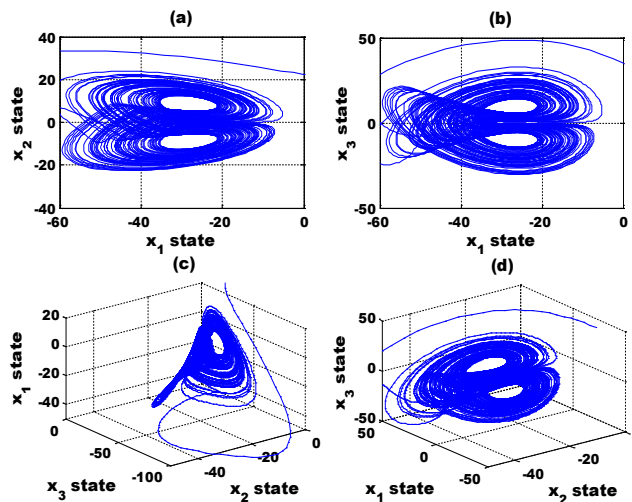


Figure 1: The phase portraits of Bhalekar-Gejji chaotic system in (a)-(d)

The error dynamics is obtained as:

$$\begin{cases} \dot{e}_1 = we_1 - y_2^2 + x_2^2 + u_1 \\ \dot{e}_2 = -\mu e_2 + \mu e_3 \\ \dot{e}_3 = ae_2 - be_3 + y_1y_2 - x_1x_2 + u_2 \end{cases} \quad (4)$$

To synchronize the master-slave chaotic system defined in (1) and (2), our aim is to design NAC such that resulting error dynamics satisfies condition (3). Designing of controller is discussed in the next Section.

## 2.2 Design Of Nac For Synchronization

To ensure the stability of error dynamics (4), the NAC controller is proposed as:

$$\begin{cases} u_1 = -x_2^2 + y_2^2 \\ u_2 = -ae_2 - y_1y_2 + x_1x_2 - \mu e_2 \end{cases} \quad (5)$$

To establish stability of error dynamics (4) using Lyapunov theory, considering positive definite Lyapunov function as:

$$V(\mathbf{e}) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (6)$$

results a negative definite function as:

$$\dot{V}(\mathbf{e}) = -we_1^2 - \mu e_2^2 - be_3^2 \quad (7)$$

Hence,  $\dot{V}(\mathbf{e}) \leq 0$  and according to Lyapunov stability theory, the states of the master and slave systems are synchronize and ensure convergence of error dynamics as  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ . Thus, according to Lyapunov stability theory  $e(t)$  always converges to zero. Hence, states of the master and slave are asymptotically stable according to (6).

## 3. APPLICATION TO SECURE COMMUNICATION

A potential approach of Synchronization of chaotic system for communication is based on chaotic signal masking and recovery. In masking scheme, a noise like signal is added at the transmitter (1) to the information bearing signal  $m$ , and at the receiver end masking is removed. A signal  $m$  is added to state  $x_1$  of the transmitter as  $s = x_1 + m$ . It is assumed that for masking signal to noise ratio of  $m(t)$  is lower than  $x_1$ .

Here, the basic idea is to use the received signal to generate masking signal at the transmitter and subtract it from the received signal to recover  $m$ . This can be done with the synchronizing master and slave system and using them as transmitter and receiver. If the receiver is synchronized with  $s$  as the drive, the  $y_1 = x_1$  and consequently  $m(t)$  is recovered as  $m = s - y_1$ . Due to the fact that output signal can be used to recover input signal, it indicates that it is possible to implement secure communication scheme with proposed chaotic system synchronization. Using above synchronization scheme, synchronizing error dynamics between transmitter and receiver can be written as;

$$\begin{cases} \dot{e}_1 = wm - y_2^2 + x_2^2 + u_1 \\ \dot{e}_2 = -\mu e_2 + \mu e_3 \\ \dot{e}_3 = ae_2 - be_3 + my_2 + x_1e_2 + u_2 \end{cases} \quad (8)$$

### 3.1 SECURE COMMUNICATION USING NAC

To ensure the convergence of error dynamics, controller is proposed as follows:

$$\begin{cases} u_1 = wm - x_2^2 + y_2^2 - k_1 e_1 \\ u_2 = -ae_2 - my_2 - x_1 e_2 - \mu e_2 \end{cases} \quad (9)$$

Control laws (9) insure the master and slave systems trajectory converges to zero and satisfies  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ , to achieve synchronization for secure communication using identical pair of chaotic systems (1) and (2) as transmitter and receiver, respectively. The modified error dynamics is written as:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -\mu e_2 \\ \dot{e}_3 = -be_3 \end{cases} \quad (10)$$

To establish the stability using Lyapunov theory, a Lyapunov function  $V(e)$  and its continuous first order derivative results a negative definite function as:

$$\dot{V}(e) = -k_1 e_1^2 - \mu e_2^2 - be_3^2 \quad (11)$$

Hence,  $\dot{V}(e) \leq 0$  and according to Lyapunov stability theory, states of master and slave are synchronized and ensure convergence of error dynamics as  $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$ , i.e. the states are asymptotically synchronized and error dynamics converges to zero and message can be retrieve successfully at the receiver end.

#### 4. RESULTS AND DISCUSSIONS

We are using MATLAB ode-45 for solving the dynamics (1) and (2) for synchronization with application to secure communication. Results are simulating with step time 0.001. Simulation is shown for 7 seconds to clear visibility.

The initial conditions for simulating the synchronization of states, controllers, and error dynamics are  $x(0) = [1 \ 2 \ 3]^T$  and  $y(0) = [5 \ 7 \ 10]^T$ . Parameters of Bhalekar-Gejji system (1) for chaotic behavior are considered as  $\alpha = 27.3$ ,  $\beta = 1$ ,  $\mu = 10$ ,  $w = -2.667$  [17].

##### 4.1 Synchronization of Chaotic System

The time response plots of master and slave synchronization states are given in Fig. 2. Required control inputs are depicted in Fig. 3. Synchronization error between the states of master and slave systems is given in Fig. 4(a). Average synchronization error  $e_s$  (avg) is used as performance criteria and given below:

$$e_s(\text{avg}) = \sqrt{e_1^2 + e_2^2 + e_3^2}$$

Figure 4(b) shows average synchronization error between the identical Bhalekar-Gejji chaotic systems.

In the upcoming subsections, results corresponding to secure communication are given based on synchronization using NAC. Message signal is masked with one of the state of master chaotic system (transmitter) and recovered at the receiver end. The resulting signal is known as transmitted signal.



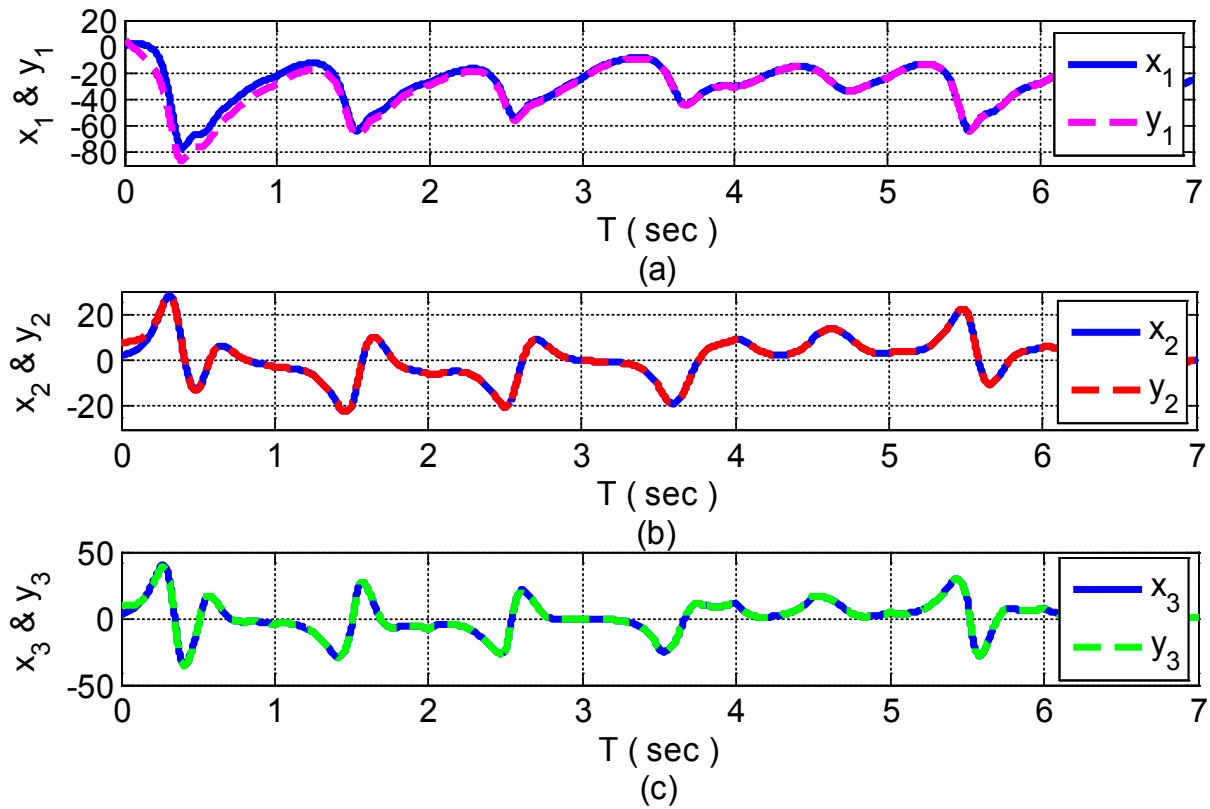


Figure 2: The time response of the states of the master (1) and slave (2) systems

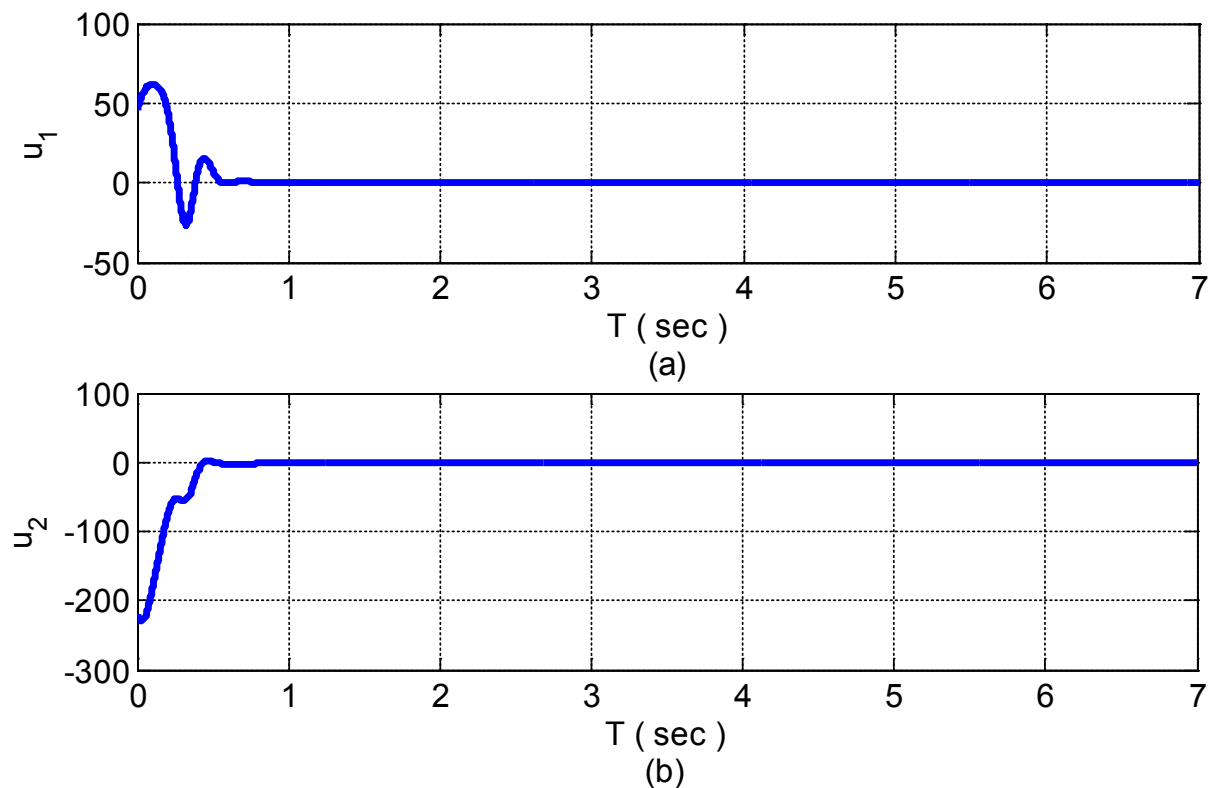


Figure 3: Required control inputs (a) (b) to achieve synchronization between Bhalekar-Gejji chaotic systems

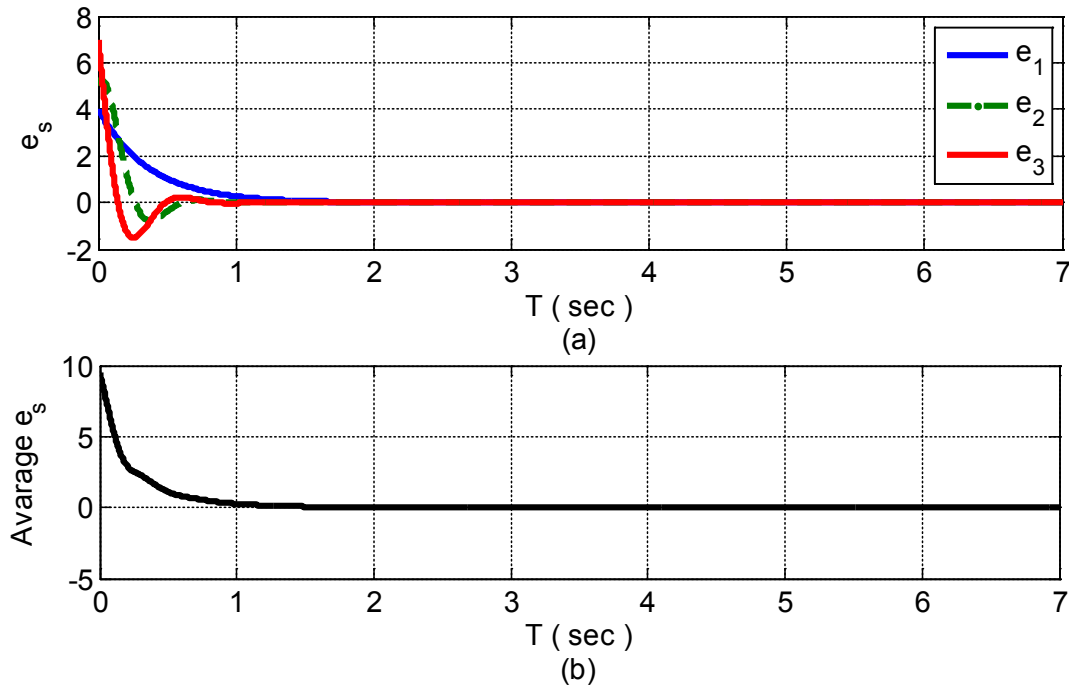


Figure 4: Synchronization error and average synchronization error in (a) and (b), respectively

### 4.2 Secure Communication When Information Signal is a Saw-Tooth Wave

A saw-tooth waveform is used as message signal  $m(t) = 10\text{sawtooth}(4\pi t)$  Information bearing signal (transmitted signal) and message signal at transmitter and receiver end is shown in Fig. 5. Synchronization errors and performance criteria  $e_s(\text{avg})$  is given in Fig. 6.

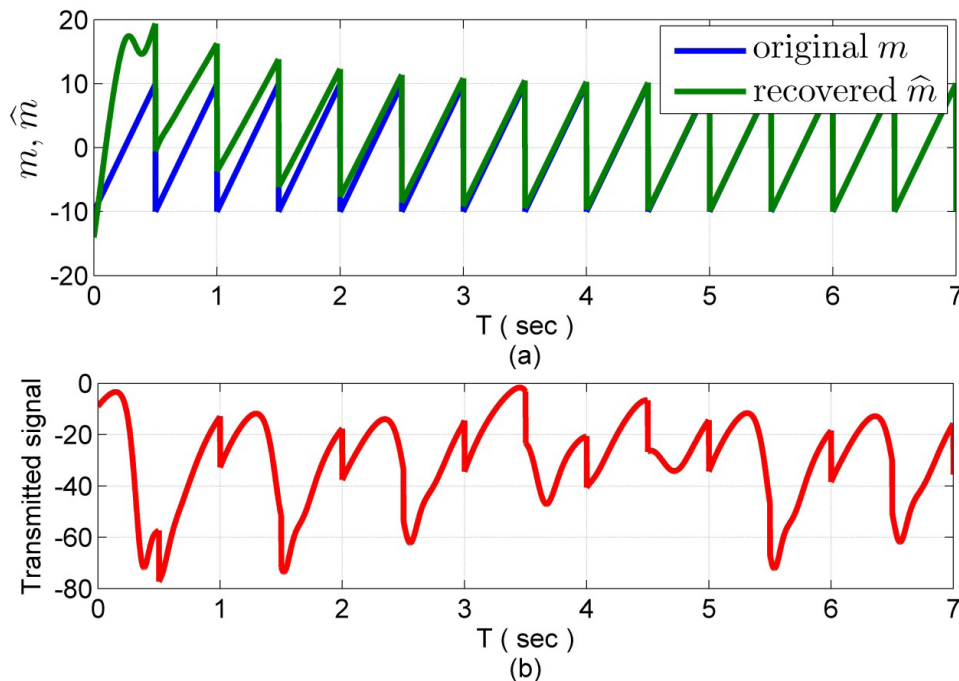


Figure 5: Information signal (a) original signal at transmitter end and recovered signal at receiver end, and (b) transmitted signal

### 4.3 Secure Communication when Information Signal is a Square Wave

A square waveform is used as message signal  $m(t) = 10\text{square}(2\pi t)$ . Information bearing signal (transmitted signal) and message signal at transmitter and receiver end is shown in Fig. 7. Synchronization errors and performance criteria  $e_s(\text{avg})$  is given in Fig. 8.

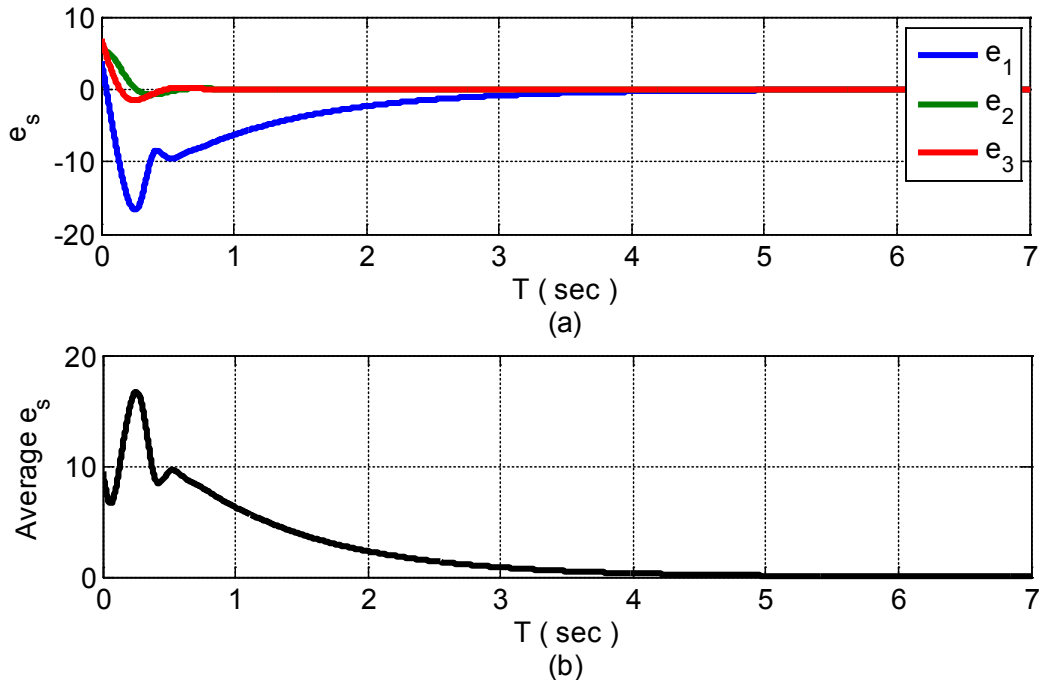


Figure 6: (a) shows synchronization errors and (b) average synchronization error  $e_s(\text{avg}) = \sqrt{e_1^2 + e_2^2 + e_3^2}$

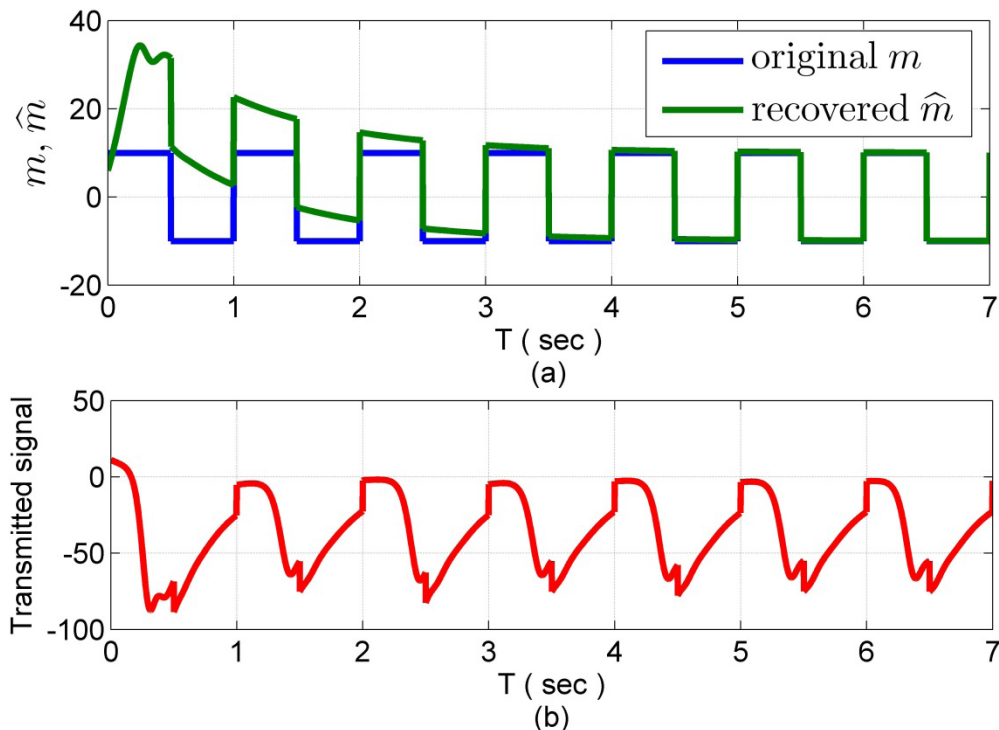


Figure 7: Shows (a) original information signal  $m$  at transmitter end and recovered information signal  $m$  at receiver end, and (b) transmitted signal  $s = x_1 + m$

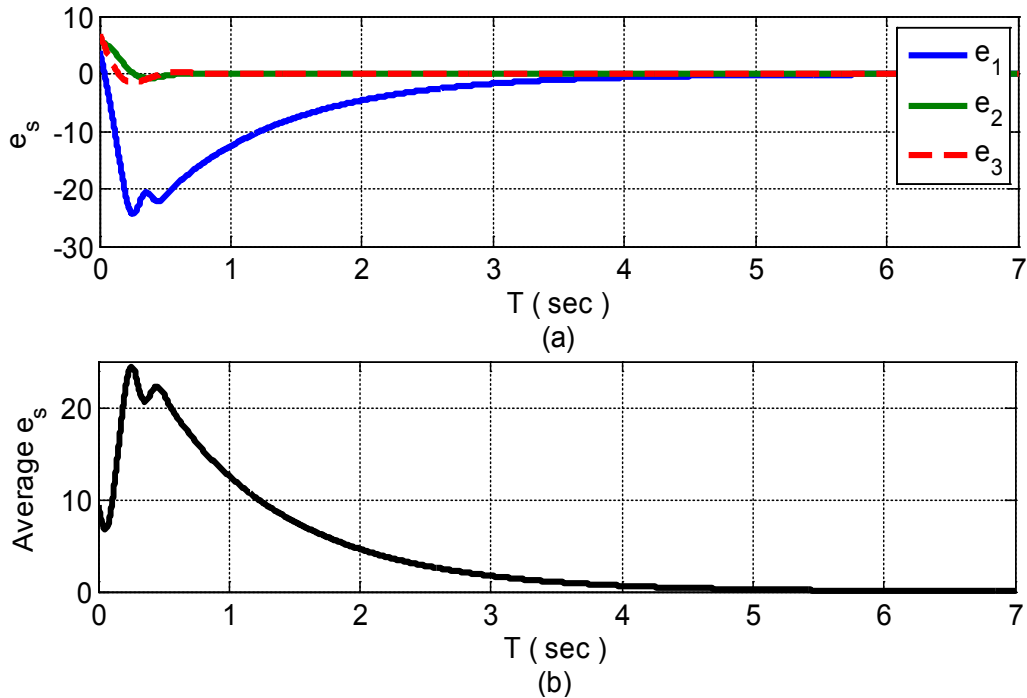


Figure 8: (a) shows synchronization errors and (b) shows average synchronization error  $e_s(avg)$

## 5. CONCLUSIONS

In this paper global synchronization scheme with application to secure communication is presented first time in literature for newly developed Bhalekar-Gejji chaotic systems. NAC is used to guaranteed the occurrence of global asymptotically stability. It has been shown that the master and slave systems are synchronized with proper NAC design. Required stability condition is derived using Lyapunov stability. Finally, numerical simulations are presented to demonstrate effectiveness of the proposed synchronization. Proposed synchronization scheme can also be used in the complex dynamical network as small world network and scale-free networks. This may be the future direction for researchers to precede and explore this work further.

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