THE GENERAL EULERIAN INTEGRAL AND ITS EVALUATION

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Abstract. *An interesting class of Eulerian integrals pertaining to Bessel function $J_v(z)$ or $J_0(z)$, which were expressed in compact forms by M.L. Glasser [2]. In view of the study of the screening properties of a charged impurity located inside and near the surface of metal subjected to a magnetic field. Afterwards the work of Glasser's was generalized and extended concerning compact form expressions for a number of Eulerian integrals pertaining to Meijer's G-function by L.T. Wille. H.M.Srivastava (1993) gave the generalization of these results. Motivated by these recent works, we aim to evaluating the general class of Eulerian integrals involving concerning to $\overline{H}$ function and S_n^m polynomials. Our main result (11) is shown to provide the key formulae from which a large number of integrals can be deduced.*

1. INTRODUCTION

The $\overline{H}$ function defined by Inayat-Hussain [4] as:

$$\overline{H}^{M,N}_{P,Q}[z] = \overline{H}^{M,N}_{P,Q}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N};(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \phi(s)\, z^s ds, \tag{1}$$

where

$$\phi(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j s)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j s)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j s)\}^{B_j}\prod_{j=N+1}^{Q}\Gamma(a_j-\alpha_j s)} \tag{2}$$

which contains fractional powers of some of the Γ-functions. Here z may be real or complex but is not equal to zero and an empty product is interpreted as unity. P, Q, M and N are integers such that $1 \le M \le Q$, $0 \le N \le P$, α_j ($j = 1, \ldots, P$), β_j ($j = 1, \ldots, Q$) are complex numbers. The exponents A_j ($j = 1, \ldots, N$) and B_j ($j = M + 1, \ldots, Q$) can take non-integer values. When these exponents take integer values, the $\bar{H}$-function reduces to the familiar H-function due to Fox.

Srivastava [6] introduced the general class of polynomials

$$S_n^m[x] = \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell} x^{\ell},\ \ell = 0, 1, 2, \ldots \tag{3}$$

where m is an arbitrary positive integer and the coefficients $A_{n,l}(n, \ell \geq 0)$ are arbitrary constants, real or complex.

2. THE GENERAL EULERIAN INTEGRAL AND ITS EVALUATION

We address the problem of closed-form evaluation of the following general Eulerian integral involving $\bar{H}$-function:

$$I\ (z) = I_{\alpha,\beta,\rho,\sigma,n}^{\gamma,\delta,\lambda,\mu,m}\left\{\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P};\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q};\end{matrix} z;\ \ell,\ w\right][y\{g(t)\}^{\rho'}]\right\}$$

$$= \int_{\ell}^{w} \frac{(t-\ell)^{\lambda}(w-t)^{\mu}}{\{f\ (t)\}^{\lambda+\mu+2}} \bar{H}_{P,Q}^{M,N}\left[z\ \{g(t)\}^{v}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right] S_n^m[y\ \{g(t)\}^{\rho'}]dt, \tag{4}$$

where

$$f(t) = w - \ell + \rho(t-\ell) + \sigma(w-t) \tag{5}$$

and

$$g(t) = \frac{(t-\ell)^{\gamma}(w-t)^{\delta}\{f\ (t)\ \}^{1-\gamma-\delta}}{\beta(w-\ell)+(\beta\rho+\alpha-\beta)+\beta\sigma(w-t)}. \tag{6}$$

Now evaluate the general Eulerian integral (4) making use of the definition (1), we first find from (4) that

$$I(z) = \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h!} A_{n,h}(y)^h \int_{\ell}^{w} \frac{(t-\ell)^{\lambda}(w-t)^{\mu}}{\{f(t)\}^{\lambda+\mu+2}} \times \left(\frac{1}{2\pi i}\int_L \phi(s)\ z^s\{g(t)\}^{vs+\rho' h} ds\right)dt \tag{7}$$

where L is a suitable contour of the Mellin-Barnes type in the complex s-plane and $f(t)$, $g(t)$, $f(s)$ are given by (5), (6) and (2) respectively.

Assuming the inversion of the order of integration in (7) to be permissible by absolute (and uniform) convergence of the integrals involved above, we have

$$I(z) = \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h\ !} A_{n,h}(y)^h \frac{1}{2\pi i}\int_L \phi(s)\left(\frac{z}{\beta^v}\right)^s \frac{1}{\beta^{h\rho'}}$$

$$\left(\int_{\ell}^{w} \frac{(t-\ell)^{\lambda+\gamma v s+\gamma\rho' h}(w-t)^{\mu+\delta v s+\delta\rho' h}}{\{f(t)\}^{\lambda+\mu+(\gamma+\delta)(vs+\rho' h)+2}} \left\{1 - \frac{(\beta-\alpha)(t-\ell)}{\beta f(t)}\right\}^{-vs-\rho' h} dt \right) ds \tag{8}$$

If $|(\beta-\alpha)(t-\ell)| < |\beta f(t)|$ $\qquad (t \in [\ell, w])$

then use can be made of the binomial expansion and we thus find from (8) that

$$I(z) = \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h\,!} A_{n,h} (y/\beta^{\rho'})^{h} \sum_{r=0}^{\infty} \left(\frac{\beta-\alpha}{\beta}\right)^{r} \frac{1}{2\pi i} \int_{L} \phi(s)$$

$$\times \frac{\Gamma(vs+\rho' h+r)}{\Gamma(vs+\rho' h) r!} \left(\frac{z}{\beta^{v}}\right)^{s} \left\{ \int_{\ell}^{w} \frac{(t-\ell)^{\lambda+\gamma v s+\gamma\rho' h+r}(w-t)^{\mu+\delta v s+\delta\rho' h}}{\{f(t)\}^{\lambda+\mu+(\gamma+\delta)\rightleftarrows(vs+\rho' h)+r+2}} dt \right\} ds \tag{9}$$

provided also that the order of summation and integration can be inverted.

The innermost integral in (9) can be evaluated by appealing to the following known extension of the Eulerian (beta-function) integral (Gradshteyn and Ryzhik, 1980 [3], p. 287, entry 3.198); see also Prudnikov *et al.* 1983, ([5], p. 301, entry 2.2.6.1):

$$\int_{\ell}^{w} \frac{(t-\ell)^{\alpha-1}(w-t)^{\beta-1}}{\{w-\ell+\rho(t-\ell)+\sigma(w-t)\}^{\alpha+\beta}} dt = \frac{(1+\rho)^{-\alpha}(1+\sigma)^{-\beta}}{w-\ell} \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{10}$$

$w \neq \ell$, $\mathrm{Re}(\alpha) > 0$, $\mathrm{Re}(\beta) > 0$, $w - \ell + \rho(t-\ell) + \sigma(w-t) \neq 0$, $(t \in [\ell, w])$

and we finally obtain the desired integral formula:

$$I(z) = (w-\ell)^{-1}(1+\rho)^{-\lambda-\gamma\rho' h-1}(1+\sigma)^{-\mu-\delta\rho' h-1} \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h\,!} A_{n,h}(y/\beta^{\rho'})^{h}$$

$$\sum_{r=0}^{\infty} \frac{\{(\beta-\alpha)/\beta(1+\rho)\}^{r}}{r\,!} \bar{H}^{M,N+3}_{P+3,Q+2}\left[z\,\{\beta(1+\rho)^{\gamma}(1+\sigma)^{\delta}\}^{-v} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(1-r-\rho' h, v;1) \\ (b_j,\beta_j)_{1,M},(1-\rho' h, v;1) \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (-\lambda-r-\gamma\rho' h,\gamma v;1),(-\mu-\delta\rho' h,\delta v;1),(a_j,\alpha_j)_{N+1,P} \\ (-\lambda-\mu-r-\gamma\rho' h-\delta\rho' h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right], \tag{11}$$

which holds true when

(a) $v > 0$; $\gamma > 0$; $\delta > 0$; $\beta \neq 0$; $\ell \neq w$; $\rho, \sigma \neq -1$ and

$w - \ell + \rho(t-\ell) + \sigma(w-t) \neq 0$ $\qquad (t \in [\ell, w])$

(b) $\mathrm{Re}(1 + \lambda + \gamma v(b_j/\beta_j) > 0$ and $\mathrm{Re}(1 + \mu + v\delta(b_j/\beta_j)) > 0$ ($j = 1, \ldots, M$), where M is a arbitrary positive integer.

(c) M, N, P, Q are positive integers constrained by $1 \leq M \leq Q$, $0 \leq N \leq P$.

(d) $|\arg(z)| \leq 1/2\pi\Omega$,

where

$$\Omega = \sum_{j=1}^{M} \left|\beta_j\right| + \sum_{j=1}^{N} \left|A_j \alpha_j\right| - \sum_{j=M+1}^{Q} \left|B_j \beta_j\right| - \sum_{j=N+1}^{P} \left|\alpha_j\right| > 0$$

(e) $|(\beta - \alpha)(t - \ell)| < |\beta\{w - \ell + \rho(t - \ell) + \rho(w - t)|$ $\qquad (t \in [\ell, w])$

(f) m is an arbitrary positive integer and the coefficients $A_{n,h}(n, h \geq 0)$ are arbitrary constants real or complex.

(g) The series on the right hand side of (11) converges absolutely.

3. APPLICATIONS

(1) In this section we specifically show how the general integral formula (11) can be applied (and suitably maneuvered) to derive various interesting (and potentially useful) results including those given by Wille (1988) [10].

First of all for $\rho = \sigma = 0$ and $z = (w - \ell)^{(\gamma+\delta-1)^v}$, (11) readily yields

$$= I_{\alpha,\beta,0,0,n}^{\gamma,\delta,\lambda,\mu,m} \left\{ \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}; \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}; \end{matrix} (w-\ell)^{(\gamma+\delta-1)v}; \ell, w \right] [y\{g(t)^{\rho'}]\right\}$$

$$= \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h\,!} A_{n,h} (y/\beta^{\rho'})^h (w-\ell)^{-1} \sum_{r=0}^{\infty} \frac{\{(\beta-\alpha)/\beta\}^r}{r!}$$

$$\times \bar{H}_{P+3,Q+2}^{M,N+3} \left[\left\{ \frac{(w-\ell)^{\gamma+\delta-1}}{\beta} \right\}^v \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(1-r-\rho'h,v;1)(-\lambda-r-\gamma\rho'h,\gamma v;1),(-\mu-\delta\rho'h,\delta v;1),(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(1-\rho'h,v;1)(-\lambda-\mu-r-(\gamma+\delta)\rho'h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right. \right], \quad (12)$$

provided that the conditions easily obtainable from those of (11) are satisfied.

Setting $\beta = \alpha = 1/\kappa$ in (12), we obtain

$$I_{1/\kappa,1/\kappa,0,0,n}^{\gamma,\delta,\lambda,\mu,m} \left\{ \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}; \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}; \end{matrix} (w-\ell)^{(\gamma+\delta-1)v}; \ell, w \right] [y\{g(t)^{\rho'}]\right\}$$

$$= \sum_{h=0}^{[n/m]} \frac{(-n)_{mh}}{h!} A_{n,h} (y\kappa^{\rho'})^h (w-\ell)^{-1}$$

$$\times \bar{H}^{M,N+3}_{P+3,Q+2}\left[\left\{\kappa(w-\ell)^{\gamma+\delta-1}\right\}^{v}\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(1-r-\rho'h,\,v;1)(-\lambda-r-\gamma\rho'h,\,\gamma v;1),(-\mu-\delta\rho'h,\delta v;1),(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(1-\rho'h,v;1)(-\lambda-\mu-r-(\gamma+\delta)\rho'h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right], \quad (13)$$

which, in the further special case when $A_j = 1 (j = 1, \ldots, N)$ and $B_j = 1$ $(j = M + 1, \ldots, Q)$, $\ell = 0$, $w = 1$, $\mu = 0$, $\gamma = \delta = v = 1$, $n = 0$,

and $\quad \alpha_j = 1 \qquad (j = 1, \ldots, \mathrm{P})$

$\quad \beta_j = 1 \qquad (j = 1, \ldots, \mathrm{Q})$

would yield one of Wille's result (Wille 1988([10], p. 601, equation (29)) on using Legendre's duplication formula for the Γ-function.

Next, we put

$$\gamma = \delta = 1 \qquad \lambda = \mu = -1/2 \qquad \alpha \to \alpha^2 \text{ and } \beta \to \beta^2$$

in the integral formula (11), and sum the resulting series by means of a known formula (Erdélyi *et al.* 1953 ([1], p. 101, eqn. 2.8(6)): applying Legendre's duplication formula as well, we thus obtain the integral

$$\int_{\ell}^{w} (t-\ell)^{-1/2}(w-t)^{-1/2}\,\bar{H}^{M,N}_{P,Q}\left[\left\{\frac{(t-\ell)(w-t)}{\alpha^2(t-\ell)+\beta^2(w-t)}\right\}^{v}\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\ \\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right]$$

$$\times S^{m}_{n}\left[y\left\{\frac{(t-\ell)(w-t)(w-\ell)^{-1}}{\alpha(t-\ell)+\beta(w-t)}\right\}^{\rho'}\right]dt$$

$$= \sqrt{\pi}\sum_{h=0}^{[n/m]}\frac{(-n)_{mh}}{h!}A_{n,h}[y/\beta^{\rho'}(\alpha+\beta)^{2\rho'}]^{h}$$

$$\times \bar{H}^{M,N+1}_{P+1,Q+1}\left[\left\{\frac{w-\ell}{(\alpha+\beta)^2}\right\}^{v}\,\middle|\,\begin{matrix}(a_j,\alpha_j;A_j),(\tfrac{1}{2}-\rho'h,v;1),(a_j,\alpha_j)_{N+1,P}\\ \\(b_j,\beta_j)_{1,M},(-\rho'h,v;1),(b_j,\beta_j)_{M+1,Q}\end{matrix}\right] \quad (14)$$

which in the further special case when

$A_j = 1 (j = 1, \ldots, N)$ and $B_j = 1 (j = M + 1, \ldots, Q)$,

$\ell = 0$, $w = 1$, $v = 1$, $n = 0$,

and $\quad \alpha_j = 1 \qquad (j = 1, \ldots, P)$

$$\beta_j = 1 \qquad\qquad (j = 1, \ldots, Q)$$

immediately yields anotheer result of Wille (1988, [10], p. 601, eqn. (22)).

If in our integral formula (11) we set $\gamma = \delta = 1/2$, $\mu = -\lambda - 2$ and $v \to 2v$ sum the resulting binomial series, and apply Legendre's duplication formula once again, we shall obtain

$$\int_\ell^w (t-\ell)^\lambda (w-t)^{-\lambda-2} \bar{H}_{P,Q}^{M,N}\left[\frac{\{(t-\ell)(w-t)\}^v}{\{\alpha(t-\ell)+\beta(w-\ell)\}^{2v}}\middle|_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}}^{(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}\right]$$

$$\times S_n^m\left[y\left\{\frac{(t-\ell)^{1/2}(w-t)^{1/2}}{\alpha(t-\ell)+\beta(w-t)}\right\}^{\rho'}\right]dt$$

$$= 2^{\left(\frac{2-\rho' h}{2}\right)}\sqrt{\pi}(w-\ell)^{-1}\sum_{h=0}^{[n/m]}\frac{(-n)_{mh}}{h\,!}A_{n,h}(y/\beta^{\rho'})^h\left(\frac{\beta}{\alpha}\right)^{1+\lambda+\frac{\rho' h}{2}}$$

$$\times \bar{H}_{P+2,Q+2}^{M,N+2}\left[(4\alpha\beta)^{-v}\middle|_{(b_j,\beta_j)_{1,M},(1-\frac{\rho' h}{2},v;1)(\frac{1}{2}-\frac{\rho' h}{2},v;1),(b_j,\beta_j)_{M+1,Q}}^{(a_j,\alpha_j;A_j)_{1,N},(-\lambda-\frac{\rho' h}{2},v;1),(\lambda+2-\frac{\rho' h}{2},v;1),(a_j,\alpha_j)_{N+1,P}}\right], \tag{15}$$

which hold true under the conditions readily obtainable from those stated with (11).

2. By applying our integral in (11) to the case of Hermite polynomials ([9], Eq. (5.5.4), p. 106 and [8], p. 153) by $m = 2$, $A_{n,h} = (-1)^h$ and $S_n^2[x] \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$, we obtain

$$\int_\ell^w \frac{(t-\ell)^\lambda (w-t)^\mu}{\{f(t)\}^{\lambda+\mu+2}} \bar{H}_{P,Q}^{M,N}\left[z\{g(t)\}^v\middle|_{(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}}^{(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}}\right]\left[y\{g(t)\}^{\rho'}\right]^{n/2} H_n\left[\frac{1}{2\sqrt{y\,\{g(t)\}^{\rho'}}}\right]dt$$

$$= \sum_{h=0}^{[n/2]}\frac{(-n)_{2h}}{h!}(-y/\beta^{\rho'})^h\sum_{r=0}^{\infty}\frac{\{(\beta-\alpha)/\beta(1+\rho)\}^r}{r!}(w-\ell)^{-1}(1+\rho)^{-\lambda-\gamma\rho' h-1}(1+\sigma)^{-\mu-\delta\rho' h-1}$$

$$\times \bar{H}_{P+3,Q+2}^{M,N+3}\left[z\left\{\beta(1+\rho)^\gamma(1+\sigma)^\delta\right\}^{-v}\middle|_{(b_j,\beta_j)_{1,M},(1-\rho' h,v;1)(-\lambda-\mu-r-\gamma\rho' h-\delta\rho' h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q}}^{(a_j,\alpha_j;A_j)_{1,N},(1-r-\rho' h,v;1)(-\lambda-r-\gamma\rho' h,\gamma v;1),(-\mu-\delta\rho' h,\delta v;1),(a_j,\alpha_j)_{N+1,P}}\right],$$

which holds true under the same conditions as those required for (11).

3. For Laguerre polynomials ([9], Eqn. (5.16), p. 101 and [8], p. 158) by $m = 1$, in (11), $A_{n,h} = \binom{n+\alpha'}{n} \frac{1}{(\alpha'+1)_h}$ and $S_n[x] \to L_n^{(\alpha')}(x)$ we obtain

$$\int_\ell^w \frac{(t-\ell)^\lambda (w-t)^\mu}{\{f(t)\}^{\lambda+\mu+2}} \bar{H}_{P,Q}^{M,N}\left[z\{g(t)\}^v \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right] L_n^{(\alpha')}[y\{g(t)\}^{\rho'}]dt$$

$$= \sum_{h=0}^{n} \binom{n+\alpha'}{n-h} \frac{(-y)^h}{h!} \sum_{r=0}^{\infty} \frac{\{(\beta-\alpha)/\beta(1+\rho)\}^r}{r!} (w-\ell)^{-1}(1+\rho)^{-\lambda-\gamma\rho'h-1}(1+\sigma)^{-\mu-\delta\rho'h-1}$$

$$\times \bar{H}_{P+3,Q+2}^{M,N+3}\left[z\left\{\beta(1+\rho)^\gamma(1+\sigma)^\delta\right\}^{-v} \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(1-r-\rho'h,v;1)(-\lambda-r-\gamma\rho'h,\gamma v;1),(-\mu-\delta\rho'h,\delta v;1),(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(1-\rho'h,v;1)(-\lambda-\mu-r-\gamma\rho'h-\delta\rho'h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right],$$

which holds true under the same conditions as those required for (11).

4. For Jacobi polynomials ([9], Eqn. (4.3.2), p. 68) by $m = 1$,

$A_{n,h} = \binom{n+\alpha'}{n} \frac{(\alpha'+\beta'+n+1)_h}{(\alpha'+1)_h}$ and $S_n[x] \to P_n^{(\alpha',\beta')}(1-2x)$ in (11), we obtain

$$\int_\ell^w \frac{(t-\ell)^\lambda (w-t)^\mu}{\{f(t)\}^{\lambda+\mu+2}} \bar{H}_{P,Q}^{M,N}\left[z\,\{g(t)\}^v \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right] \times P_n^{(\alpha',\beta')}\left[1-2y\{g(t)\}^{\rho'}\right]dt$$

$$= \sum_{h=0}^{n} \binom{n+\alpha'}{n-h} (-y)^h \begin{bmatrix} \alpha'+\beta'+n+h \\ h \end{bmatrix} \sum_{r=0}^{\infty} \frac{\{(\beta-\alpha)/\beta(1+\rho)\}^r}{r!} (w-\ell)^{-1}(1+\rho)^{-\lambda-\gamma\rho'h-1}(1+\sigma)^{-\mu-\delta\rho'h-1}$$

$$\times \bar{H}_{P+3,Q+2}^{M,N+3}\left[z\left\{\beta(1+\rho)^\gamma(1+\sigma)^\delta\right\}^{-v} \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(1-r-\rho'h,v;1)(-\lambda-r-\gamma\rho'h,\gamma v;1),\ (-\mu-\delta\rho'h,\delta v;1),(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(1-\rho'h,v;1)(-\lambda-\mu-r-\gamma\rho'h-\delta\rho'h-1,(\gamma+\delta)v;1),(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right]$$

which holds true under the same conditions as those required for (11).

4. SPECIAL CASES

(1) On taking $A_j (j = 1, \ldots, N) = B_j\ (j = M + 1, \ldots, Q) = 1$ and $n = 0$ in (11) the result reduces to a known result derived by Srivastava H.M. and Raina R.K. ([7], p. 693, Eqn. (15)).

(2) Taking $A_j (j = 1, \ldots, N) = B_j (j = M + 1, \ldots, Q) = 1$ and $\alpha_j (j = 1, \ldots, P) = \beta_j (j = 1, \ldots, Q)$ with $n = 0$ our result (15) reduces to another result obtained by Srivastava H.M. and Raina R.K. in ([7], p. 695, Eqn. (20) or (21)).

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