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Synthesis of a distributed control system

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Abstract : This article presents the concept of systems with distributed parameters and investigates the method of distributed controller synthesis and a homogeneous control object. This article analyzes main methods of transferring heat energy. On the basis of the heat-transfer equation, the function of initial heating has been obtained, and the process has been mathematically simulated, and the results obtained have been analyzed. The practical results of this research make it possible to draw a conclusion about the possibility of building a silicon-carbide heating element made in the shape of an isotropic rod.
Keywords : Green's function, thermal field, sampling interval, control object, analysis, synthesis.

1. INTRODUCTION

With the appearance of machines and mechanisms, people addressed the issue of controlling them automatically with minimum human interference. At first these were just technical developments of craftsmen, but over time this process has become a technical science. It was called Automatic Control Theory. Automatic Control Theory (ACT) is the science that studies the processes taking place in objects of any nature, and methods of influencing them. The object is a physical body, in which a process or a phenomenon occurs,which needs to be controlled with the use of a control device. Objects may be of two types: controlled and uncontrolled. The control device sends a command to the process controller, which acts on the object, and makes it reach target values. The object is subjected to disturbance input, which can be internal (noise) and external (load). Automatic control systems (ACS) may be open-circuited (Figure 1) and close-circuited (Figure 2) (Tikhonov, and Samara, 1965).

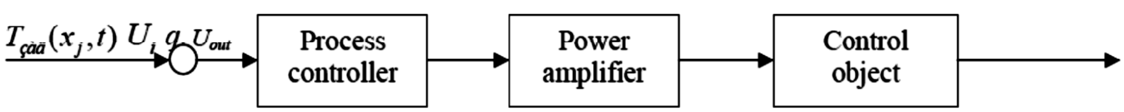

Fig. 1. ACS with open-circuit control system.

An open-circuit system is characterized by the absence of feedback. This system operates strictly in linear mode. This system does not feature dynamic changing of the input data. Traditionally, automatic control systems are classified by the number of other characteristics that cover many areas of science and technology; we will list just a few of them as an example.

By the nature of controlling the object:

- control system;
- regulating system.

By the nature of the action on the object:

- continuous systems (the linear law is acting) of action;
- discrete systems (the differential law is acting).

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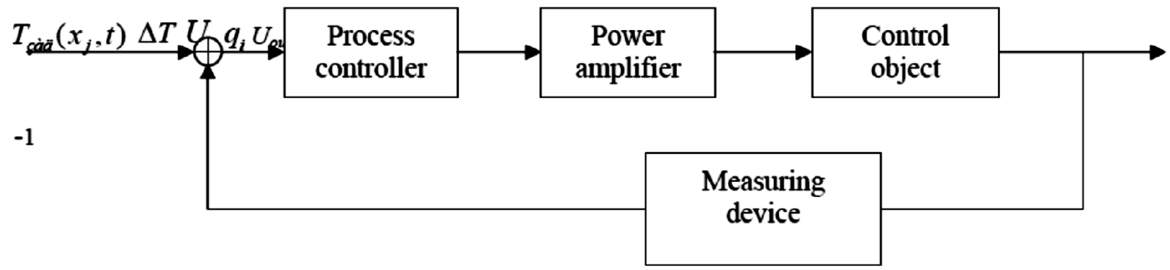

Fig. 2. ACS with a close-circuit control system

By the degree of optimization, self-tuning and adaptation to the conditions of action on the object:

- extreme systems;
- self-adjusting systems;
- intelligent systems.

Besides, the automatic control systems can be classified by the type of mathematical model that describes operation of the control object. However, there is a fundamental classification of automatic control systems. The fundamental classification is made by the type of the mathematical model. Classification of automatic control systems by the type of the mathematical model is shown in Figure 3. Linear automatic control systems are systems where dynamic behavior (the behavior of the system with regard to time or any other variable characteristics) of components is described by linear equations (algebraic, differential, differential). For this behavior, it is necessary that the characteristics (static) of all components of the system are linear, or linearized in case of nonlinearity.

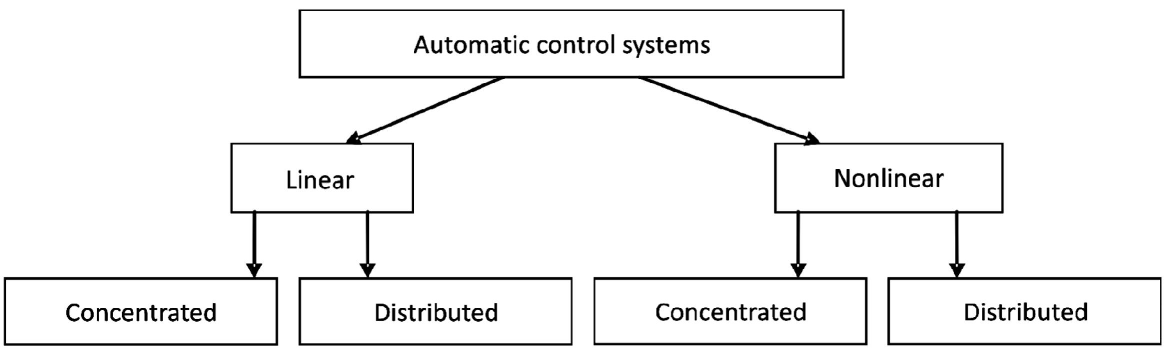

Fig. 3. Classification of automatic control systems by the mathematical model of the object.

Geometrical functions of linear systems will have the form of linear dependencies, which are shown in Fig.4.

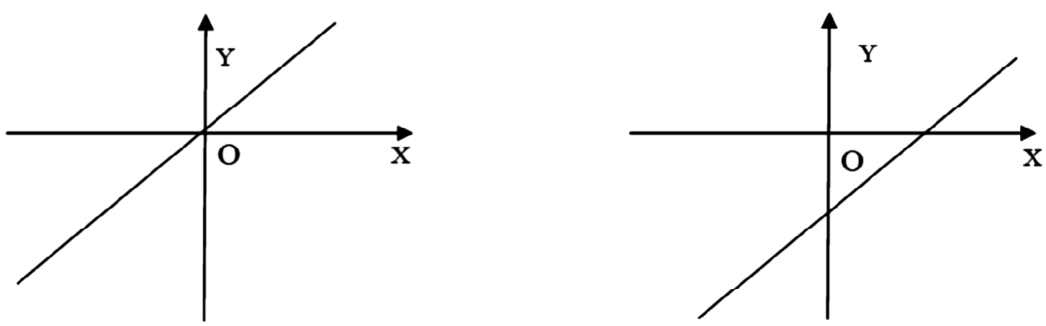

Fig. 4. Linear Automatic Control Systems.

Non-linear automatic control systems are systems where dynamic behavior (the behavior of the system with regard to time or any other variable characteristics) of components is described by non-linear equations (multiplication of variables, derivatives, second or other higher degree, etc.). Geometrically, such an automatic control system will have the form shown in Fig. 5. A concentrated system or a system with concentrated parameters is a system, where spatial parameters do not have span, or they can be neglected within this system. The state of such systems is characterized by a finite set of functions, the argument of which is only the variable denoting the time of this system. The behavior of such a system is described by a finite number of ordinary differential equations.

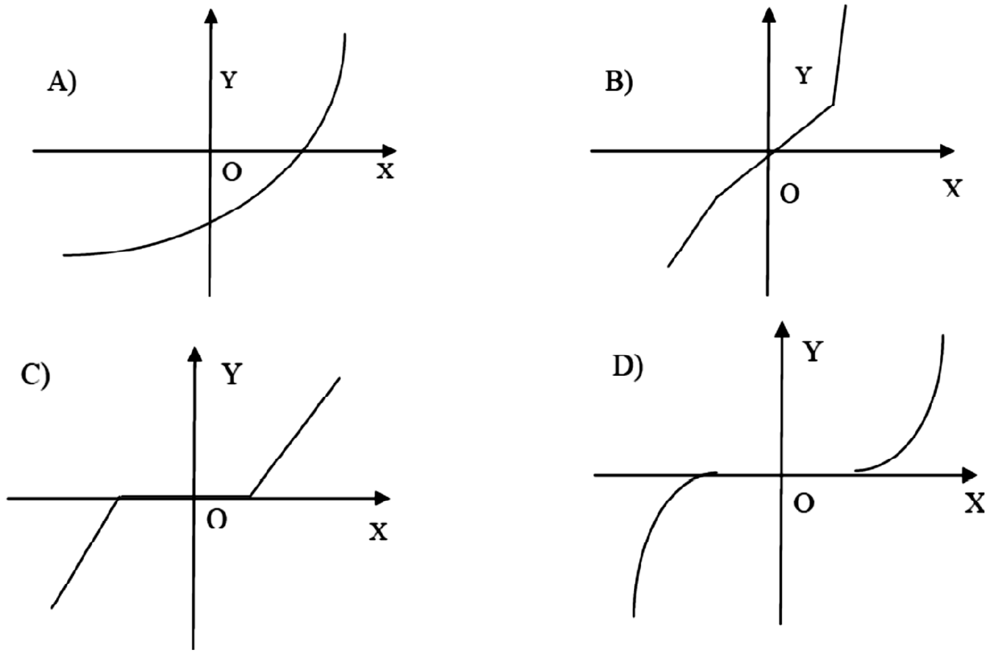

Fig. 5. Non-linear Automatic Control Systems.

The function describing the state of an object with distributed parameters Q(*x, t*) anddefined in certain area $\overline{D}$ by variable $x \in \overline{D}$ will have operator form (Kolesnikov, et al. (2007); Kolesnikov, et al. (2007); Zarembo, and Kolesnikov (2006); Zarembo, and Kolesnikov (2006); Kolesnikov (2009)).

$$L[Q(x, t)] = f(x, t).$$

$$x \in \overline{D},\ t > 0.$$

where D is the open part of area $\overline{D}$; L is the defined linear integral-differential operator, the specific form of which is determined by the content of the described process; $f(x, t)$ is the function that characterizes external influence on the process.

In practical problems, control objects with distributed parameters are described using differential equations, which are essentially a mathematical form of presenting the fundamental law of perdurability of matter in the elementary volume. In this case, variable L can be viewed as a differential operator. For obtaining unique solution of the equation, the initial and boundary conditions are to be specified. These describe the laws or interaction functions at certain points, namely, at the ends of the control object. In general case, the initial conditions can be described by some linear operator N (Zarembo, et al. (2004); Rapoport, and Pleshivtseva (2010); Pleshivtseva, and Rapoport (2009); Rapoport (2006); Rapoport, and Pleshivtseva (2006); Rapoport (1996);Ilyushin, et al. (2014); Ilyushin (2011)).

$$N[Q(x, t)] = Q_0(x),\ x \in \overline{D},\ t = 0$$

The boundary conditions characterizing interaction of Q(*x*, *t*) with external environment can be written in the following form:

$$\tilde{A}[Q(x, t)] = g(x, t),\ x \in \partial\overline{D},\ t > 0,$$

where G is the corresponding linear operator; $g(x, t)$ is external influence; and $\partial\overline{D}$ is the border of the area $\overline{D}$.

Equationswith given linear differential operators L, N, G (constituting the boundary problem) can be considered as the base model. This model is intended for mathematical description of objects with distributed parameters with controllable output status function Q(*x*, *t*) and external inputs $f(x, t)$ and $g(x, t)$. In most cases, controllable state functions of objects with distributed parameters represent spatially-temporal characteristics of fields of different physical nature. In many cases, they can be described by linear differential equations from mathematical physics.

As an example, let us consider the mathematical model of the heat conduction process in a finite size plate (Chernishev, 2009).

$$\frac{\partial T}{\partial \tau} = a^2\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

$$0 < x < L_x ;\ 0 < y < L_y ;\ 0 < z < L_z ;$$

Boundary and initial conditions

$$T(x, y, L_z, \tau) = U(x, y, \tau) ;$$

$$\frac{\partial T(x, y, 0, \tau)}{\partial z} = 0 ;$$

$$T(x, 0, z, \tau) = T(x, L_y, z, \tau) = T(0, y, z, \tau) = T(L_x, y, z, \tau) = 0 ;$$

$$T(x, y, z, 0) = 0;$$

where $T(x, y, z, t)$ is the temperature field of the plate; L_x, L_y, L_z are given numbers; τ is time; a^2 is the temperature conductivity coefficient of the plate material; $U(x, y, t)$ is the control action.

For the simplest case of spatial distribution $Q(x, t)$ along one coordinate x that varies in the interval $[x_0, x_1]$, a general linear differential equation of the second order can be written as follows:

$$L[Q(x, t)] = A(x,t)\frac{\partial^2 Q}{\partial t^2}+2B(x,t)\frac{\partial^2 Q}{\partial x \partial t}+C(x,t)\frac{\partial^2 Q}{\partial x^2}+A_1(x,t)\frac{\partial Q}{\partial t}+B_1(x,t)\frac{\partial Q}{\partial x}+$$

$$+ C_1(x, t)Q = f(x, t)$$

$$A(x,t)\frac{\partial^2 Q}{\partial t^2}+A_1(x,t)\frac{\partial Q}{\partial t} = C(x,t)\frac{\partial^2 Q}{\partial x^2}+B_1(x,t)\frac{\partial Q}{\partial x}+C_1(x,t)Q+f(x,t,u(x,t))\, x_0 < x < x_1 ;\ t > 0 ;$$

with initial and boundary conditions :

$$Q(x, 0) = Q_0^{(0)}(x) ;\ \frac{\partial Q(x,0)}{\partial t} = Q_0^{(1)}(x) ;\ x_0 \le x \le x_1$$

$$\alpha(x_0,t)Q(x_0,t)+\beta(x_0,t)\frac{\partial Q(x_0,t)}{\partial x} = g_0(t,u_0(t)),\ t > 0 ;$$

$$\alpha(x_1,t)Q(x_1,t)+\beta(x_1,t)\frac{\partial Q(x_1,t)}{\partial x} = g_1(t,u_1(t)),\ t > 0.$$

The presented spatially distributed model of the control object, unlike concentrated systems, has the spatial coordinate. A distributed system is a system where dynamic behavior of segments is described by differential equations in deferential derivatives.

2. THE MAIN METHODS OF HEAT ENERGY TRANSFER. TRANSFER METHODS

At the present stage of development of informatization technological processes and control systems, more and more attention is paid to thermal processes in various environments and control systems. All of them are based on the ideal model of an infinite mono-crystal as the generally accepted standard of a solid body. According to the classical understanding of interaction between thermal processes, all particles of a body are interconnected into a lattice. If these forces are considered from the point of view of a mathematical model, the distance between the molecules, the oscillations around the equilibrium positions in the crystal lattice influences strength of interaction between the molecules (Chernyshev, 2010; Chernyshev, et al., 2010; Chernyshev, 2008).

Theory of thermal conductivity in solids is developed on the basis of an infinite crystal model. Let us analytically resolve the problem of heat conduction by generating a differential equation of heat conductivity. Let us study a system consisting of a one-dimensional temperature field, where heat is propagated in one direction along one of the axes,where the temperature coefficients shall be considered independent (from coordinates and time).

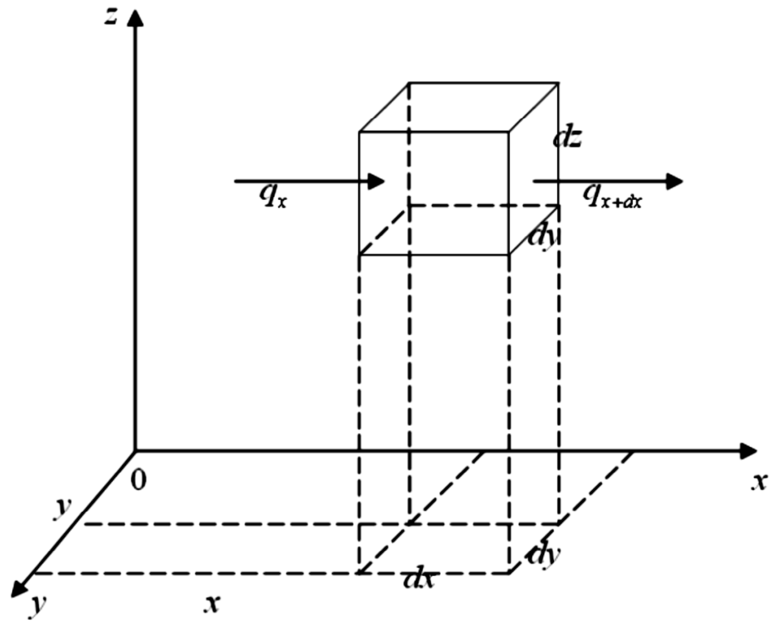

Fig. 6. Heat flow through an object that occupies elementary volume.

An isotropic plate can serve as such an object. Let us take its coordinates d_x, d_y, d_z as spatial dimensions of the plate. Let us denote the amount of heat via the spatial variable $q_x\, d_y\, d_z$. The left side $d_y\, d_z$ will be the point of heat source application. The boundary condition for the heat dissipation will be expressed via variables $q_x +_{dx}$ *dy dz*.

Therefore, in the event of situation $q_x > q_{x+dx}$ the plate will be heated. From this it follows that the difference between the flows is [16]

$$q_x dydz - q_{x+dx} dydz = c\gamma \frac{\partial \mathrm{T}}{\partial \tau} dxdydz$$

From the presented equality it can be seen that value q_{x+dx} is the necessary function of x. In order to obtain the solution of the heat transfer equation, this function should be expanded in a Taylor expansion, after which it is necessary to set boundary conditions.

$$q_{x+dx} \approx q_x + \frac{\partial q_x}{\partial x} dx$$

Then:

$$-\frac{\partial q_x}{\partial x} dxdydz = c\gamma \frac{\partial T}{\partial \tau} dxdydz\,;$$

According to

$$q_x = -\lambda \frac{\partial T}{\partial x}$$

$$c\gamma \frac{\partial \mathrm{T}}{\partial \tau} = \lambda \frac{\partial^2 \mathrm{T}}{\partial x^2}$$

or

$$\frac{\partial \mathrm{T}}{\partial \tau} = a \frac{\partial^2 \mathrm{T}}{\partial x^2}\,.$$

The resulting equation is called a differential equation, and makes it possible to simulate the behavior of the temperature field in all isotropic objects. For the purpose of modeling, it is necessary to know the coordinates of the object and its thermal conductivity. In considering the objects with large dimensions, distributed objects with three-dimensional coordinates in particular, the equation is transformed into the following form (Tikhonov and Samara 1965; Chernyshev, 2010; Chernyshev, 2008):

$$\frac{\partial T}{\partial \tau} = a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) = a\nabla^2 T$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is the Laplace operator.

If we consider the cases where the heat source is inside the body, for example, evaporation, and if $\omega(\mathrm{W}/m^3)$ is the specific power rating,then $\omega dxdydz$ is the amount of heat released per unit of time. Then, for such a task, a mathematical model of heat propagation in the object will take the form of the following differential equation:

$$\frac{\partial T}{\partial \tau} = a\nabla^2 T + \frac{\omega}{c\gamma}$$

Considering this system, one can see that if the Ostrogradsky-Gauss formula is applied to the given equation, there is a possibility to obtain the general differential equation of heat conductivity.

Let us consider an object with arbitrary volume V, and some limited by surface S. The temperature field propagation in this area shall occur according to the thermal conductivity equation. Then the amount of heat passing through S in a unit of time will take the form of an integral along surface S

$$\int_{(S)} \lambda grad\ T \cdot dS = \int_{(S)} \lambda \cdot 1_n \cdot grad\ T \cdot dS$$

In the absence of heat sources, the integral will be taken by volume V:

$$\frac{\partial}{\partial \tau}\int_{(V)} c\gamma T dv = \int_{(V)} c\gamma \frac{\partial T}{\partial \tau} dv$$

For this case, according to the energy conservation principle, the difference in heat loss and changes in the internal energy will be equal to the following equation:

$$\int_{(V)} c\gamma \frac{\partial T}{\partial \tau} dv = \int_{(S)} 1_n \lambda \cdot grad\ T \cdot dS$$

Let us use the Ostrogradsky-Gauss transformation

$$\int_{(S)} 1_n \lambda \cdot grad\ T \cdot dS = \int_{(V)} div(\lambda\ grad\ T)dv$$

$$\int_{(V)} c\gamma \frac{\partial T}{\partial \tau} dv = \int_{(V)} div(\lambda grad\ T)dv$$

$$c\gamma \frac{\partial T}{\partial \tau} = div(\lambda grad\ T)$$

$$\frac{\partial T}{\partial \tau} = a\nabla^2 T = a div(grad\ T)$$

The last equation is called a differential heat conductivity equation. The equations obtained describe the law of heat propagation in the object.

Now let us consider heat transfer methods. There are three ways of heat transfer between the source and the body - these are heat conductivity, radiation and convection. Let us take a look at each of them.

The effect of heat conductivity only occurs as a result of physical contact between the interacting objects, so this method of heat transfer is acceptable for stationary liquids and solids only. Based on that, physical contact makes it possible for the kinetic energy of the molecules of the warmer substance to migrate to the molecules of the colder body.

This process is possible due to physical collision of molecules with different kinetic energies. Upon collision of molecules, vibrations in warmer body become less intense, and fluctuations of molecules in the colder substance increase.

If we consider a mathematical model of heat propagation in a three-dimensional object, the model will be described by the differential equation of the parabolic type

$$\frac{\partial U}{\partial t} = v \cdot \frac{\partial^2 U}{\partial x^2} - a \cdot \frac{\partial U}{\partial x} + b \cdot U + f(t,x)$$

where t is the temperature at the point with coordinates x, y, z at moment τ; and a is thermal conductivity.

Radiation (irradiation) is a form of heat exchange between two objects where the bodies are relatively distant from each other. That is, there is no direct contact between the bodies. In this case, heat exchange will occur through electromagnetic oscillations (ultraviolet, infrared and light rays).

Heat transfer through radiation occurs between bodies with different temperatures, when they are in direct view of each other. By its nature, this type of heat exchange does not require physical contact, moving flow of fluid, exchange of gases, or any other medium to transfer energy.

The term "radiation" is explained by the fact that energy is transferred from body to body in the form of electromagnetic waves. Quantity of (absorbed, reflected, transmitted) radiation energy depends on the nature of body, structure, surface, and color. Light materials with polished surfaces, on the basis of laws of waves reflection and refraction, reflect radiation better. Materials with rough dark surface absorb the maximum amount of radiation energy (in the laboratory conditions, black sphere is used for measuring the level of heat transfer by radiation).

Radiation does not penetrate through non-transparent materials (wood, metal). Some waves are reflected, and the rest of them are absorbed. In case of increasing temperature (by types of radiation), electromagnetic waves (in the infrared part of the spectrum) are generated that are then transferred to colder bodies. Radiant integral flux (emitted per unit of body surface) is called the integrated density of radiation

$$E = \frac{dQ}{dF}, \ W/m^2;$$

where dQ is the radiant flux, W, emitted from the element surface dF, m^2. The radiant flux over the entire surface can be expressed as

$$Q = \int_F EdF, \ W.$$

where F is the entire body surface, m^2, A is the absorption coefficient, R is the reflection coefficient, and D is the coefficient of permeability.

The third method of heat transfer between the source and the body is **convection**. Same as heat conductivity, it occurs only in moving fluids. In this case, the thermal energy is transferred from one location to another by currents (formed in the liquid). Convection currents are formed naturally when the fluid is heated. They can be caused by mechanical action (pump, screw or vane). It is known that convection currents are the result of changing density of the liquid. This effect is associated with expansion of its heated part (the liquid is heated, it expands and its volume increases). Then, the heated portion of the fluid becomes more lightweight, and starts rising to the top, which in turn initiates descent of the colder fluid. In the end, the fluid circulates until the entire fluid has equal temperature. Convection, like thermal conductivity, depends on the initial speed of the molecules, density, etc.

Based on the Newton's law, the ratio that describes convection will have the following form

$$q = hA\,(T_W + T_0),$$

where the value of the heat flow q is measured in watts, the area of body A to which the convection is applied will be measured in square meters (m^2), and the temperature of the appropriate environment T_0 and source T_W – in Kelvin. The convective heat transfer coefficient h depends on the environment, the initial speed of the molecules, and the shape of the heat source, and is measured in units of W/($m^2 \times$ K).

2.1. Determination of the temperature field parameters on the basis of the Green's function

The temperature field is the value of temperature distribution at a certain moment in the considered space (material system). A varying temperature field is called non-stationary (transient). An unchanging temperature field is called stationary (steady-state).

Let us consider the first boundary-value problem given by the following equations (Kolesnikov, et al. (2007); Zarembo, and Kolesnikov (2006); Zarembo, et al. (2006); Zàrembo, et al. (2004); Rapoport, et al. (2010); Pleshivtseva, et al. (2009); Rapoport (2006); Rapoport, and Pleshivtseva (2006); Rapoport (1996); 1Ilyushin, et al. (2014); Chernishev (2009); Chernishev (2009); Chernishev (2010); Chernishev, et al. (2010); Chernishev (2008)):

$$\begin{cases} u_t = a^2 u_{xx} + f(x, t), \ 0 < x < l, \ 0 < t \le \mathrm{T}, \\ u(0, t) = \mu_1(t), \ 0 \le t \le \mathrm{T}, \\ u(l, t) = \mu_2(t), \ 0 \le t \le \mathrm{T}, \\ u(x, 0) = \varphi(x), \ 0 \le x \le l. \end{cases} \tag{1}$$

Let us consider the following issues: existence and uniqueness of the solution, stability, use of Green's function. In this case many solutions may be observed, such as:

$$\begin{aligned} &\tilde{u}(x, t) = const, \ (x, t) \in \mathrm{Q}_T = \{(x, t) : (0, l) \times (0, \mathrm{T}]\}, \\ &\tilde{u}(0, t) = \mu_1(t), \ 0 \le t \le \mathrm{T}, \\ &\tilde{u}(l, t) = \mu_2(t), \ 0 \le t \le \mathrm{T}, \\ &\tilde{u}(x, 0) = \varphi(x), \ 0 \le x \le l. \end{aligned}$$

Suppose the function is continuous. According to the definition of the solution of the first boundary problem, it is known that the solution of the first boundary problem $\tilde{u}(x, t)$ is reduced to three conditions.

Let us find a solution of the equation with zero boundary conditions, taking into account the homogeneous heat conductivity equation:

$$\begin{cases} (1) \ u_t = a^2 u_{xx}, \ \ 0 < x < l, \ 0 < t \le T, \\ (2) \ u(0, t) = 0, \ \ 0 \le t \le T, \\ (3) \ u(l, t) = 0, \ \ 0 \le t \le T, \\ (4) \ u(x, 0) = \varphi(x), \ \ 0 \le x \le l. \end{cases} \tag{2}$$

The solution will be found according to the following algorithmc:

- let us build $\tilde{u}(x, t)$ function;
- let us prove that under certain restrictions this function is the solution of the first boundary problem.

Let us introduce a new function is equal to:

$$\upsilon(xt) = \mathrm{X}(x)\mathrm{T}(t)$$

Let us substitute into the main equation and get:

$$\mathrm{X}(x)\mathrm{T}'(t) = a^2\mathrm{X}''(x)\mathrm{T}(t)$$

Let us divide both parts of the equation:

$$\frac{\mathrm{T}'(t)}{a^2\mathrm{T}(t)} = \frac{\mathrm{X}''(x)}{\mathrm{X}(x)}$$

From the equation it can be seen that in different parts of the equation there are different variables, they are equal to some constant, which we shall denote as λ.

$$\frac{\mathrm{T}'(t)}{a^2\mathrm{T}(t)} = \frac{\mathrm{X}''(x)}{\mathrm{X}(x)} = -\lambda$$

From that it follows : $\quad \mathrm{X}''(x) + \lambda\mathrm{X}(x) = 0 \quad (3)$

$$\mathrm{T}'(t) + a^2\lambda\mathrm{T}(t) = 0 \tag{4}$$

Writing $u(x, t)$ we get:

$$\begin{cases} u(0,t)=0, \\ u(l,t)=0, \text{ with } t\in[0,\mathrm{T}] \end{cases}$$

we'll get the equation of representation in the form of multiplication,

$$\begin{cases} \mathrm{X}(0) = 0, \\ \mathrm{X}(l) = 0, \end{cases}$$

Combining $\mathrm{X}''(x)+\lambda\mathrm{X}(x)=0$ with the obtained system, we will receive the Sturm – Liouville problem:

$$\begin{cases} \mathrm{X}''(x)+\lambda\mathrm{X}(x)=0, \\ \mathrm{X}(l)=0 \\ \mathrm{X}(l)=0 \end{cases}$$

It is necessary to find all λ for which nontrivial solutions of this system exist. From the Differential equations course it is known that:

$$\begin{cases} \lambda_n=\left(\dfrac{\pi n}{l}\right)^2 \\ \mathrm{X}_n(x)=c_n^1\sin\left(\dfrac{\pi n}{l}x\right) \end{cases}$$

$n\in N$ are eigenvalues (with $s_n^{\ 1}$ – some constants).

Substituting λ_n in (4), we get:

$$\mathrm{T}'(t) + a^2\lambda_n T_n(t) = 0$$

$$\mathrm{T}_n = c_n^2\exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\}$$

Combining $\mathrm{X}_n(x)$ and $\mathrm{T}_n(t)$, we obtain:

$$\upsilon(x.t) = \mathrm{T}_n(t)\mathrm{X}_n(x)=c_n\sin\left(\frac{\pi n}{l}x\right)\exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\}$$

According to the boundary conditions 2 and 3:

$$\upsilon(x,t) = \sum_{n=1}^{\infty}\upsilon_n(x.t)$$

It should be noted that this equation satisfies the boundary conditions, and in case of uniform convergence of a number of derivatives - to the equation of heat conductivity as well. Let us choose the constants to satisfy the initial condition:

$$\varphi = \upsilon(x,0)=\sum_{n=1}^{\infty}\upsilon_n(x,t)=\sum_{n=1}^{\infty}c_n\sin\left(\frac{n\pi}{l}x\right)$$

Let us multiply the equality by $\sin\left(\dfrac{n\pi}{l}x\right)$ and replace the variable x with s, and index the resulting equation by s. Let us also obtain the final formula that describes behavior of the thermal field with specified boundary conditions. Let us define this function with x coordinate at certain moment t. Let us define the resulting dependence by formula $\upsilon(x, t)$.

$$\int_0^l \varphi(s)\sin\left(\frac{m\pi}{l}s\right)ds = \sum_{n=1}^{\infty} c_n \int_0^l \sin\left(\frac{m\pi}{l}s\right)\sin\left(\frac{n\pi}{l}s\right)ds$$

$$\int_0^l \sin\left(\frac{n\pi}{l}x\right)\sin\left(\frac{m\pi}{l}x\right)dx = \begin{cases} 0, n \neq m \\ \frac{l}{2}, n = m \end{cases} \Rightarrow \int_0^l \varphi(s)\sin\left(\frac{m\pi}{l}s\right)ds = \frac{l}{2}c_m \Rightarrow$$

$$\tilde{n}_m = \frac{2}{l}\int_0^l \varphi(s)\sin\left(\frac{m\pi}{l}s\right)ds$$

$$\upsilon(x,\,t) = \sum_{n=1}^{\infty}\frac{2}{l}\left(\int_0^l \varphi(s)\sin\left(\frac{n\pi}{l}s\right)ds\right)\sin\left(\frac{\pi n}{l}x\right)\exp\{-a^2\left(\frac{\pi n}{l}\right)^2 t\}\ 1.5$$

The resulting equation is the solution of the first boundary problem. Let us consider a few of its properties. To do so, let us consider the first boundary problem.

$$\begin{cases} u_t = a^2 u_{xx}, \quad 0 < x < l,\ 0 < t \leq T, \\ u(0,\, t) = 0, \quad 0 \leq t \leq T, \\ u(l,\, t) = 0, \quad 0 \leq t \leq T, \\ u(x,\, 0) = \varphi\ (x), \quad 0 \leq x \leq l. \end{cases}$$

$$\upsilon(x,\,t) = \int_0^l \mathrm{G}(x,a,t)\varphi(a)ds$$

Where $$\mathrm{G}(x,\,a,\,t) = \sum_{n=1}^{\infty}\frac{2}{l}\sin\left(\frac{\pi n}{l}s\right)\sin\left(\frac{\pi n}{l}s\right)\exp\left\{-a^2\left(\frac{\pi n}{l}\right)^2 t\right\}$$

– is Green's function for the first boundary problem.

$$\left[{}^0\mathrm{C} = \sum_{n=1}^{\infty} {}^0C/\text{м} * \frac{2}{\text{м}} * e^{\frac{\text{м}^2/c}{\text{м}^2} * c} \right]$$

The Green's function has the following properties :

1. $\mathrm{G}(x,a,t) = \mathrm{G}(a,x,t)$
2. $\mathrm{G}(x,a,t) \in \mathrm{C}^{\infty}(\mathrm{R}\times\mathrm{R}\times\mathrm{R}^{+})$

All these properties reflect behavior of the arbitrary shape temperature field in the object. Also, the kind of the Green's function differs from the type of boundary conditions, as well. For considering the function for different boundary conditions, let us represent it in the following form:

$$\mathrm{G}(x,\,y,\,\varepsilon,\,\eta) = p(\varepsilon)\sum_{n=1}^{\infty}\frac{u_n(x)u_n(\varepsilon)}{\|u_n\|^2}\Psi_n(y,n;\lambda_n)$$

Where $$p(x) = \frac{1}{a(x)}\exp\left[\int\frac{d(x)}{a(x)}dx\right]$$

$$\|u_n\|^2 = \int_{x_1}^{x_2} p(x)u_n^2(x)dx$$

In this case, λ_n, u_n are the values and functions of a homogeneous boundary problem represented by the following form:

$$a(x)\cdot u''_{xx} + b(x)\cdot u'_x + [\lambda + c(x)]u = 0$$
$$s_1 \cdot u'_x - k_1 \cdot u = 0, x = x_1$$
$$s_2 \cdot u'_x - k_2 \cdot u = 0, x = x_2$$

In the case of the description of this boundary value problem by differential equations, it is possible to consider the various Green's functions for various boundary conditions. The explicit form of the Green's function for various boundary conditions will be manifested in changing function $Ø_n$. Table 1 presents a row of function $Ø_n$ values for different boundary conditions by variable y. It should be noted that this is just one of the varieties of the Green's function. The choice of Green's function depends on the boundary conditions of the object in question. The function there has other names, in some literature it is called "pulse transition function" or "fundamental solution of the heat conductivity boundary problem".

Table 1. Functions Ψ_n for different boundary conditions of Green's function.

Area	*Boundary conditions*	*Function* $\Psi_n(y,n;\lambda_n)$
$-\infty < y < \infty$	$\lvert\omega\rvert < \infty, y \to \pm\infty$	$\frac{1}{2\beta_n} e^{-\beta_n \lvert y-\eta \rvert}$
$0 \le y < \infty$	$\omega = 0, y = 0$	$\frac{1}{\beta_n}\begin{cases} e^{-\beta_n y} sh(\beta_n \eta), y > \eta \\ e^{-\beta_n \eta} sh(\beta_n y), y < \eta \end{cases}$
$0 \le y < \infty$	$\partial_y \omega = 0, y = 0$	$\frac{1}{\beta_n}\begin{cases} e^{-\beta_n y} ch(\beta_n \eta), y > \eta \\ e^{-\beta_n \eta} ch(\beta_n y), y < \eta \end{cases}$
$0 \le y < \infty$	$\partial_y \omega - k_3\omega = 0, y = 0$	$\frac{1}{\beta_n(\beta_n + k_3)}\begin{cases} e^{-\beta_n y}[\beta_n ch(\beta_n \eta) + k_3 sh(\beta_n \eta)], y > \eta \\ e^{-\beta_n \eta}[\beta_n ch(\beta_n y) + k_3 sh(\beta_n y)], y < \eta \end{cases}$
$0 \le y \le h$	$\omega = 0, y = 0$ $\omega = 0, y = h$	$\frac{1}{\beta_n sh(\beta_n h)}\begin{cases} [sh(\beta_n \eta) sh(\beta_n (h-y)], y > \eta \\ [sh(\beta_n y) sh(\beta_n (h-\eta)], y < \eta \end{cases}$
$0 \le y \le h$	$\partial_y \omega = 0, y = 0$ $\partial_y \omega = 0, y = h$	$\frac{1}{\beta_n sh(\beta_n h)}\begin{cases} [ch(\beta_n \eta) ch(\beta_n (h-y)], y > \eta \\ [ch(\beta_n y) ch(\beta_n (h-\eta)], y < \eta \end{cases}$
$0 \le y \le h$	$\omega = 0, y = 0$ $\partial_y \omega = 0, y = h$	$\frac{1}{\beta_n sh(\beta_n h)}\begin{cases} [sh(\beta_n \eta) ch(\beta_n (h-y)], y > \eta \\ [sh(\beta_n y) ch(\beta_n (h-\eta)], y < \eta \end{cases}$

2.2. Methods of linearizing nonlinear distributed control systems

In practice, all systems in most cases are non-linear. Analysis of such systems is considerably complex. There is a wide range of nonlinear systems, which can be linearized under certain assumptions, *i.e.* made linear in the mathematical sense.

To do so, it is necessary to consider sufficiently small deviations of the object state function and its derivatives from some stationary mode. With smooth functional dependencies that describe nonlinear effects, linearization technique may be applied during simulation of the object. This technique consists in decomposing nonlinear dependencies close to the stationary regime into a Taylor series. With further regard to its linear members only.

The result is a linear equation that in first approximation models the nonlinear behavior of the control object with satisfactory accuracy.

After that, all ways of describing linear objects with distributed parameters may be used in regard to the linearized model.

One-dimensional differential equation of second order with nonlinear operator can be written as

$$\mathrm{L}\left(x,t,\mathrm{Q}(x,t),\frac{\partial^2 \mathrm{Q}}{\partial t^2},\frac{\partial Q}{\partial t},\frac{\partial^2 Q}{\partial x^2},\frac{\partial Q}{\partial x},u(x,t)\right)=0;\quad x_0<x<x_1\ ;\ t>0$$

where $Q(x,t)=Q^*(x,t)+\Delta Q(x,t)-$ is the function of the object state; $Q^*(x,t)$ is some distribution corresponding to external input effects $u^*(x,t)$, satisfying the equation; and $\Delta Q(x,t)-$ is the relatively small variation relating to $Q^*(x,t)$.

If function L is at least twice differentiable in all of its arguments, the result of equation linearization will be for $\Delta Q(x,t)$:

$$\left(\frac{\partial L}{\partial Q}\right)\Delta Q(x,t)+\left(\frac{\partial L}{\partial \ddot{Q}}\right)\frac{\partial^2\Delta Q(x,t)}{\partial t^2}+\left(\frac{\partial L}{\partial \dot{Q}}\right)\frac{\partial\Delta Q(x,t)}{\partial t}+\left(\frac{\partial L}{\partial Q''}\right)\frac{\partial^2\Delta Q(x,t)}{\partial x^2}+$$

$$+\left(\frac{\partial L}{\partial Q'}\right)\frac{\partial\Delta Q(x,t)}{\partial x}+\left(\frac{\partial L}{\partial u}\right)\Delta u(x,t)=0\,;$$

where $$\ddot{Q}=\frac{\partial^2 Q}{\partial t^2},\ \dot{Q}=\frac{\partial Q}{\partial t},\ Q''=\frac{\partial^2 Q}{\partial x^2},\ Q'=\frac{\partial Q}{\partial x},\quad \Delta u=u-u^*.$$

Corresponding derivatives at Q = Q* take the role of the coefficients A, B, A_1, B_1, C_1.

Let us consider a typical linear second order equation which models the behavior of state function Q(x, t). For spatially one-dimensional object with distributed parameters it can be written in the form:

$$A\frac{\partial^2 Q}{\partial t^2}+A_1\frac{\partial Q}{\partial t}=C\frac{\partial^2 Q}{\partial x^2}+B_1\frac{\partial Q}{\partial x}+C_1Q+f(x,t,u(x,t));x_0<x<x_1;t>0$$

with initial and boundary conditions:

$$Q(x,0)=Q_0^{(0)}(x)\ ;\quad \frac{\partial Q(x,0)}{\partial t}=Q_0^{(1)}(x)\ ;x_0\le x\le x_1;$$

$$\alpha(x_0,t)Q(x_0,t)+\beta(x_0,t)\frac{\partial Q(x_0,t)}{\partial x}=g_0(t,u_0(t)),\ t>0;$$

$$\alpha(x_1,t)Q(x_1,t)+\beta(x_1,t)\frac{\partial Q(x_1,t)}{\partial x}=g_1(t,u_1(t)),\ t>0.$$

Each input function f,g_0,g_1 can have a corresponding control function $u(x,t)$, $u_0(t)$, $u_1(t)$.

The boundary controls $u_0(t)$, $u_1(t)$ for a one-dimensional object are influences that vary with time t.

Controls $u(x,\ t)$ can be both independent from the spatial coordinates of control (focused), and spatial-temporal influences.

The basic ration that connects the output of the object at a given initial state with the input influences can be represented in the following integral form:

$$Q(x,t)=\int_{x_0}^{x_1}N_0(x,\xi,t)Q_0^{(0)}(\xi)d\xi+\int_{x_0}^{x_1}N_1(x,\xi,t)Q_0^{(1)}(\xi)d\xi+$$

$$+\int_0^t\int_{x_0}^{x_1}G(x,\xi,t,\tau)f(\xi,\tau,u(\xi,\tau))d\xi d\tau+\int_0^t K_0(x,t,\tau)g_0(\tau,u_0(\tau))d\tau+$$

$$+\int_0^t K_1(x,t,\tau)g_1(\tau,u_1(\tau))d\tau,\quad x\in(x_0,x_1),t>0$$

where ξ and τ are variables of integration, by spatial coordinate and time, respectively.

The first two integrals in the spatial variable describe the influence on Q(x, t), initial distributions $Q_0^{(0)}(x)$ and $Q_0^{(1)}(x)$. The last two integrals in time take into account focused input influences g_0 and g_1 by boundary conditions.

The double integral over the spatial-temporal domain of changes in the spatial and temporal arguments of a distributed input influence f reflects its contribution to object response. The output of object Q(x, t) is connected to external influences f, g_0, g_1 and initial state Q0(x) by corresponding kernels of linear integral operators N0, N1, G, K0, K1, which reflect inner properties of the object relating to respective inputs.

As follows from the general theory of linear equations, all these nuclei can be expressed in a finite form only via one of them - function G (x,ξ, t, τ) called Green's function, which is a fundamental main characteristic of the object.

If in a boundary problem the initial conditions are zero, and the boundary conditions are homogeneous, *i.e.* $Q_0^{(0)}(x) \equiv Q_0^{(1)}(x) = 0$, $x \in [x_0; x_1]$, $g_1(x, u_1(t)) = 0$, and function f is represented as the product of Delta-functions: $f(x, t, u(x,t)) = \delta(x - \xi_0)\delta(t - \tau_0)$, then, in accordance with properties of the Delta functions, we will obtain:

$$Q(x,t) = \int_0^t \int_{x_0}^{x_1} G(x,\xi,t,\tau)\ \delta(\xi - \xi_0)\delta(\tau - \tau_0) d\xi d\tau = G(x, \xi_0, t, \tau_0)$$

Thus, Green's functionis the solution of the boundary value problem : $G(x, \xi_0, t, \tau_0)$

$$A(x,t)\frac{\partial^2 Q}{\partial t^2} + A_1(x,t)\frac{\partial Q}{\partial t} = C(x,t)\frac{\partial^2 Q}{\partial x^2} + B_1(x,t)\frac{\partial Q}{\partial x} + C_1(x,t)Q +$$
$$+\delta(x - \xi_0)\delta(t - \tau_0);\ x_0 < x < x_1; t > 0;$$
$$Q(x,0) = \frac{\partial Q(x,0)}{\partial t} = 0; x_0 \le x \le x_1;$$
$$\alpha(x_0,t)Q(x_0,t) + \beta(x_0,t)\frac{\partial Q(x_0,t)}{\partial x} = 0,\ t > 0;$$
$$\alpha(x_1,t)Q(x_1,t) + \beta(x_1,t)\frac{\partial Q(x_1,t)}{\partial x} = 0,\ t > 0.$$

and describes reaction of a controlled distributed system with zero initial and boundary conditions at any point $x \in [x_0; x_1]$.

At any moment $t \ge 0$ on a point pulse influence of the form of Delta functions applied to point $\xi_0 \in [x_0; x_1]$ at the moment $t = \tau_0 \ge 0$.

3. EXPERIMENT

Modeling systems has a huge complex of software and hardware resources. There are many methods of calculation. However, the fundamental approach to solving this problem is to consider different approaches. There are two fundamental approaches in modeling nonlinear systems - these are analytical and software approaches.

The analytical method is used when it is impossible to obtain specific values. This happens in case of lacking computational resources and hardware. In some cases, an analytical solution is considered the only correct solution of a certain automatic control system.

This is mainly due to the fact that analytical solution shows the dependence between some variables, and the dynamic relationship between various variables.However, it is necessary to obtain simulation results for solving specific control tasks. In case of analytical approach, the final result of the modeling will be a formula or a function. For obtaining specific outcomes the program approach is used.

The software approach is software modeling of a physical process. This modeling may be made in a number of software products such as:

- MATLAB;
- Mathcad;
- Maple;
- Programming languages, etc.

When modeling in Mathcad software systems, modeling occurs for a given range of values, and consistent values are obtained by the time of computation run. The disadvantage of this system is impossibility to correctly process obtained results, in particular for problems of thermal conductivity on the basis of Green's function.

However, an important advantage is that when short intervals of time are set, this environment makes it possible to simulate behavior of spatially distributed processes. However, it should be noted that when external modules are connected, the capabilities of simulation and mathematical calculations expand greatly.

For implementation of technical computation, there are several software products focused on the distributed nature of computation in relation to time, such as MATLAB, Maple. Due to the presence of built-in programming languages, these software products make it possible to model the time intervals of any length.

In case of defining too large and cumbersome calculations, the systems can operate without user interaction and groups of computers can be merged into a single cluster. In the MATLAB system, there is also the possibility of integration with external modules. In the whole, these systems are powerful enough to perform calculations for any engineering systems, such as:

- Digital processing of signals, images and data;
- Control systems;
- Financial analysis;
- Analysis and synthesis of geographical maps, including three-dimensional maps.

This is far from a complete list of plug-in modules. Each of these modules contains a list of sub-modules combined with this class of problems.

In solving complicated tasks in non-industrial conditions, it is customary to use programming languages.Using programming languages provides a fairly flexible approach to solving many problems. For example, using programming languages makes it possible to gradually upload calculated data. This allows savinga part of the obtained results in case of system disruption.

Also, using programming languages makes it possible to structure the output information, which greatly facilitates its subsequent processing.Delphi programming language is used in this dissertation research. This choice is explained by appropriate mathematical mechanism of the match library for executing current mathematical operations. Another advantage of this language is low data workload, which ensures quick and reliable operation.

Considering various approaches and methods of modeling nonlinear distributed systems, there can't be the best or the worst way. For various classes of problems there are corresponding approaches. So, in cases where there is no possibility to get the result the analytical approach is used; and in cases of exact calculations - programming languages are used.

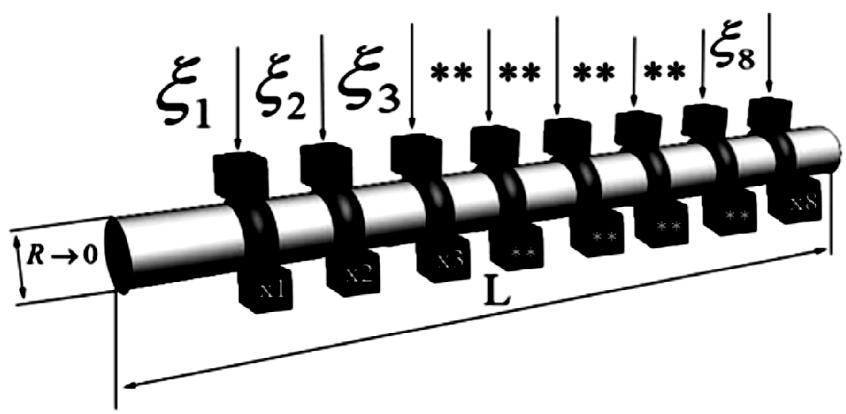

Fig. 7. Control object, a cylindrical rod.

Let us consider discrete control of an isotropic cylindrical rod with radius R, length *l* and material thermal conductivity a^2 (see Fig. 7.). Let us place sectional heating element ξi, on the lateral boundary of the rod. Control over this sectional heating element will be ensured by relay elements. The ends of the rod are to be insulated.

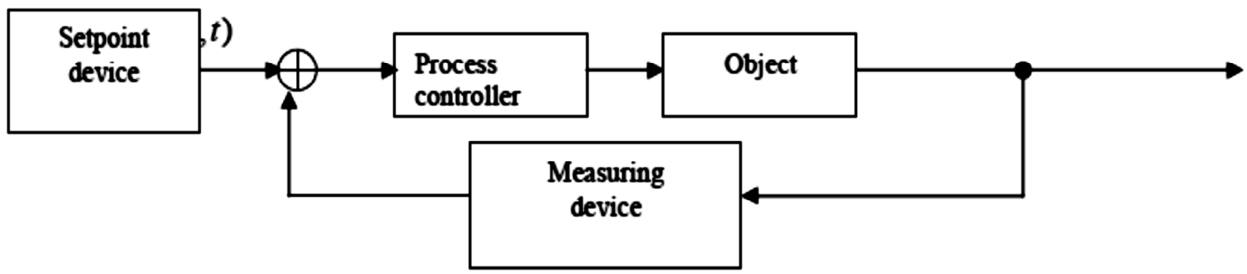

Fig. 8. Structural diagram of a closed-circuit regulation system.

For regulating this system, the control device should create influences for changing temperature field from the specified value. Such deviations should be recorded at certain points at certain moments. Then they can be recorded. Thus, it is necessary to calculate the place and time of enabling heating elements.

The problem of stabilizing temperature field is reduced to keeping temperature changes $T(x,t)$ within T_{back}. This function will be implemented by pulse heating elements.

Let us consider $U(x, t)$ defined in $-\infty < x < \infty, t \ge 0$, satisfying conditions

$$\frac{\partial U}{\partial t} = a^2\left(\frac{\partial^2 U}{\partial x^2}\right), -\infty < x < \infty, t > 0$$

$$U(x,0) = \varphi(x), -\infty < x < \infty,$$

where function $\varphi(x)$ specifies initial temperature distribution. Let us transform the temperature distribution equation according to Fourier. This transformation will be made with variable x

$$\begin{cases} \dfrac{\partial U}{\partial t} = -a^2 p^2 U \\ U(p,0) = \hat{O}(p) \end{cases}$$

$$\left\{ U(p,t) = e^{-a^2 p^2 i} * c, \right.$$

With

$$\hat{O}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varphi(y) e^{-ipy} dy$$

$$U(p,0) = C = \hat{O}(p)$$

$$U(p,t) = \hat{O}(p) e^{-a^2 p^2 i}$$

We obtain:

$$U(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-a^2 p^2 i} \left(\int_{-\infty}^{\infty} \varphi(y) e^{-ipy} dy \right) e^{-ipx} dp = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(y) \left(\int_{-\infty}^{\infty} e^{-ipy} e^{-a^2 + ipx} dp \right) dy$$

$$(-a^2 p^2 - 2\frac{ipy}{2} + \frac{(y-x)^2}{4a^2 t}) - \frac{(y-x)^2}{4a^2 t} - (-a^2 p^2 t - 2\frac{ip(y-x)}{2} + \frac{i^2 (y-x)^2}{4a^2 t}) - \frac{(y-x)^2}{4a^2 t} -$$

$$-(a\sqrt{tp} + \frac{i\ (y-x)}{2a\sqrt{t}})^2 - \frac{(y-x)^2}{4a^2 t};$$

$$\int_{-\infty}^{\infty} e^{-(a\sqrt{tp}+\frac{i(y-x)}{2a\sqrt{t}})^2-\frac{(y-x)^2}{4a^2t})} dp = \frac{e^{-\frac{(y-x)^2}{4a^2t}}}{a\sqrt{t}} \int_{-\infty}^{\infty} e^{-U^2} dU = \frac{\sqrt{\pi}}{a\sqrt{t}} e^{-\frac{y^2}{4a^2t}}$$

$$U = a\sqrt{tp} + \frac{iy}{2a\sqrt{t}}$$

$$dU = a\sqrt{t}dp$$

$$U(x,y) = \frac{1}{2a\sqrt{t\pi}} \int_{-\infty}^{\infty} \varphi(y) e^{-\frac{(y-x)^2}{4a^2t}} dy$$

$$G(y,x,t) = \frac{1}{2a\sqrt{t\pi}} e^{-\frac{(y-x)^2}{4a^2t}}$$

Green's function for the heat conductivity equation.

$$U(x,y) = \int_{-\infty}^{\infty} G(x,y,t)\varphi(y)dy$$

This solution is called the heat conductivity equation solution for zero boundary conditions.

Let us have a look at the control object (see Fig.1.8.) described by the following mathematical model:

$$T(0,r,t) = T(l,r,t) = 0\,;$$

$$T(x,R,t) = u(x,t)\,;$$

$$\frac{\partial T(x,0,\tau)}{\partial r} = 0\,.$$

$$\frac{\partial T}{\partial t} = a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right);(5)$$

$$0 < r < R\,; 0 < x < l\,;$$

Where T*(x, r; t)* is the temperature field of the control object; a^2 is the specified coefficient of thermal conductivity of the control object material; R, l are present numbers; $u(x,t)$ is the control action, x, r are the spatial coordinates of the control object; and t is time.Function output for a given mathematical model will be function T*(x,r,t)*, where r is a specified value from the range ($0 < r < r$). From the boundary conditions it can be seen that the boundaries of the object have zero temperature, the input influence is extended along the boundary of the rod, which is necessary to satisfy the condition of the temperature fields symmetry.

Suppose the diameter of the rod is negligible. With this assumption, the temperature in arbitrary points of the isotropic rod can be considered the same. Let us assume that the control object is spatially one-dimensional. Then distribution of the temperature field along the isotropic cylinder can be described by an infinite Fourier series, Green's function.

$$T(x,t,\xi,\tau) = \frac{2}{l}\sum_{n=1}^{\infty} \exp\left[-\left(\frac{\pi n a}{l}\right)^2 (t-\tau)\right] \sin\frac{\pi n}{l}x \sin\frac{\pi n}{l}\xi \; (6)$$

Where n is the number of the member in the Fourier series; l is the length of the rod; t is time; x is the point (X-axis coordinate) of the temperature sensor location; ξ is the point (X-axis coordinate) of the heating element location; τ is the time of engaging point source, and a^2 is the specified coefficient of thermal conductivity of the control object material.

Thus, formula 1.6 makes it possible to calculate behavior of the temperature field at an arbitrary point on the isotropic rod at an arbitrary moment. However, in order to study the temperature field over time, it is necessary to obtain the function of initial heating.

Since the temperature value is the sum of the Green's function values at current moment and the function of initial heating, the diagram of the temperature field values at the initial moment can be observed in Table 2.

Thus, for analyzing temperature field it is necessary to use the formula that considers the primary heating function:

$$T(x_j, t) = \sum_{i=1}^{d} \sum_{n=1}^{k} \frac{2}{l} \exp\left[-\left(\frac{\pi n a}{l} \right)^2 t \right] \sin\frac{\pi n}{l} x_j \sin\frac{\pi n}{l} \xi_i +$$
$$+\sum_{p} \sum_{n=1}^{k} \frac{2}{l} \exp\left[-\left(\frac{\pi n a}{l} \right)^2 \left(t - \tau_p\right) \right] \sin\frac{\pi n}{l} x_j \sin\frac{\pi n}{l} \xi_{z(p)} \; ; \quad (7)$$

Modeling the control system with the help of this function, the observer has the ability to monitor the propagation of the temperature field at any point of one-dimensional control object within any period of time.

Table 2. Table with number of sensors calculation results.

Thermal conductivity of the material A2 = 0.00001. Cylinder length 1 = 10.

d = 9	***d = 8***	***d = 7***	***d = 6***	***d = 5***
tmas[1.690] = 0.89	tmas[1.690] = 0.81	tmas[1.690] = 0.77	tmas[1.690] = 0.82	tmas[1.690] = 0.99
tmas[2.690] = 0.90	tmas[2.690] = 0.77	tmas[1.690] = 0.60	tmas[1.690] = 0.55	tmas[1.690] = 0.99
tmas[3.690] = 0.89	tmas[3.690] = 0.83	tmas[1.690] = 0.74	tmas[1.690] = 0.45	tmas[1.690] = 0.99
tmas[4.690] = 0.90	tmas[4.690] = 0.76	tmas[1.690] = 0.74	tmas[1.690] = 0.55	tmas[1.690] = 0.99
tmas[5.690] = 0.90	tmas[5.690] = 0.83	tmas[1.690] = 0.60	tmas[1.690] = 0.82	tmas[5.690] = 0.64
tmas[6.690] = 0.89	tmas[6.690] = 0.77	tmas[1.690] = 0.77	tmas[1.690] = 0.40	
tmas[7.690] = 0.90	tmas[7.690] = 0.81	tmas[1.690] = 0.28		
tmas[8.690] = 0.89	tmas[8.690] = 0.02			
tmas[9.690] = 0.82				
d = 14	***d = 13***	***d = 12***	***d = 11***	***d = 10***
tmas[1.690] = 1.35	tmas[1.690] = 1.27	tmas[1.890] = 1.19	tmas[1.690] = 1.09	tmas[1.690] = 0.99
tmas[2.690] = 1.47	tmas[2.690] = 1.34	tmas[2.890] = 1.21	tmas[2.690] = 1.10	tmas[2.690] = 1.00
tmas[3.690] = 1.32	tmas[3.690] = 1.25	tmas[3.890] = 1.18	tmas[3.690] = 1.09	tmas[3.690] = 0.99
tmas[4.690] = 1.44	tmas[4.690] = 1.34	tmas[4.890] = 1.22	tmas[4.690] = 1.10	tmas[4.690] = 1.00
tmas[5.690] = 1.39	tmas[5.690] = 1.26	tmas[5.890] = 1.17	tmas[5.690] = 1.09	tmas[5.690] = 0.99
tmas[6.690] = 1.36	tmas[6.690] = 1.31	tmas[6.890] = 1.22	tmas[6.690] = 1.09	tmas[6.690] = 1.00
tmas[7.690] = 1.44	tmas[7.690] = 1.31	tmas[7.890] =1.17	tmas[7.690] = 1.10	tmas[7.690] = 0.99
tmas[8.690] = 1.36	tmas[8.690] = 1.26	tmas[8.890] = 1.22	tmas[8.690] = 1.09	tmas[8.690] = 1.00
tmas[9.690] = 1.39	tmas[9.690] = 1.34	tmas[9.890] = 1.18	tmas[9.690] = 1.10	tmas[9.690] = 0.99
tmas[10.690] = 1.44	tmas[10.690] = 1.25	tmas[10.890] = 1.21	tmas[10.690] = 1.09	tmas[10.690] = 0.91
tmas[11.690] = 1.32	tmas[11.690] = 1.34	tmas[11.890] = 1.19	tmas[11.690] = 0.87	
tmas[12.690] = 1.47	tmas[12.690] = 1.27	tmas[12.890] = 0.76		
tmas[13.690] = 1.35	tmas[13.690] = 0.58			
tmas[14.690] = 0.37				

Thus, formula 7 shows behavior of the temperature field with regard to system uptime. This equation is modeled in any programming language and in any software environment.

4. DISCUSSION

Relevance of the made research is determined by the complexity of implementation of nonlinear control systems for objects with distributed parameters. The controlled values of such systems depend not only on time, but also on the distribution in the spatial area occupied by the object. In this regard, the class of control actions essentially expands, primarily through the possibility of including spatial and temporal controls described by the functions of several variables – time and spatial coordinates into them.

Features of systems with distributed parameter require creation of an apparatus for their analysis and synthesis based on non-traditional mathematical tools for the classical control theory. There are various forms of describing models of systems with distributed parameters: differential equations in local derivatives; structural representation of systems with distributed parameters, which relies on a fundamental solution of the boundary problem; presentation of distributed objects in the form of complex transfer coefficients in the eigenvector functions of the operator of the object.

For analysis of the control objects that are described by nonlinear partial equations in local derivatives, approximation methods are used most frequently. However, it should be noted that to date the method of distributed systems approximation with a specially chosen concentrated system has not been developed, thus, in many problems the approximation process is unstable, relative to errors in intermediate calculations. Many authors have been recently developing the considered systems and synthesis methods, due to their relevance and the demand for these technical solutions in practice. While many works end with some stages of systems modeling, suggesting further parametric synthesis, which is related to solving several problems, the proposed method is different in the fact that it has been brought to its logical end, i.e., control algorithms have been derived.

The scientific value of this work consists in developing theoretical foundations for analysis and synthesis of nonlinear distributed control systems.

The practical relevance of the work lies in the fact that the developed method of calculating location for heating elements installation, depending on the value of the temperature field, makes it possible to consider the possibility of installing sectional pulsed heaters in electric tunnel ovens of conveyor type. Analysis of assessing performance of the developed software package by stabilization of the temperature field showed:

1. The possibility of bringing the furnace to the required temperature mode through the use of pulse heating elements;
2. The possibility of stabilizing the temperature field within the allowable values. Dependence of the temperature mode on section length has been considered.

5. CONCLUSIONS

This article discusses the basic concepts and models in the theory of distributed control systems. Methods of modeling and transition from concentrated systems to systems with distributed parameters and methods of linearizing these systems have been shown. This makes it possible to show the possibilities of control systems linearization, and of transition from concentrated systems to systems with distributed parameters.

The mathematical model of the heat conductivity equation has been described, and on its basis the solution of the heat conductivity problem for one-dimensional control object, *i.e.*, the Green's function, has been obtained. This makes it possible to perform synthesis of closed-loop system for controlling the temperature field. The model of observing behavior of the temperature field makes it possible to definitely define the value of the temperature field at a point in the isotropic rod with regard to a given time interval. This solution can be applied to all objects for which Green's function there.

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