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A COMMUNICATION NETWORK WITH RANDOM PHASES OF TRANSMISSION

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ABSTRACT

In this paper we develop and analyze a communication network with the assumption that the arrival of request in a communication network follows a poisson process and the transmission through the transmitter is in phases. Each phase is having exponentially distributed processing times. The number of phases required by a packet is random and follows a Binomial distribution with parameters n and p, where n is the maximum number of phases in the transmitter, and p is the Probability that the transmission of the packet terminates at a particular phase. Then the inter transmission times required by the packets in the transmitter follows a compound Erlangian ^ Binomial distribution. Assuming that the network is in steady state, the probability generating function of the buffer size distribution, the average number of packets in the buffer, the throughput of the communication network, the average delay time distribution and average delays etc., are derived and analyzed with respect to the parameters. This communication network gives closure approximation than the communication network with Erlangian transmission times having fixed number of phases. Sensitivity of the network with respect to the parameters is also discussed. This network is useful for performance evaluation of ATMs, Communication networks, CPU scheduling etc,.

Keywords: *Communication network, Probability generating functions, Erlangian^ Binomial distribution, Random phases of transmission*

INTRODUCTION

The demand for data voice communication is growing rapidly in many different fields. To satisfy this rapidly growing demand by many users, various kinds of effective communication networks have been developed. With the development of sophisticated technological innovations in recent years, a wide variety of

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communication networks are designed and analyzed with effective switching techniques. In general a realistic and high speed transmission of a data or voice over the transmission lines is a major issue of the communication systems.

It is generally known that various queueing models are developed for communication networks in order to analyse the natural phenomenon more close to reality. It is customary to consider that the arrival and transmission processes in a communication networks are homogeneous, but many practical situations the service time required by a process is non-homogeneous for example in ATM networks, communication networks and CPU Scheduling the packet requires various phases of transmission (i.e. the transmission phases in the transmitter are random). There is a similarity between communication network and queueing model.

Much work has been reported in literature regarding queueing models with heterogeneous service time. Kiran Singh[1] (1977) developed a two servers system in which the service time distribution of two servers are different. Albrights C [2] (1977) developed a class of M/G/S queueing reward system with heterogeneous services and processes. Neuts[3] (1978) studied a M/M/1 queue in which the arrival and service rates vary. Kaufman[4] (1981) developed a heterogeneous server model which was useful in modeling facility trunk circuits. Kambo and Bhaliak[5] (1982) analyzed bulk arrival heterogeneous queues in N environments. Ramachandram[6] (1989) developed optimal control of queues in tandem with heterogeneous servers. Charterjee and Mucherjee[7] (1990) studied and analysed a bulk service heterogeneous queueing model with infinity capacity. Madhu Jain andGhimine[8] (1996) studied the limited multiserver queueing systems with additional servers under restrictions of no passing. Wang P.P., Gray and Scott[9,10] (1996, 97) studied a poisson queueing model with heterogeneous exponential servers in which the server is only activated when the queue length exceeds a fixed value. They have also studied on the number of packets lost in a finite state dependent queue. Jain J.L., Gupta J.M. and Singh T.P.[11] (1998) made a survey of the literature on queueing systems with finite calling populations. Srinivasa Rao K and Gopinath[12] (2000) developed a single server finite source and finite capacity queueing model with two state input source. Srinivasa Rao K., Shobha, T. and Srinivasa Rao,P.[13] (2000) developed an interdependent queueing model with controllable arrival rates. Aftab Begum and Maheswari[14] (2002) have extended this model to multipleserver interdependent queueing model with controllable arrival rates. Mishra and Pal[15] (2001) have studied the transient state analysis of busy period distributions in $GI/M^{a,b}/M$; queues. Recently P.Srinivasa Rao, K.Srinivasa Rao and J.Lakshminarayana[16] (2002) have studied a communication network with a mixture of erlangian service time distribution. But no effort is made to develop and analyse a communication network or a queueing model with Erlangian transmission (Service) times having random number of phases, which are very usful in some situations arise places like data voice communications, where transmission time of the packets follows a compound

K-Erlangian-binomial distribution. Since the number of phases required for transmission completion are in random and follows a binomial distribution. We can approximate Erlangian ^ Binomial distributions for transmission times.

Very little work has been reported regarding queueing models with random number of phases in service times which are very useful in developing the optimal operating policies of computer communication systems, data voice transmission, transportation systems. In this paper an attempt is made to develop a communication network with poisson arrivals and the transmission time follows Erlangian ^ Binomial distribution. Using the Laplace transformations, the probability generating function of the system size is derived. The system behavior is analyzed through the system characterized.

A COMMUNICATION NETWORK WITH RANDOM PHASES OF SERVICE

The schematic diagram representing the a communication network with random phases of service fig.1

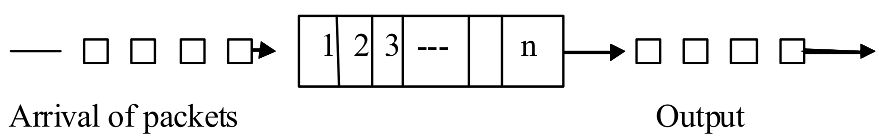

Fig 1

Consider a single node infinite capacity FIFO discipline communication network with assumption that the arrival process is Poisson and the transmission time of a packet follows an Erlangian distribution with k phases of transmission each having 1/μ mean transmission time. It is also further considered that k is a random variable and follows a Binomial distribution with parameters n and p (where n is the maximum number of phases, p is the probability that a packet may require next phases transmission). The service time of a packet follows a compound Erlangian ^ Binomial distribution with probability density function of the form

$$dB(t) = e^{-\mu t} \mu \sum_{k=1}^{n} \frac{(\mu t)^{k-1}}{(k-1)!} \binom{n}{k} \rho^{k} (1-\rho)^{n-k} \quad (2.1)$$

The mean transmission time of the packet is E(t) is given by

$$E(t) = \int_{0}^{\infty} t \, dB(t) = \frac{np}{\mu} \quad (2.2)$$

The variance of the transmission is

$$Var(t) = E(t^2) - [E(t)]^{2l}$$

$$Var(t) = \int_0^\infty t^2 \, dB(t) - [E(t)]^2 \tag{2.3}$$

if K_i is the probability that there are i packet arrivals during a transmission time t then

$$K_i = \int_o^\infty \frac{e^{-\lambda t}(\lambda t)^u}{i!} dB(t)$$

Its probability generating function is

$$K(z) = \sum_{i=0}^{\infty} K_i Z^i$$

$$K(z) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{p\mu}{\lambda + \mu - \lambda z} \right)^k (1-\rho)^{n-k} \tag{2.4}$$

Let P_{ij} be the probability that the number of packets in the network immediately after a transmission is j given that the number of packets is i and

$$\lim_{t \to \infty} P_{ij} = P_j \,, \quad P(z) = \sum_{j=1}^{\infty} P_j z^j$$

Therefore the probability generating function of the number of packets in the network at any time is

$$P(Z) = \frac{(1-\lambda E(t))(1-z)K(z)}{K(z)-z}$$

This implies

$$P(Z) = \frac{\left(1 - \frac{\lambda np}{\mu}\right)(1-z)}{1 - z\left(\frac{p\mu}{\lambda + \mu - \lambda z} + (1-p)\right)^{-n}} \,, \; |z| < 1 \text{ and } \lambda np < \mu \tag{2.5}$$

CHARACTERISTICS OF THE COMMUNICATION NETWORK

The probability that there are i packets in the network can be obtained by expanding P(Z) given in equation 2.5 and collecting the coefficients of z^i.

The Probability that the network is empty is

$$p(0)=\left(1-\frac{\lambda np}{\mu}\right),\ \lambda np < \mu \tag{3.1}$$

The average number of packets in the network can be obtained as

$$L=p'(z)=\frac{d}{dz}\left[\frac{\left(1-\frac{\lambda np}{\mu}\right)(1-z)}{1-z\left(\frac{p\mu}{\lambda+\mu-\lambda z}+(1-p)\right)^{-n}}\right] \tag{3.2}$$

The average number of packets in the buffer is

$$L_q = p'(z)-(1-p(0)) \tag{3.3}$$

For different values of λ, μ, n and p the probability of the network is empty, p_o, The average number of packets in the network, L, and the average number of packets in the buffer, L_q are computed are given in Table 1

Table 1
The values of p_0, L and L_q for different values of λ, μ, n and p

λ	μ	n	p	P_0	L	L_q
1	5	2	0.1	0.960	0.043	0.0030
1	5	2	0.2	0.920	0.085	0.0050
1	5	2	0.4	0.840	0.168	0.0080
1	5	2	0.1	0.960	0.043	0.0030
1	5	4	0.1	0.920	0.085	0.0048
1	5	6	0.1	0.880	0.125	0.0051
1	5	2	0.1	0.960	0.043	0.0037
1	7	2	0.1	0.971	0.032	0.0034
1	9	2	0.1	0.978	0.025	0.0031
1	9	2	0.1	0.978	0.025	0.001
2	9	2	0.1	0.956	0.047	0.002
3	9	2	0.1	0.933	0.065	0.0025

From the above table and the equations (3.1), (3.2) and (3.3), it is observed that average number of packets in the network (L), average number of packets in the buffer (L_q) decrease with the transmission rate μ, when other parameters are fixed. It is also observed that average number of packets in the network (L), average number of packets in the buffer (L_q) are increasing functions of packet arrival rate ë when other parameters are fixed. It is also observed that average number of packets in the network (L), average number of packets in the buffer (L_q) increase with the number of phases n, when other parameters are fixed.

DELAY TIME DISTRIBUTION

In this section the delay time distribution is derived. Here we use the Laplace transforms (LT) of the service and waiting times. Let B*(s) and W*(s) be the Laplace transforms of the transmission time and delay time distributions respectively.

We have

$$K(z) = \sum_{k=0}^{n} \binom{n}{k} \left(\frac{p\mu}{\lambda + \mu - \lambda z} \right)^{k} (1-p)^{n-k}$$

$$B^*(s) = \int_0^\infty e^{-st}\, e^{-\mu t}\, \mu \sum_{k=1}^{n} \frac{(\mu t)^{k-1}}{(k-1)!} \binom{n}{k} p^k (1-p)^{n-k} \tag{4.1}$$

$$W^*(s) = \frac{(1-K'(1))s\, B^*(s)}{s - \lambda(1 - B^*(s))} \tag{4.2}$$

From the convolution properties of transformations,

$$W^*(s) = W_q^*(s)\ B^*(s)$$

Therefore

$$W_q^*(s) = \frac{(1-K'(1))s}{s - \lambda(1 - B^*(s))} \tag{4.3}$$

The mean delay time of a packet in the network is

$$W = \frac{d}{ds}\left(W^*(s)|_{s=0}\right) \tag{4.4}$$

The mean delay time of the packets in the buffer for transmitter is

$$W_q = \frac{d}{ds}\left(W_q^*(s)|_{s=0}\right) \tag{4.5}$$

For different values of λ, μ, n and p the mean delay time of a packet in the network W and the delay time of a packet in the buffer W_q are computed are given in Table 2

Table 2
The values of W and W_q for different values of λ, μ, n and p

λ	μ	n	P	W	W_q
1	3	2	0.1	0.078	0.412
1	3	2	0.2	0.188	0.521
1	3	2	0.3	0.347	0.680
1	3	2	0.1	0.078	0.412
1	3	3	0.1	1.115	4.115
1	3	4	0.1	5.067	14.734
5	7	2	0.1	0.087	0.459
5	8	2	0.1	0.070	0.370
5	9	2	0.1	0.057	0.302
3	9	2	0.1	0.026	0.137
4	9	2	0.1	0.046	0.240
5	9	2	0.1	0.057	0.302

From the table 2 and equations (4.4) and (4.5) it is observed that mean delay time of the packet in the network (W) and mean delay time of the packet in the buffer (W_q) increases with packet arrival time λ, when other parameters are fixed. The values of W and W_q are decreasing functions of the service rate μ, when other parameters are fixed. The values of W and W_q are increasing when p is increasing, when other parameters are fixed.

CONCLUSIONS

The developed model is useful for performance evaluation of ATMs, Communication networks, CPU scheduling etc, where the random number of phases are required to complete the transmission. This model is also useful in developing an optimal statistical multiplex for data voice transmission through satellite communications. This model reduces the congestion in buffers and avoids noise.

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