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Surface Reconstruction using Feature based Approach with Radial Basis Function Neural Network

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ABSTRACT

Current work presents an approach for reconstruction of surfaces using feature points and the Radial basis function (RBF). Features of the surface are extracted at the transmission end and the surface is reconstructed using a RBF neural network at the receiver end. A greedy way of addition of neurons is used in RBF network. The inherent capabilities of a RBF net allows for the reconstruction even when the feature points are contaminated with noise. The simulation results depicting the feature based reconstruction for noisy and non-noisy feature sets are presented for some standard surfaces. Comparisons of the current approach are shown with the traditional interpolation and the reconstruction with dense point cloud data sets. The results demonstrate the effectiveness of the proposed scheme in terms of accuracy, efficiency and robustness to noise.

Keywords: Feature Points, Point Cloud, Radial Basis Function, Surface Reconstruction, Traditional Interpolation

I. INTRODUCTION

Modeling of curves and surfaces from a set of unorganized points with no information about the connectivity of points is a problem with lot of practical significance and is encountered in many fields of computer graphics, image processing and multi-media applications. This derives the need to design a method for effectively storing, retrieving and transmitting these curves and surfaces. The representation of surfaces using control points is an effective way of saving space. It helps in representing the surfaces mathematically thus leading to an overall systematic representation of these surfaces. The current work deals with the reconstruction of surfaces from unorganized points using feature points as control points. Piecewise feature extraction is used here for extracting features. The feature points are stored and can be used for transmitting a surface. On the receiver end the surface is reconstructed using a radial basis function neural network. This leads to an efficient and accurate reconstruction as the process reduces the informational complexity of the RBF network. The surfaces could also be reconstructed even when the feature points are contaminated with noise while transmitting them. RBF neural networks are known to have universal approximation capabilities as discussed in detail in section 2 and this leads to an effective reconstruction at the receiver end.

The problem of curve and surface reconstruction has been solved in various ways using RBF networks. A comprehensive survey of RBF neural networks for the solution of fitting problems is given in [1, 2]. RBF networks perform better then back propagation and generalized regression neural networks for reconstructing surfaces from point cloud data as given in [3]. The application of RBF networks to the problem of interpolation and curve fitting is given in detail by Sheng et al [4]. Zarita Zainuddin [5] has applied RBFNN

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to function approximation problem and also has compared the results by using various types of basis functions at the hidden layer of the RBF network. In this work periodic, exponential and piecewise continuous functions have been approximated using radial basis function networks. C. Enachescu [6] has given various experimental results for demonstrating the approximation capabilities of RBF networks for approximating smooth and continuous functions. A comprehensive survey of RBF networks for classification and function approximation problems is given by Wu et al [7]. Liu [8] has used RBF neural network to approximate a B-spline surface. For this approximation RBF centers are determined by k-means clustering algorithm. The results show that the method can not only approximate a B-spline surface from a point cloud with noise but can also repair an incomplete surface and a surface with wrong input points. Also the surface obtained with the method is a smooth surface and the convergence is fast and robust. A greedy algorithm for reducing the number of centers is proposed by Carr et al [9]. It gives a non-interpolating approximation to the data set and thus smoothly approximates the given surface. Also the method can deal with non-uniformly sampled data and can also fill holes in the data.

Other approaches used for curve and surface reconstruction include the approaches based on Delaunay triangulation and principal component analysis. The drawback of Delaunay based approaches is that they use the entire data set for reconstruction which leads to high complexity when the sample is dense [10]. Also with this approach the reconstructed surface interpolates the data points due to which it is not considered efficient for noisy data sets. The principal component analysis based approaches cannot deal with missing data which usually can happen in practical life applications [10].

The major advantage of the approach presented in this paper is that it reduces the data set of the surface to a few selective feature points leading to an efficient storage, retrieval and transmission of data. The method can reconstruct the surfaces even when the feature set contains noise.

In this approach the surface is divided in to pieces along the xy plane and the feature points are extracted in each of the pieces along z axis. The features which are considered in this work are the one which are invariant under geometric transformations. This in turn optimizes the parameters of a RBF network for reconstruction. A RBF network solves a system of equations in order to train the network from the set of examples. If the training examples are more then it would definitely increase the complexity of the training process. Using feature points as training samples thus reduces the informational complexity of the network. In other words the presented scheme reduces the complexity of the reconstruction process in terms of time and space. Also it reduces the bandwidth requirement for transmission of surfaces. Further it is helpful in animation, as rather than applying the transformations on the entire surface, the transformations could be applied to the control points and then reconstruction process could be carried out from these control points. Effectiveness of the method is shown with the help of some standard surfaces and also a comparison is drawn with reconstruction from the feature points using traditional fitting and reconstruction from point cloud data using traditional fitting techniques. The robustness of the method to noise is shown by adding noise to the feature set and the results are compared with the traditional surface fitting.

This paper is organized as follows. Section II describes the concepts related to this problem including various features and the universal approximation with radial basis function neural networks. The detailed algorithm designed for the solution of the problem is presented in section III. Simulation results along with the analysis and comparisons are shown in section IV. Section V gives the conclusions and future scope of the presented work.

II. BASIC CONCEPTS

In the proposed method we have combined two techniques for reconstructing surfaces – feature selection and RBF reconstruction. The selection of features is critical as the selected features should be such that they should preserve the original shape of the surface and also they should be geometric invariant. Hoffman and Richards [11] have included the maxima, minima and zeros as the critical points which are invariant under translation, rotation and uniform scaling. Freeman [12] defines the critical points as the curvature maxima, minima, end points and points of intersection.

In the present work with extensive experimentation we have determined that using maxima and minima is sufficient to model the shapes of surfaces from the smaller pieces of the surface taken along xy direction.

Radial basis function reconstruction is used in the second phase to reconstruct the original surface from the feature point set. A RBF is a feed forward network which works in a supervised learning mode. It has three layers – the input layer, the hidden layer and the output layer. It has local activation function rather than a global activation function in a multi layer perceptron (MLP). Also it overcomes the disadvantages of a MLP getting trapped in local minima and its non convergence in case of high non-linearities [13]. The general architecture of a RBF neural network is shown in figure 1.

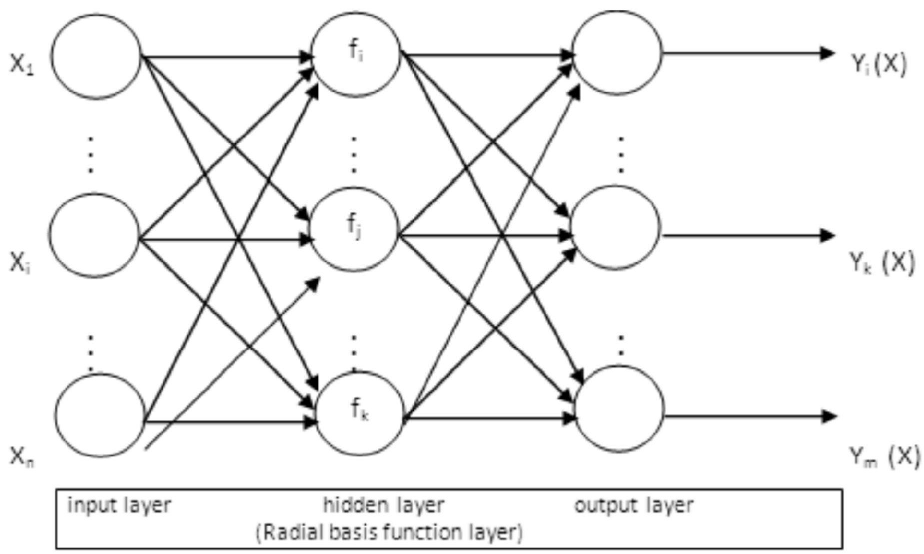

Figure 1: Architecture of a RBF neural network

As discussed in section I RBF networks satisfy universal approximation theorem [14] which forms the very basics of neural networks. This theorem states that a single feed forward neural network with single hidden layer having a finite number of neurons can approximate any continuous function on compact subsets on Rn. So given a continuous function g “ $[0, 1]^m$, we can always find a function G such that:

$|G(x) - g(x)| < e \ \forall$ x in $[0, 1]^n$, and $\exists\ \alpha \in$ Rn, b $\in$ Rm and W $\in R^{n\times m}$ such that $G(x) = \alpha.\ h(Wx + b)$; where h is a continuous, bounded, monotonically increasing and a non-constant function.

The radial basis function has to be selected appropriately in order to achieve universal approximation with RBF networks. The major contributions in providing conditions for selecting radial basis functions have been given in Schoenberg Interpolation theorem [15], Micchelli interpolation theorem [16] and the theorem given by Park and Sandberg [17]. In the most recent of these i.e. Park and Sandberg theorem the condition established for radial basis functions is that the function should be integrable, bounded, continuous almost everywhere and its integration should not be zero.

Also the selection of center points is crucial in RBF networks. If the centers are taken to be the entire input data set then it results in exact interpolation of the data. In order to have better approximation the centers have to be chosen carefully. A number of methods [18, 19] have been designed in literature for selecting centers for RBF networks.

In this paper we have used Gaussian basis function as radial basis function because it satisfies the conditions given by Park and Sandberg. Also instead of using the entire data set as centers we have used a greedy way of selection of centers to approximate the underlying surface from the extracted feature points. The complete algorithm is given the next section.

III. ALGORITHM

The main motivation for developing this algorithm was to design an approach for reconstructing surfaces which uses very less number of points so that we do not have to use all the vertices of the surface for storing and transferring it. But these vertices should lead to an efficient reconstruction of the given surface. If the number of vertices is less then we can have reduced storage requirement, low bandwidth for transmission and less time for reconstruction. With these ideas we have presented a hybrid approach based on feature extraction and RBF reconstruction. The entire process is carried out as per the following steps:

i) The initial input is the set of data points lying on the surface such that each point belongs to R^3.

ii) Along the xy plane, the surface is divided into equal pieces and features are extracted from each of the pieces. As discussed in the previous section following invariant features are considered: the maxima and minima of z coordinates in each of the pieces along xy plane.

iii) A RBF neural network is created and trained as follows:

- The input layer consists of three neuron, one for each of the coordinates of the data points which here is the set of feature points.
- The hidden layer is composed of Gaussian basis neurons which is given as

  $g(\|X-c_j\|) = \exp(-\|X-c_j\|^2 / 2\sigma^2)$; where

  X is the input vector with three components,

  cj is the center of the j^{th} radial basis function,

  σ is the parameter controlling the smoothness properties of the interpolating function, and j=1 to n, n is the number of hidden nodes
- The selection of hidden layer neurons is based on the following greedy approach for selection of centers [20]:
  - Firstly a subset of the given data set is considered as centers
  - At each of the centers, the error is evaluated and checked with the maximum tolerance.
  - If it is less, then the method stops and gives the centers of RBF network
  - More neurons are added near the nodes where the error is large
  - The process is repeated till the error is less.
- The parameter spread is taken same for all the hidden layer nodes. The rule that works for this parameter is that it should be approximately equal to twice the distance between the centers of RBF.
- In the output layer of the network which comprises of 3 neurons, the output is computed as:

$$G(x) = \sum_{j=1}^{C} (w_j \, g(\| X - c_j \|)$$

  where g is the Gaussian basis activation function.

iv. Noise is added experimentally to the feature point data set.

v. Surface is reconstructed by training the RBF network as done in iii.

vi. Traditional fitting is used to reconstruct surface from noisy data points and feature points without noise.

IV. SIMULATION RESULTS

The algorithm presented in previous section is applied to some standard surfaces and the results are shown in figure 2-4. The first surface which is reconstructed is given by $z = 3/(x^2 + y^2)$ and is shown in figure 2(a), points on the surface are shown in figure 2(b). The projected view of these points in x-y plane in shown in figure 2(c). The points of minima and maxima in each of the pieces is shown in figure 2 (d) and 2 (e) respectively. The surface reconstructed using these feature points and RBF network is shown in figure 2(f). Noise is then added experimentally to the feature points of the surface which are shown in figure 2(g). The reconstructed surface using traditional surface fitting and the proposed method are shown in figure 2(h) and 2(i) respectively. The reconstructed surface using feature points which are without any noise with traditional surface fitting is shown in figure 2(j). Initially the number of points is 200, shown in 2(b) and 2(c). The pieces of the surface are taken at a gap of 0.4 so that the number of minimum and maximum points extracted is 25 each, making the total number of points to be 50. A RBF net with spread 0.3 is used to reconstruct the surface from 50 feature points as training set. The noise of 0.1 is then added to feature points and the surface is reconstructed from the noisy feature points. The details of the parameters are shown with each of the surface. Results of the second surface given by $z = -5 / (1 + x^2 + y^2)$, are shown in figure 3(a-j) and the results of third surface given by $z = \sin^2 x + \sin^2 y$, are shown in figure 4(a-i).

Results for surface given by $z = 3/(x^2 + y^2)$

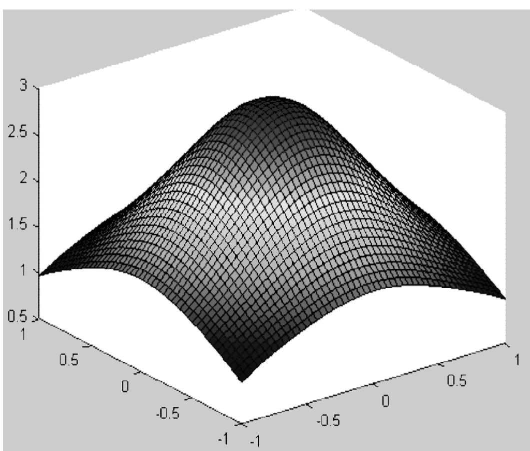

Figure 2 (a): Original surface

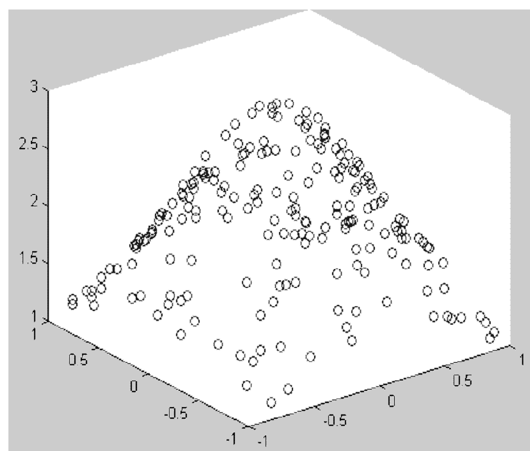

Figure 2(b): Arbitrary points of a surface

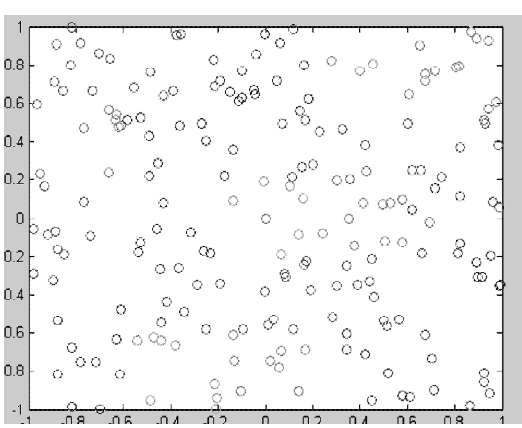

Figure 2(c): Projection of points in xy plane

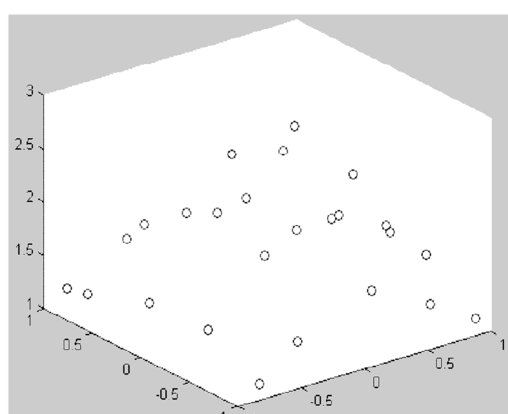

Figure 2(d): Points of minima

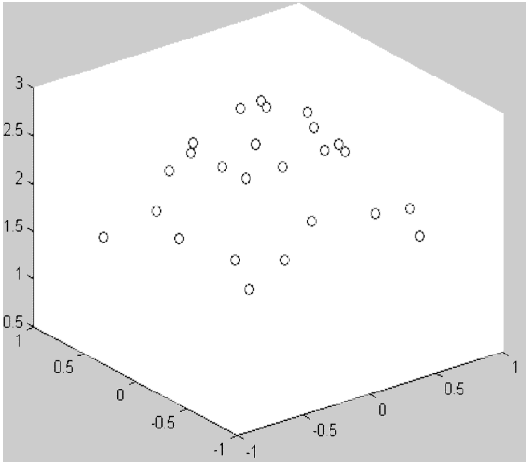

Figure 2(e): Points of maxima

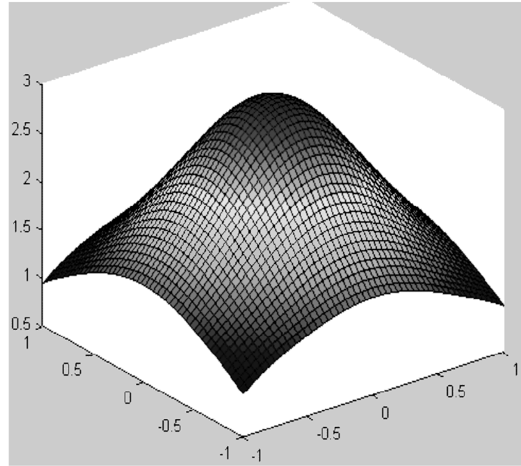

Figure 2(f): Reconstructed surface using feature Points and RBF

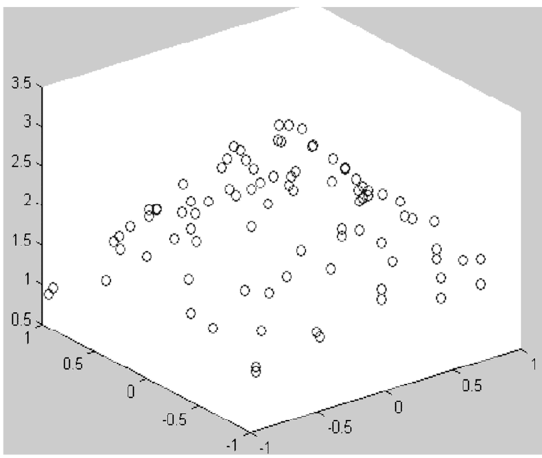

Figure 2(g): Noisy point feature points

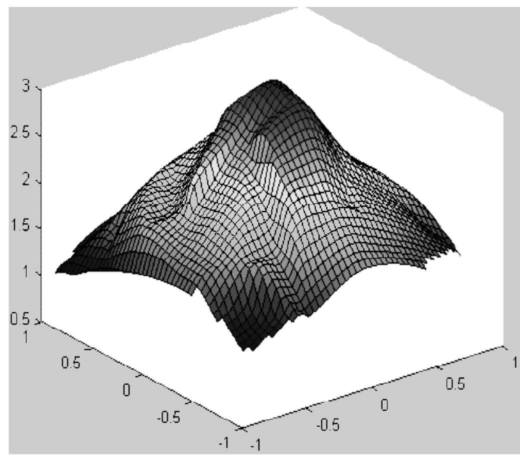

Figure 2(h): Surface reconstructed from noisy data set of features by using traditional surface fitting

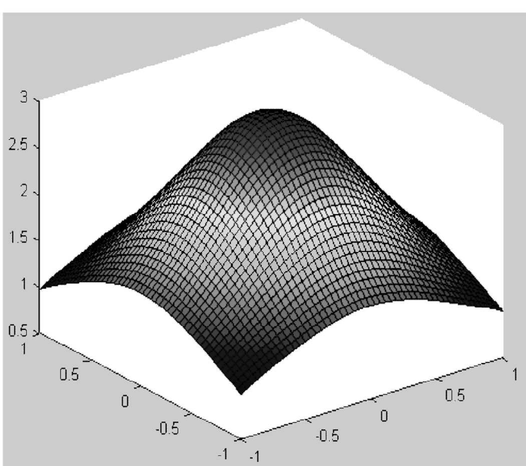

Figure 2(i): Surface reconstructed from noisy data set of feature points using RBF

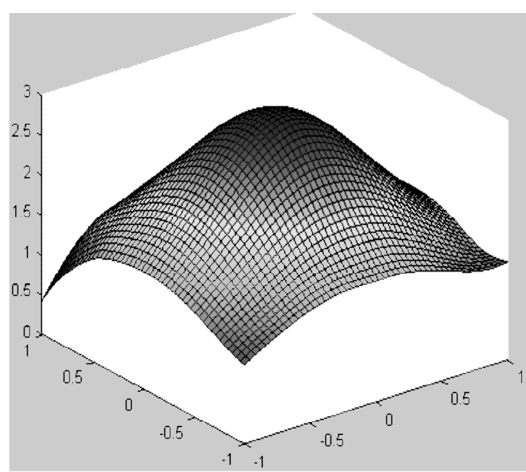

Figure 2(j): Surface reconstructed from feature set (without noise) by using surface fitting

Parameters for above reconstructed surface given by ($z = 3/(x^2 + y^2)$:

No. of total points = 200
Segment gap = 0.4
No. of Minimum points=25
No. of Maximum points=25
Spread = 0.3
Noise added = 0.1

Results for surface given by z = -5 / (1 + x2 + y2):

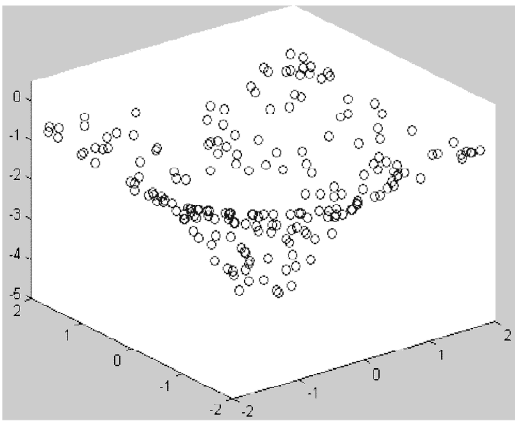

Figure 3(a): Arbitrary points of a surface

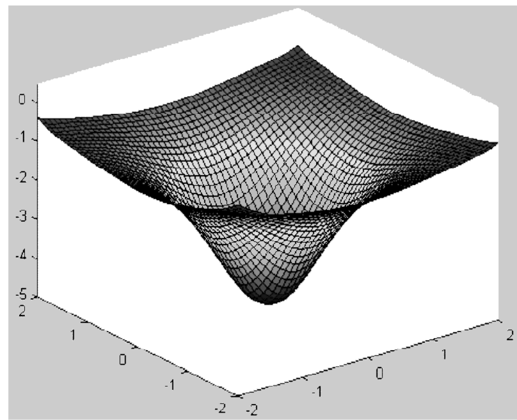

Figure 3(b): Original surface

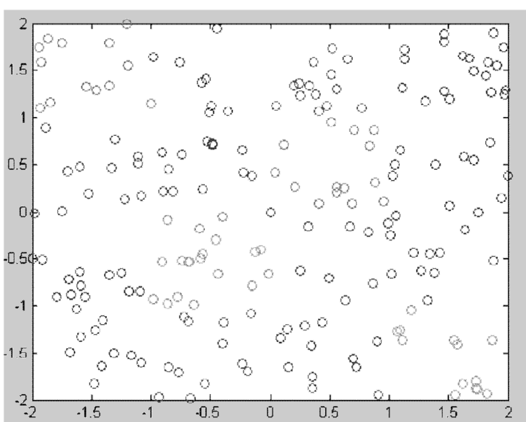

Figure 3(c): Projection of points in xy plane

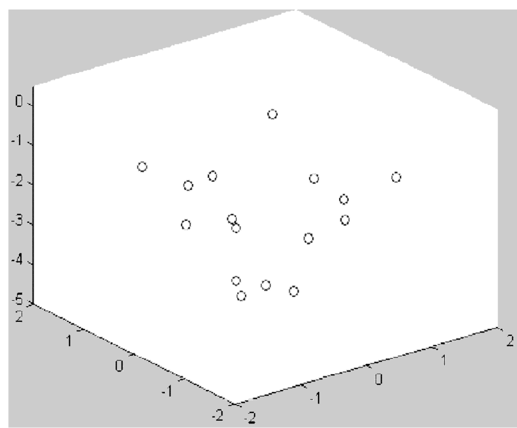

Figure 3(d): Points of minima

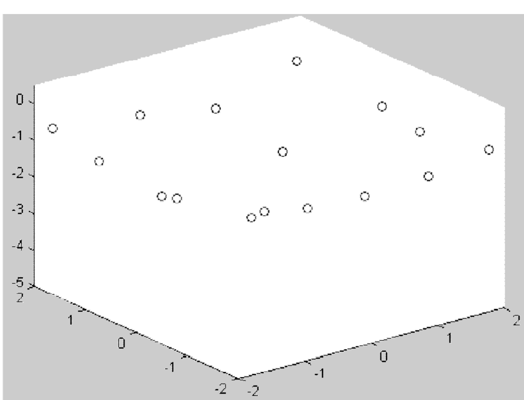

Figure 3(e): Points of maxima

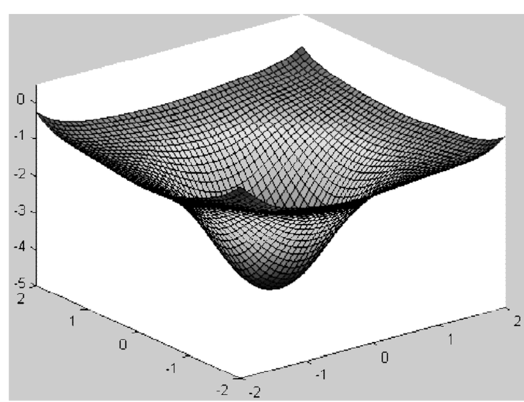

Figure 3(f): Reconstructed surface using feature Points and RBF

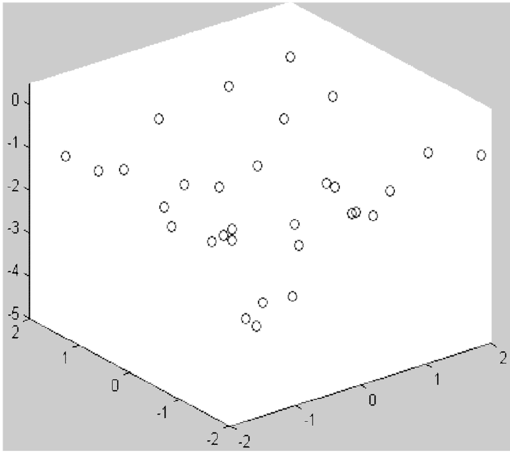

Figure 3(g): Noisy point feature points

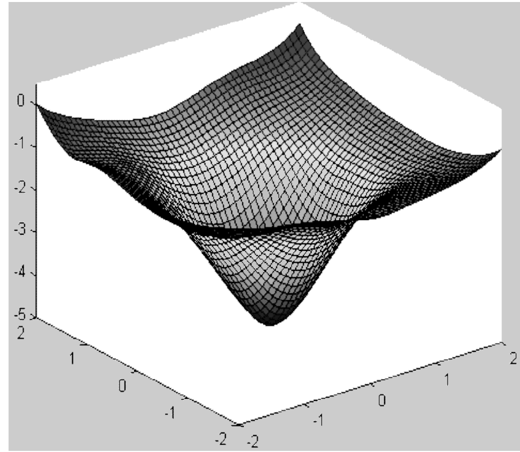

Figure 3(h): Surface reconstructed from noisy data set of features by using traditional surface fitting

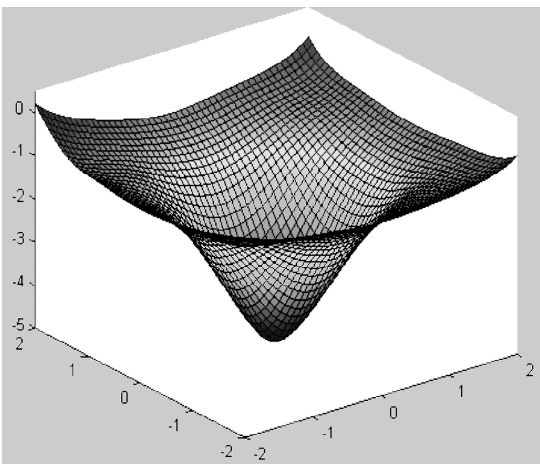

Figure 3(i): Surface reconstructed from noisy data set of feature points using RBF

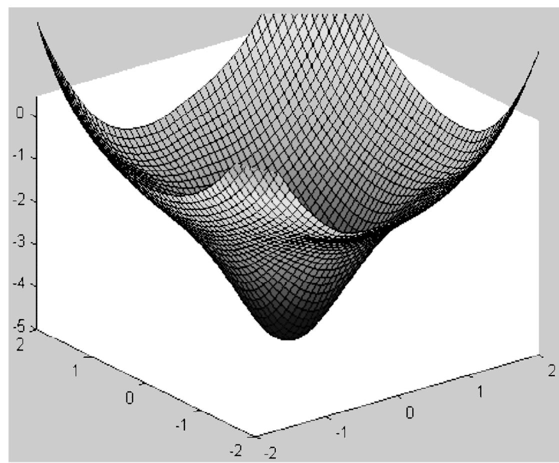

Figure 3(j): Surface reconstructed from feature set (without noise) by using surface fitting

Parameters for above reconstructed surface given by (z = -5 / (1 + x2 + y2))

No. of total points = 200

Segment gap = 1

No. of Minimum points=16

No. of Maximum points=16

Spread = 0.3

Noise added = 0.1

Results for surface given by z = sin2x + sin2y:

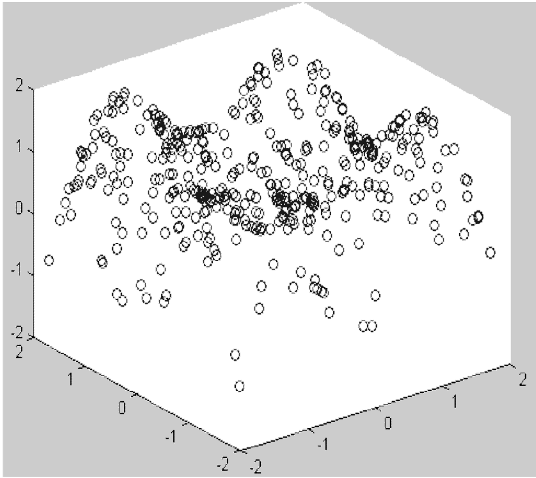

Figure 4(a): Arbitrary points of a surface

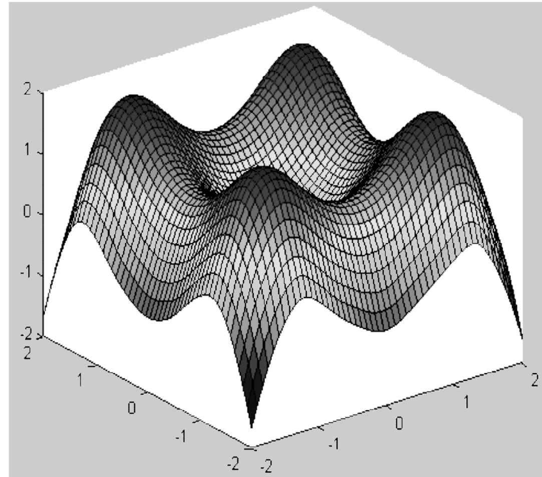

Figure 4(b): Original surface

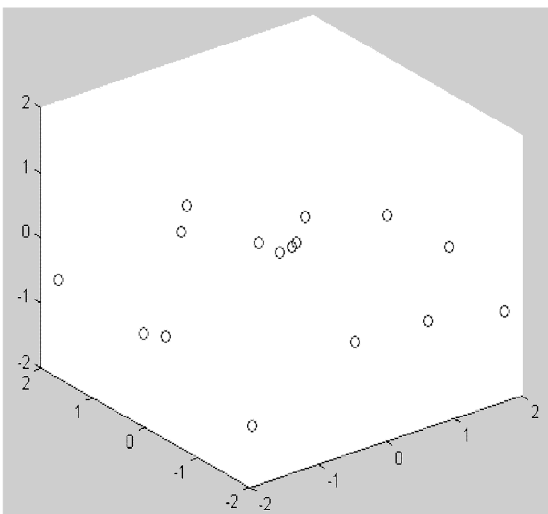

Figure 4(c): Points of minima

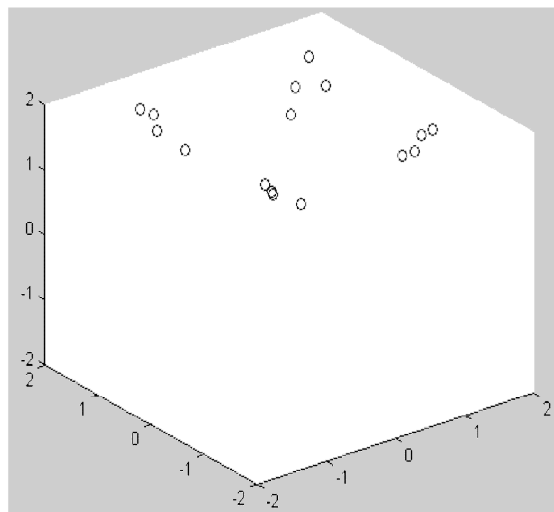

Figure 4(d): Points of maxima

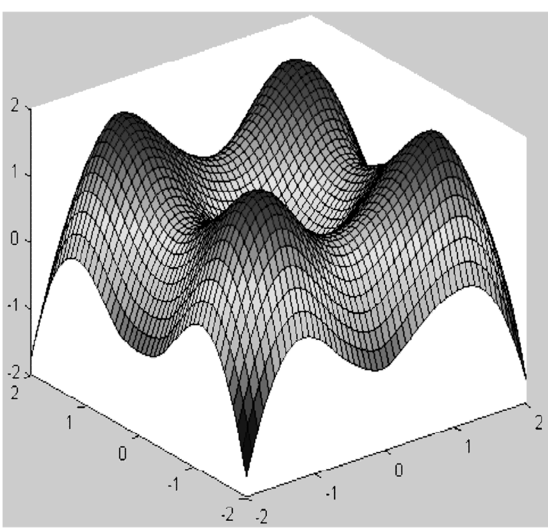

Figure 4(e): Reconstructed surface using feature Points and RBF

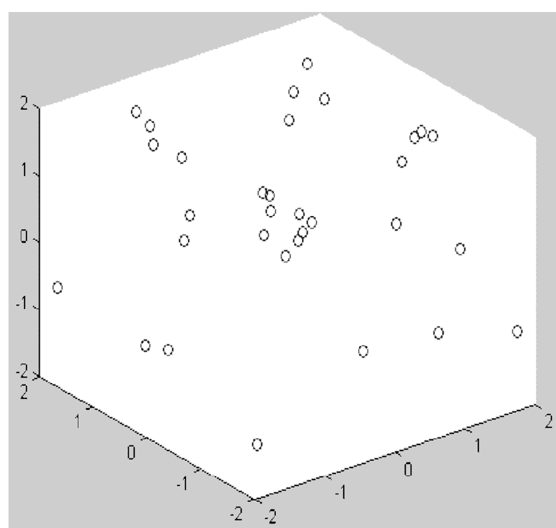

Figure 4(f): Noisy point feature points

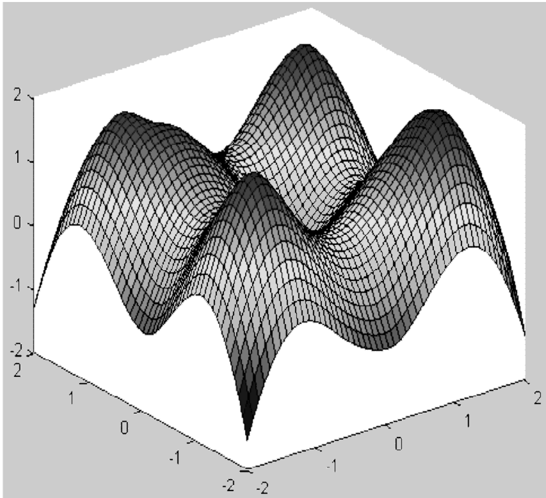

Figure 4(g): Surface reconstructed from noisy data set of features by using traditional surface fitting

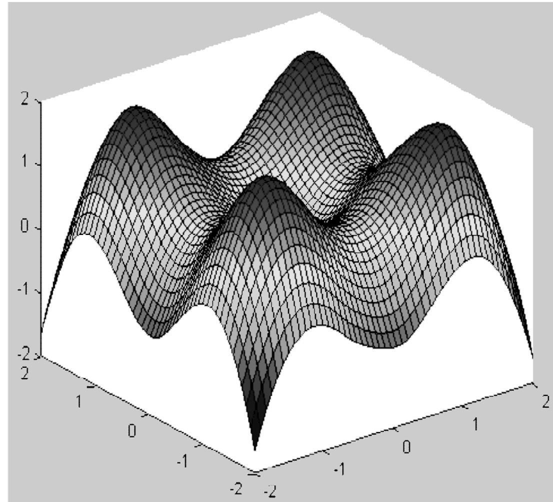

Figure 4(h): Surface reconstructed from noisy data set of feature points using RBF

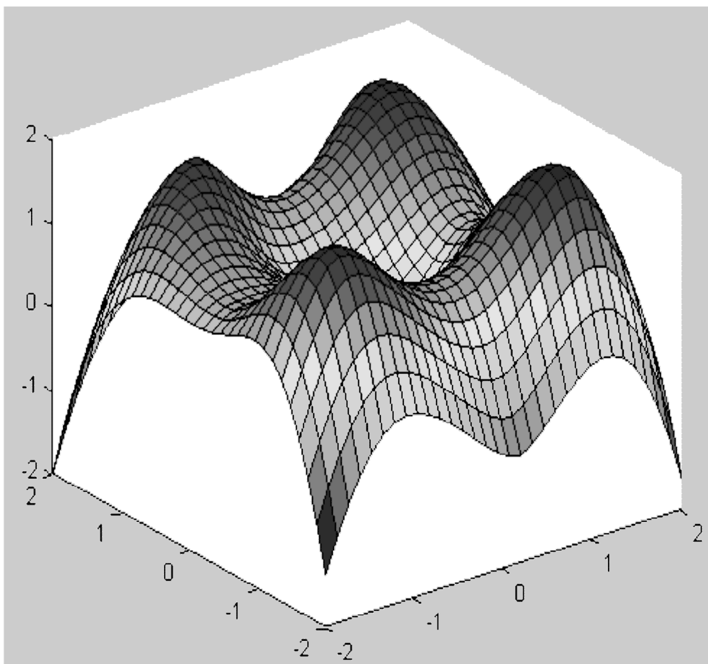

Figure 4(i): Surface reconstructed from feature set (without noise) by using surface fitting

Parameters for above reconstructed surface given by (z = sin2x + sin2y):

No. of total points = 600

Segment gap = 1

No. of Minimum points=16

No. of Maximum points=16

Spread = 0.3

Noise added = 0.2

III. ANALYSIS OF RESULTS

Simulation results show that feature based RBF reconstruction is able to reconstruct surfaces accurately. Whereas if we use simple fitting techniques then we cannot reconstruct the exact shape of the surface by using only the feature points. The surface diverges from the original shape when the number of points is reduced to feature point set. Also with noise in the feature points we are able to reconstruct the original surface whereas simple fitting distorts the surface greatly in the presence of noise in feature points.

The existing approaches of reconstruction based on RBF neural networks use a large data set for training the neural network to learn the shape of the surface for reconstruction. Liu et al [8] have used RBF networks for reconstructing only B-Spline surfaces. The data set that is used in their work is a large data set which trains the RBF network fitting B-Spline surfaces. Yang et al [3] have also used RBF networks for reconstructing some standard surfaces. The surfaces are reconstructed well but at the cost of a large number of training samples.

The advantage of our method is that it can reconstruct surface with same accuracy as with a large data set but uses very few training samples for reconstruction. Thus in addition to reducing the informational complexity of RBF networks it optimizes the control points to reduce time, space and transmission cost.

IV. CONCLUSION

In the current work an approach for reconstructing surface is presented which is based on extracting the control points of surface, and then reconstructing them using the same points as a training set to a RBF neural network. The selection of feature points is based on the idea of using the points which are invariant under a set of transformations. These points are selected as the training set for RBF network. RBF approximation is carried out by using an incremental greedy way of addition of centers of RBF network in the hidden layer. The simulation results clearly show that instead of storing the entire surface it is effective to store only the control points of the surface. The surfaces could be efficiently reconstructed from those control points and the reconstructed shape is a good approximation to the original shape. The proposed method is robust to noise in the data sets as the surface could be reconstructed well from the noisy data set of feature points. Noise can occur frequently when a surface is transmitted. Thus we can say that this method is able to reduce memory requirement and evaluation time of the entire process without the cost of accuracy. The method is also helpful in animation, as rather than applying the transformations on the entire surface, the transformations could be applied to the control points and the surfaces could be generated from those control points using the inverse process presented in the current work. The method is tested on various surfaces with lot of variations in the curvature. Ongoing research includes the identification of more detailed features in surfaces.

REFERENCES

[1] Seng Poh Lim and Habibollah Haron, "Surface reconstruction techniques: a review", *Artificial Intelligence Review*, Springer, vol. 42(1), pp. 59-78, 2014.

[2] Yue Wu, Hui Wang, Biaobiao Zhang and K.L. Du, "Using Radial Basis Function Networks for Function Approximation and Classification," *ISRN Applied Mathematics*, vol. 2012, Article ID 324194, 34 pages, 2012. doi:10.5402/2012/324194.

[3] Sibo Yang, T.O. Ting, K.L. Man and Sheng-Uei Guan, "Investigation of Neural Networks for Function Approximation", *Procedia Computer Science*, Elsevier, vol. 17, pp. 586–594, 2013.

[4] Z. B. Sheng and Xiao Rong Tong, "The Application of RBF Neural Networks in Curve Fitting", *Advanced Materials Research*, vol. 490, pp. 688-692, 2012.

[5] Zarita Zainuddin and Ong Pauline, "Function approximation using artificial neural networks", *WSEAS Transactions on Mathematics*, vol. 7(6), pp. 333-338, 2008.

[6] C. Enăchescu, "Approximation capabilities of neural networks", *JNAIAM*, vol. 3(3-4), pp. 221-230, 2008.

[7] Y. Wu, Hui Wang, Biaobiao Zhang and K-L. Du, "Using radial basis function networks for function approximation and classification", *ISRN Applied Mathematics*, 2012.

[8] Xumin Liu, Houkuan Huang and Weixiang Xu "Approximate B-Spline Surface Based on RBF Neural Networks", *Lecture Notes in Computer Science*, Springer, vol. 3514, pp. 995-1002, 2005.

[9] J. C. Carr, R. K. Beatson, J. B. Cherrie, T. J. Mitchell, W. R. Fright, B. C. McCallum and T. R. Evans, "Reconstruction and representation of 3D objects with radial basis functions", i*n Proc. of the 28th annual conference on Computer graphics and interactive techniques*, pp. 67-76, ACM, 2001.

[10] Kavita Khanna and Navin Rajpal, "Reconstruction of curves from point clouds using fuzzy logic and ant colony optimization", *Neurocomputing*, vol. 161, pp. 72-80, 2015.

[11] D. D. Hoffman and W. A. Richards, "Representing Smooth Plane Curves for Recognition: Implications for Figure-Ground Reversal", *in Proc. AAAI*, 1982, pp. 5-8.

[12] H. Freeman, "Shape Description via the use of Critical Points", *Pattern Recognition*, vol. 10, pp. 159-166, 1978.

[13] Kavita and Navin Rajpal, "Comparative analysis of feed forward and radial basis function neural networks for the reconstruction of noisy curves", in proc. *Optimization, Reliabilty, and Information Technology (ICROIT), 2014 International Conference on*, pp. 353-358. IEEE, 2014.

[14] Simon Haykin, *Neural networks: a comprehensive foundation*, Mc Millan, New Jersey, 2010.

[15]] I. J. Schoenberg, "On best approximations of linear operators", *IJ Schoenberg Selected Papers*, *Contemporary Mathematicians*, Springer, 1988, pp. 397-405, 1988.

[16] C. A. Micchelli, "Interpolation of scattered data: distance matrices and conditionally positive definite functions," *Constructive Approximation*, vol. 2(1), pp. 11–22, 1986.

[17] J. Park and I. W. Sandberg, "Universal Approximation Using Radial-Basis-Function Network", *Neural Computation*, vol. 2(3), pp. 246–257, 1991.

[18] B. Walczak and D. L. Massart, "Local modelling with radial basis function networks", *Chemometrics and Intelligent Laboratory Systems*, Elsevier, vol. 51, pp. 219-238, 2000.

[19] Irina Diaconi³a and Florin Leon, "A learning model for intelligent agents using radial basis function neural networks with adaptive training methods", *BULETINUL INSTITUTULUI POLITEHNIC DIN IAªI*, pp.9-20, 2011.

[20] J. C. Carr, R. K. Beatson, J. B. Cherrie, T. J. Mitchell, W. R. Fright, B. C. McCallum and T. R. Evans, "Reconstruction and representation of 3D objects with radial basis functions", *in Proc. of the 28th annual conference on Computer graphics and interactive techniques*, ACM, 2001, pp. 67-76.