I J A B E R, Vol. 14, No. 5, (2016): 3137-3153

SUPERSYMMETRIC 3-3-1 MODEL WITH MSSM - LIKE HIGGS

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Abstract: *We present a supersymmetric 3-3-1 model with exotic quarks and a charged lepton as an extension of the MSSM model with anomaly free three generations. The scalar sector is studied with six triplet Higgs and the mass spectrum for light Higgs scalars are found to obey MSSM like predictions.The trilinear Higgs term in 3-3-1 is found to be consistent with the bilinear* $\mu H_u H_d$ *term of MSSM and play an important role in the tree-level mass spectrum of scalars.*

1. INTRODUCTION

New physics beyond the Standard Model (SM) and its supersymmetric extensions (minimal extension of Standard Model or MSSM [1]and next-to-minimal supersymmetric extension of the Standard Model NMSSM [2,3]) form interesting challenges to the LHC and linear colliders. An interesting class of models based on $SU(3)\otimes SU(3)\otimes U(1)$ gauge symmetry have been studied recently with supersymmetric extensions [4]. While several versions of 3-3-1 models exist in literature, these can be characterized by their embedding in larger gauge symmetry groups as $\left[SU(3)\right]^3$ [5], $SU(6)\otimes U(1)$ [6] and 3-4-1 gauge symmetry [7].

The supersymmetric version has been considered recently for 3-3-1 models [8] including a right-handed neutrino[9] and a non-exotic anti-lepton [10].The exotic bosons for these models have non-zero lepton numbers. In this work we present a supersymmetric version of a $SU(3)C\otimes SU(3)L\otimes U(1)_X$ model which predicts additional exotic charged quarks and a charged lepton [11]. We consider a three-generation 3-3-1 model [12] without bilepton gauge bosons derived from 3-3-1-1 gauge symmetry which is a subgroup of $SU(4)_{PS}\otimes SU(4)_W$. The coupling constants of $SU(3)_L$ and $U(1)_X$, g and g_X are related to electroweak mixing

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angle θ_W $\frac{g_X^2}{g^2} = \frac{\sin^2_{\theta_W}}{1-4\sin^2_{\theta_W}}$. The anomaly cancellation takes place within three generations of fermions which transform differently for the first two and the third generation. The SUSY version of the model has an extended scalar sector with three Higgs triplets along with three new scalars in adjoint representation to cancel chiral anomalies generated by Higgsinos. The mass spectrum for the Higgs scalars is obtained for neutral, pseudoscalar and charged cases. The lightest scalar h_1^0 is found to have a mass within the acceptable range of 114-128 GeV. Since the gauge bosons and scalars have B-L = 0, we have an analogous situation as in MSSM in which the $\mu H_u H_d$ term in superpotential is replaced by the trilinear Higgs term $\varepsilon\rho\eta\chi$ [8].

In Sec.2 we present a general formulation of the model along with the connection with MSSM and NMSSM models. In Sec.3 we present the supersymmetric 3-3-1 model along with the scalar potential and constraint equations..Sec.4 deals with the mass spectrums of the neutral scalar, pseudoscalar, single and double- charged scalars.Sec.5 deals with the numerical analysis of our work. In Sec.6 we discuss the.interactions of Higgs scalars with gauge bosons and fermions. Finally Sec.7 is a short discussion on results and conclusions.

2. THE SUPERSYMMETRIC 3-3-1 MODEL

The 3-3-1 model with exotic charged quarks and a charged lepton [9] can be embedded in $SO(12)$ - derived $SU(4)_{PS} \otimes SU(4)_W$ group with $SU(4)_{PS} \to SU(3)_C \otimes U(1)_{B-L}$ [13]. In addition, $SU(4)_W \to SU(3)_L \otimes U(1)_{Y_1}$ such that $U(1)_{B-L} \otimes U(1)_{Y_1} \to U(1)_X$ gives $3_C - 3_L - 1_X$ symmetry group with $U(1)_X$ charge defined by $X = Y_1 + \frac{(B-L)}{2}$.

The pattern of symmetry breaking in the model is given by [12]

$$
\begin{aligned}
&SU(3)_C \otimes U(1)_{B-L} \otimes SU(4)_W \to SU(3)_C \otimes SU(3)_L \otimes U(1)_{Y_1} \otimes U(1)_{B-L} \\
&SU(3)_C \otimes SU(3)_L \otimes U(1)_X \xrightarrow{M_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_{X'} \otimes U(1)_X \\
&\xrightarrow{M_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_{X'} \otimes U(1)_X \\
&\xrightarrow{M_\eta, M_\rho} SU(3)_C \otimes U(1)_{em}
\end{aligned} \tag{1}
$$

The $SU(2)_L$ weak isospin group is embedded in $SU(3)_L$ which decomposes as $SU(2)_L \otimes U(1)_{X'}$ [14] with $X' = -\sqrt{3}T_8 = -T_{3R} + Y_1$. Here T_{3R} and Y_1 are $SU(4)_W$ generators.

The fundamental representation of $SU(3)_L \otimes U(1)_{Y_1}$,

$$\left(3_L, \frac{1}{2}\right) = \left(2_L, X', \frac{1}{2}\right) + \left(1_L, X', \frac{1}{2}\right) \text{ with } X' = \frac{1}{2}, -1 \tag{2}$$

For $X' = -1$, we consider $T_{3R} = \frac{3}{2}, Y_1 = \frac{1}{2}$ which is different from $T_{3R} = \frac{1}{2}, Y_1 = -\frac{1}{2}$

- Under $SU(2)_L \otimes U(1)_{3R} \otimes U(1)_{Y_1}$ the fundamental triplet 3_L and singlets $1_L, 1_L'$ decompose as

$$3_L = \left(3_L, \frac{1}{2}\right) = \left(2_L, 0, \frac{1}{2}\right) \oplus \left(1_L, \frac{3}{2}, \frac{1}{2}\right)$$

$$1_L = \left(1_L, -\frac{1}{2}\right) = \left(1_L, -\frac{1}{2}, -\frac{1}{2}\right); 1_L' = \left(1_L, -\frac{3}{2}\right) = \left(1_L, -\frac{3}{2}, -\frac{3}{2}\right) \tag{3}$$

The electric charge operator

$$\frac{Q}{e} = T_{3L} + \sqrt{3}T_8 + \sqrt{6}T_{15L} + \frac{(B-L)}{2} I_4 = T_{3L} - X' + X = T_{3L} + \frac{Y}{2} \tag{4}$$

where $T_{i,} (i = 3,8,15)$ are diagonal generators of $SU(4)_W, Y_1 = \sqrt{6}T_{15L}, I_4$ is the 4×4[4] identity matrix, and Y denotes hypercharge.

- **Matter multiplets**

Lepton $\psi_{\alpha L} = \begin{pmatrix} \nu_{l\alpha} \\ e_{l\alpha} \\ P_\alpha \end{pmatrix} \sim (1_C, 3_L, 0), l_\alpha = e, \mu, \tau; P_\alpha = P_e, P_\mu, P_\tau$

Quark $Q_i = \begin{pmatrix} d_i \\ u_i \\ D_i \end{pmatrix} \sim \left(3_C, 3_L^*, -\frac{1}{3}\right), i = 1,2; Q_3 = \begin{pmatrix} t \\ b \\ T \end{pmatrix} \sim \left(3_C, 3_L, \frac{2}{3}\right)$

The singlet leptons are

$l_R^C \sim (1_C, 1_L, -1); P_R^C \sim (1_C, 1_L, 1); l = e, \mu, \tau; P = P_e, P_\mu, P_\tau$.

The right-handed neutrinos are singlets of the model and do not contribute to anomaly cancellation. The singlet quarks

$$u_R^C \sim \left(3_C^*, 1_L, -\frac{2}{3}\right); d_{Ri}^C \sim \left(3_C^*, 1_L, \frac{1}{3}\right); D_{Ri}^C \sim \left(3_C^*, 1_L, \frac{4}{3}\right)$$

$$t_R^C \sim \left(3_C^*, 1_L, -\frac{2}{3}\right); b_{Ri}^C \sim \left(3_C^*, 1_L, \frac{1}{3}\right); T_R^C \sim \left(3_C^*, 1_L, -\frac{5}{3}\right) \tag{5}$$

- **Higgs triplets**

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (1,3,0); \begin{pmatrix} \eta^0 \\ \eta_1^- \end{pmatrix} \sim \left(1_C, 2_L, -\frac{1}{2}, 0\right); \eta_2^+ \sim (1_C, 1_L, 1, 0)$$

$$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1,3,1); \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix} \sim \left(1_C, 2_L, \frac{1}{2}, 1\right); \rho^{++} \sim (1_C, 1_L, 2, 1) \tag{6}$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1,3,-1); \begin{pmatrix} \chi^- \\ \chi^{--} \end{pmatrix} \sim \left(1_C, 2_L, -\frac{3}{2}, -1\right); \chi^0 \sim (1_C, 1_L, 0, -1)$$

The vacuum expectation values which are nonzero include

$$\langle \eta^0 \rangle = v; \langle \rho^0 \rangle = u; \langle \chi^0 \rangle = V \tag{7}$$

We now introduce chiral superfields $\hat{\varphi}$ by extending the particle content to include squarks, sleptons and higgsinos. The superpartner for a given particle f is $\tilde{f}$. The scalar sector contains additional fermion partners or higgsinos $\tilde{\eta}, \tilde{\rho}, \tilde{\chi}$. For anomaly cancellation for higgsinos, we introduce a set of three additional scalars η', ρ', χ' with higgsinos $\tilde{\eta}', \tilde{\rho}', \tilde{\chi}'$ transforming in adjoint representations

$$\tilde{\eta}' = \begin{pmatrix} \tilde{\eta}'^0 \\ \tilde{\eta}_1'^+ \\ \tilde{\eta}_2'^- \end{pmatrix} \sim \left(1, 3^*, 0\right); \begin{pmatrix} \tilde{\eta}'^0 \\ \tilde{\eta}_1'^+ \end{pmatrix} \sim \left(1_C, 2^*_L, \frac{1}{2}, 0\right); \tilde{\eta}_2'^- \sim \left(1_C, 1_L, 1, 0\right)$$

$$\tilde{\rho}' = \begin{pmatrix} \tilde{\rho}'^- \\ \tilde{\rho}'^0 \\ \tilde{\rho}'^{--} \end{pmatrix} \sim \left(1, 3^*, -1\right); \begin{pmatrix} \tilde{\rho}'^- \\ \tilde{\rho}'^0 \end{pmatrix} \sim \left(1_C, 2^*_L, -\frac{1}{2}, -1\right); \tilde{\rho}'^{--} \sim \left(1_C, 1_L, -2, -1\right)$$

$$\tilde{\chi}' = \begin{pmatrix} \tilde{\chi}'^+ \\ \tilde{\chi}'^{++} \\ \tilde{\chi}'^0 \end{pmatrix} \sim \left(1, 3^*, +1\right); \begin{pmatrix} \tilde{\chi}'^+ \\ \tilde{\chi}'^{++} \end{pmatrix} \sim \left(1_C, 2^*_L, +\frac{3}{2}, +1\right); \tilde{\chi}'^0 \sim \left(1_C, 1_L, 0, +1\right) \tag{8}$$

The vacuum expectation values which are nonzero include

$$\langle \eta' \rangle = v', \langle \rho' \rangle = u', \langle \chi' \rangle = V' \tag{9}$$

- The extended electroweak sector can be compared with minimal supersymmetric standard model (MSSM). The MSSM scalar doublet H_d transforms as $\left(\eta_0, \eta_1^-\right)$ but H_u is different from $\left(\rho^+, \rho^0\right)$ while χ^0 is $SU(2)_L$ singlet field. These transform under $SU(2)_L \otimes U(1)_{T_{3R}} \otimes U(1)_{Y_1}$ symmetry

$$\begin{pmatrix} H_d{}^0 \\ H_d{}^- \end{pmatrix} \sim \left(2_L, -\frac{1}{2}, 0\right); \begin{pmatrix} H_u{}^+ \\ H_u{}^0 \end{pmatrix} \sim \left(2_L, \frac{1}{2}, 0\right) \tag{10}$$

The μ term $\varepsilon_{ab} H_d^a H_u^b \to \varepsilon_{ab} \lambda \hat{N} H_u^a H_d^b \to \varepsilon_{ijk} \kappa_1 \eta^i \rho^j \chi^k$ in the MSSM, NMSSM and present model. This is also evident in the case of pseudoscalar masses for which $\kappa_1 V$ plays the role of m_{12}^2 in MSSM model as will be shown later.

The superpotential and constraint equations along with superfields have been discussed in literature [8] There are 27 chiral superfields. The vector superfields for the gauge bosons of each of $SU(3)_C, SU(3)_L$ and $U(1)_X$ are denoted by Gluon g^b, gluino λ_C^b; $SU(3)_L$ gauge boson V_L^b and gauginos λ_L^b and $U(1)_X$ gauge boson V_X and gaugino λ_X.

3. HIGGS SECTOR AND SUSY 3-3-1

The total supersymmetric 3-3-1 Lagrangian density may be written as,

$$L_{SUSY\,3-3-1} = L_{SUSY} + L_{SOFT} \tag{11}$$

where SUSY Lagrangian density is

$$L_{SUSY} = L_{GAUGE} + L_{MATTER} + L_{SCALAR} \tag{12}$$

We consider the scalar part for the present purposes

$$L_{SCALAR} = \int d^4\theta \left[\begin{array}{l} \hat{\eta}^\dagger e^{(2g\hat{V})}\hat{\eta} + \hat{\rho}^\dagger e^{(2g\hat{V}+g_X\hat{V}_X)}\hat{\rho} + \hat{\chi}^\dagger e^{(2g\hat{V}-g_X\hat{V}_X)}\hat{\chi} + \hat{\eta}'^\dagger e^{(2g\hat{V})}\hat{\eta}' \\ +\hat{\rho}'^\dagger e^{(2g\hat{V}-g_X\hat{V}_X)}\hat{\rho}' + \hat{\chi}'^\dagger e^{(2g\hat{V}+g_X\hat{V}_X)}\hat{\chi}' \end{array} \right] + \int d^2\theta W + \int d^2\bar{\theta}\,\overline{W} \tag{13}$$

Where g and g_X are the gauge couplings of $SU(3)_L$ and $U(1)_X$.

The superpotential
$$W = \frac{W_2}{2} + \frac{W_3}{3} \tag{14}$$

The bilinear terms exclude lepton number violating terms

$$W_{2(bilinear)} = \mu_\eta \hat{\eta}\hat{\eta}' + \mu_\rho \hat{\rho}\hat{\rho}' + \mu_\chi \hat{\chi}\hat{\chi}' \tag{15}$$

The trilinear terms include only the following B-L conserving terms

$$\begin{aligned} W_{3(trilinear)} = {} & \sum_a k_a^e \hat{L}_a \hat{\rho}' \hat{l}_R^c + \sum_a k_a^P \hat{L}_a \hat{\chi}' \hat{P}_R^c + \sum_{i\alpha} [k_{i\alpha}^d \hat{Q}_i \hat{\eta} \hat{d}_{R\alpha}^c \\ & + k_{i\alpha}^u \hat{Q}_i \hat{\rho} \hat{u}_{R\alpha}^c] + \hat{Q}_3 \sum_\alpha [k_\alpha^b \hat{\rho}' \hat{d}_{R\alpha}^c + k_\alpha^b \hat{\eta}' \hat{u}_{R\alpha}^c] \\ & + k^T \hat{Q}_3 \hat{\chi}' \hat{T}_R^c + \sum_{i\beta} (k_{i\beta}^D \hat{Q}_i \hat{\chi} \hat{D}_{R\beta}^c) + f_1 \varepsilon \hat{\rho}\hat{\eta}\hat{\chi} + f_1' \varepsilon \hat{\rho}' \hat{\eta}' \hat{\chi}' \end{aligned} \tag{16}$$

The indices $a=1,2,3; i=1,2; \alpha=1,2,3$ and $\beta=1,2$ refer to three generations of leptons and quarks. We consider only the scalar sector of eqn. (16). The coefficients $\mu_\rho, \mu_\eta, \mu_\chi$ have mass dimensions while all the coefficients in W3 are dimensionless.

3.1. The Scalar Potential

The scalar potential of the model (involving $\phi_a = \eta, \eta', \rho, \rho', \chi, \chi'$ fields) is given by

$$V_H = V_F + V_D + V_{Soft} \tag{17}$$

Where $V_F = -L_F = \sum_a F_a^* F_a ; F_a^* = -\dfrac{\partial W}{\partial \phi_a}$ denote auxiliary fields.

$$\begin{aligned} V_F = \sum\nolimits_{ijk} [& \left| \frac{\mu_\eta}{2}\eta_i' + \frac{f_1}{3}\varepsilon_{ijk}\rho_j\chi_k \right|^2 + \left| \frac{\mu_\rho}{2}\rho_i' + \frac{f_1}{3}\varepsilon_{ijk}\chi_i\eta_k \right|^2 + \left| \frac{\mu_\chi}{2}\chi_i' + \frac{f_1}{3}\varepsilon_{ijk}\rho_j\eta_k \right|^2 \\ & + \left| \frac{\mu_\eta}{2}\eta_i + \frac{f_1'}{3}\varepsilon_{ijk}\rho_j'\chi_k' \right|^2 + \left| \frac{\mu_\rho}{2}\rho_i + \frac{f_1'}{3}\varepsilon_{ijk}\chi_i'\eta_k' \right|^2 + \left| \frac{\mu_\chi}{2}\chi_i + \frac{f_1'}{3}\varepsilon_{ijk}\rho_j'\eta_k' \right|^2 \end{aligned} \tag{18}$$

$$V_D = -L_D = \frac{1}{2}\left(D^\alpha D^\alpha + DD\right), \text{ where } D^\alpha = \frac{g}{2}\varphi_i^\dagger \lambda_{ij}^\alpha \varphi_j ; D = g_X \varphi_i^\dagger X \varphi_i \tag{19}$$

Here $\varphi = \eta, \rho, \chi; \lambda^\alpha (\alpha = 1,2,....8)$ are $SU(3)_L$ generators and X denotes $U(1)_X$ charge.

$$\begin{aligned} V_D = -L_D = & \frac{g_X^2}{2}(\rho^\dagger\rho - \chi^\dagger\chi - \rho'^\dagger\rho' + \chi'^\dagger\chi')^2 \\ & + \frac{g^2}{8}\sum\nolimits_{i,j}(\eta_i^\dagger\lambda_{ij}^a\eta_j + \rho_i^\dagger\lambda_{ij}^a\rho_j + \chi_i^\dagger\lambda_{ij}^a\chi_j - \eta_i'^\dagger\lambda_{ij}^{*a}\eta_j' - \rho_i'^\dagger\lambda_{ij}^{*a}\rho' - \chi_i'^\dagger\lambda_{ij}^{*a}\chi_j')^2 \end{aligned} \tag{20}$$

$$\begin{aligned} V_{Soft} = -L_{soft}^{scalar} = & m_\eta^2\eta^\dagger\eta + m_\rho^2\rho^\dagger\rho + m_\chi^2\chi^\dagger\chi + m_{\eta'}^2\eta'^\dagger\eta' + m_{\rho'}^2\rho'^\dagger\rho' + m_{\chi'}^2\chi'^\dagger\chi' \\ & + \kappa_1\varepsilon\eta\rho\chi + \kappa_1'\varepsilon\eta'\rho'\chi' + H.c. \end{aligned} \tag{21}$$

Where κ_1, κ_1' have dimension of mass. The complete scalar potential V_H given by eqn. (17) includes the neutral Higgs field $(X_i^0 = \rho, \eta, \chi, \rho', \eta', \chi')$ terms as well as charged Higgs terms

$$V_F = \frac{g_X^2}{2}(\rho^\dagger\rho - \chi^\dagger\chi - \rho'^\dagger\rho' + \chi'^\dagger\chi')^2 + \frac{g^2}{8}[\frac{4}{3}\{(\eta^\dagger\eta)^2 + (\rho^\dagger\rho)^2 + (\chi^\dagger\chi)^2\}$$
$$+\{(\eta'^\dagger\eta')^2 + (\rho'^\dagger\rho')^2 + (\chi'^\dagger\chi')^2\} - \frac{4}{3}\{ \tag{22}$$

We introduce the expansion of neutral scalar fields

$X_i^0 = \frac{1}{\sqrt{2}}(v_{X_i} + \xi_{X_i} + i\zeta_{X_i})$ where the vacuum expectation values [VEV] include

$$\begin{aligned} v = v_\eta = \langle \eta^0 \rangle, u = v_\rho = \langle \rho^0 \rangle, V = v_\chi = \langle \chi^0 \rangle; \\ v' = v_{\eta'} = \langle \eta'^0 \rangle, u' = v_{\rho'} = \langle \rho'^0 \rangle, V' = v_{\chi'} = \langle \chi'^0 \rangle \end{aligned} \tag{23}$$

The parameter $\tan\beta = u/v$ corresponds to the MSSM parameter $\tan\beta = \langle h_2 \rangle / \langle h_1 \rangle$ of vev's of two Higgs doublets. The CP even real fields are ξ_{X_i} while CP-odd imaginary fields are ζ_{X_i}.

3.2. The constraint equations in 3-3-1 model

The requirement of vanishing of linear terms in fields, $\frac{\partial V_H^{\min}}{\partial v_{X_i}} = 0$ give the constraint six equations. The structure of one of the constraint equation is,

$$m_\eta^2 + \frac{\mu_\eta^2}{4} = \frac{g^2}{12}[2v^2 - v'^2 - u^2 + u'^2 - V^2 + V'^2] - \frac{1}{6v\sqrt{2}}[\mu_\rho f_1 u'V - \mu_\chi f_1 uV'$$
$$+\mu_\eta f_1' u'V'] - \frac{|f_1|^2}{18}(V^2 + u^2) - \frac{\kappa_1}{2\sqrt{2}}\frac{uV}{v} \tag{24}$$

To obtain mass terms, we use terms quadratic in scalars

$$M_{ij}^2 = \frac{\partial^2 V_H^{\min}}{\partial\phi_i \partial\phi_j} \tag{25}$$

Where $\phi_i = \eta, \eta', \rho, \rho', \chi, \chi'$

We introduce the following parameters which show the deviation of the model from MSSM predictions for scalar masses.

$$
\begin{aligned}
X_\eta &= \frac{1}{6v\sqrt{2}}[\mu_\rho f_1 u'V - \mu_\chi f_1 uV' + \mu_\eta f_1' u'V'];\\
X'_\eta &= \frac{1}{6v'\sqrt{2}}[\mu_\rho f_1' uV' - \mu_\chi f_1' u'V + \mu_\eta f_1 uV];\\
X_\rho &= \frac{1}{6u\sqrt{2}}[\mu_\eta f_1 v'V - \mu_\chi f_1 vV + \mu_\rho f_1' v'V'];\\
X'_\rho &= \frac{1}{6u'\sqrt{2}}[\mu_\eta f_1' vV' - \mu_\chi f_1' v'V' + \mu_\rho f_1 vV];\\
X_\chi &= \frac{1}{6V\sqrt{2}}[\mu_\eta f_1' vu' - \mu_\chi f_1 uv + \mu_\rho f_1' uv']
\end{aligned}
\tag{26}
$$

4. MASS SPECTRUM IN THE NEUTRAL SCALAR SECTOR

The neutral scalar CP-even sector do not have Goldstone bosons and include three pairs of massive physical fields. In the basis of $(\xi_\eta, \xi_\rho, \xi_{\eta'}, \xi_{\rho'}, \xi_\chi, \xi_{\chi'})$ after imposing the constraint equations we obtain 6 x 6 mass matrix

$M_H^2 = \begin{pmatrix} M_{4\eta\rho}^2 & 0 \\ 0 & M_{2\chi\chi'}^2 \end{pmatrix}$ where the 4 x 4 and 2 x 2 submatrices are

$M_{4\eta\rho}^2 = \begin{pmatrix} M_{\eta\eta'}^2 & \cdots \\ & M_{2\chi\chi'}^2 \end{pmatrix}$ and $M_{2\chi\chi'}^2 = \begin{pmatrix} M_{\chi\chi}^2 & M_{\chi\chi'}^2 \\ M_{\chi\chi'}^2 & M_{\chi'\chi'}^2 \end{pmatrix}$

We consider the 4x4 mass matrix $M_{4\eta\rho}^2$ as two submatrices in the bases of (ξ_η, ξ_ρ) and $(\xi_{\eta'}, \xi_{\rho'})$ which give two pairs of massive neutral CP-even physical Higgs fields (H_1^0, h_1^0) and (H_2^0, h_2^0).

In terms of the mixing angles α_1, α_2

$$
\begin{aligned}
H_1^0 &= \cos\alpha_1 \xi_\eta + \sin\alpha_1 \xi_\rho, H_2^0 = \cos\alpha_2 \xi_{\eta'} + \sin\alpha_2 \xi_{\rho'}\\
h_1^0 &= -\sin\alpha_1 \xi_\eta + \cos\alpha_1 \xi_\rho, h_2^0 = -\sin\alpha_2 \xi_{\eta'} + \cos\alpha_2 \xi_{\rho'}
\end{aligned}
\tag{27}
$$

The physical masses of Higgs bosons are

$$\begin{aligned} m_{h_1}^2 &= \frac{1}{2}(M_1 - \sqrt{M_1^2 + M_1'}) \\ m_{H_1}^2 &= \frac{1}{2}(M_1 + \sqrt{M_1^2 + M_1'}) \end{aligned} \tag{28}$$

The physical masses of second pair of Higgs bosons are

$$\begin{aligned} m_{h_2}^2 &= \frac{1}{2}(M_2 - \sqrt{M_2^2 + M_2'}) \\ m_{H_2}^2 &= \frac{1}{2}(M_2 + \sqrt{M_2^2 + M_2'}) \end{aligned} \tag{29}$$

The physical masses of third pair of Higgs bosons are

$$\begin{aligned} m_{h_3}^2 &= \frac{1}{2}(M_3 - \sqrt{M_3^2 + M_3'}) \\ m_{H_3}^2 &= \frac{1}{2}(M_3 + \sqrt{M_3^2 + M_3'}) \end{aligned} \tag{30}$$

An interesting correspondence with MSSM soft bilinear term is that $\frac{\kappa_1}{2\sqrt{2}}V$ now plays the role of m_{12}^2 in the matrix elements for masses of lightest Higgs.

4.1. Mass spectrum in neutral pseudoscalar sector

The 6 x 6 mass square matrix in the pseudoscalar sector in basis

$(\zeta_\eta, \zeta_\rho, \zeta_{\eta'}, \zeta_{\rho'}, \zeta_\chi, \zeta_{\chi'})$

$M_{PH}^2 = \begin{pmatrix} M_{4\eta\rho}^2 & 0 \\ 0 & M_{2\chi\chi'}^2 \end{pmatrix}$ where the 4 x 4 and 2 x 2 submatrices are

$$M_{4\eta\rho}^2 = \begin{pmatrix} M_{\eta\eta}^2 \cdots & \\ & M_{2\rho'\rho'}^2 \end{pmatrix} \text{ and } M_{2\chi\chi'}^2 = \begin{pmatrix} M_{\chi\chi}^2 & M_{\chi\chi'}^2 \\ M_{\chi\chi'}^2 & M_{\chi'\chi'}^2 \end{pmatrix} \tag{31}$$

We consider the 4 x 4 matrix $M^2_{4\eta\rho}$ as two 2 x 2 submatrices with basis $(\zeta_\eta, \zeta_\rho), (\zeta_{\eta'}, \zeta_{\rho'})$. In the first basis, (ζ_η, ζ_ρ)

$$M^2_{\zeta_\eta \zeta_\rho} = \begin{pmatrix} A_P & B_P \\ B_P & C_P \end{pmatrix}$$

$$M^2_{\eta\eta} = -X_\eta - \frac{\kappa_1}{2\sqrt{2}} \frac{uV}{v} = A_P$$

$$M^2_{\eta\rho} = \frac{\mu_\chi f_1}{6\sqrt{2}} V' - \frac{\kappa_1}{2\sqrt{2}} V = B_P \tag{32}$$

$$M^2_{\rho\rho} = -X_\rho - \frac{\kappa_1}{2\sqrt{2}} \frac{vV}{u} = C_P$$

The non-vanishing trace and a vanishing determinant imply a massless Goldstone boson and a massive CP-odd pseudoscalar. Thus physical fields include a neutral CP-odd pseudoscalar A1 and a Goldstone boson G_1^0.

$$A_1^0 = \sin\beta_1 \zeta_\eta + \cos\beta_1 \zeta_\rho$$
$$G_1^0 = -\cos\beta_1 \zeta_\eta + \sin\beta_1 \zeta_\rho$$

Where $$\tan 2\beta_1 = \frac{2B_P}{A_P - C_P}; m^2_{A_1} = A_P + C_P \tag{33}$$

Similarly we have another neutral CP-odd pseudoscalar A_2, A_3, A_3' and a Goldstone boson G_2^0.

4.2. Mass spectrum for single charged scalar

In the single charged sector, two charged Goldstone bosons are obtained with one each for masses of ordinary and exotic gauge bosons along with six massive scalars.

The basis for 8 x 8 mass square matrix is $(\eta_1^+, \rho^+, \eta_1'^+, \rho'^+, \eta_2'^+, \chi'^+)$

$$M_{sc}^2 = \begin{pmatrix} M_{4\eta\rho}^2 & & \\ & M_{2\eta_2\chi}^2 & \\ & & M_{2\eta_2'\chi'}^2 \end{pmatrix}$$

where 4 x 4 matrix

$$M_{4\eta\rho}^2 = \begin{pmatrix} M_{\eta_1^+\eta_2^-}^2 & M_{\eta_1^+\rho^-}^2 & M_{\eta_1^+\eta_1'^-}^2 & M_{\eta_1^+\rho'^-}^2 \\ & M_{\rho^+\rho^-}^2 & M_{\rho^+\eta_1'^-}^2 & M_{\rho^+\rho'^-}^2 \\ & & M_{\eta_1'^+\eta_1'^-}^2 & M_{\eta_1'^+\rho'^-}^2 \\ & & & M_{\rho'^+\rho'^-}^2 \end{pmatrix} \tag{34}$$

We consider the base (η_1^+, ρ^+) for which 2 x 2 mass-squared matrix gives non-vanishing trace and vanishing determinant.The physical fields include a charged Goldstone boson and massive charged Higgs $(G_1^\pm, H_1^\pm)$ where

$$H_1^\pm = \sin\gamma_1\eta_1^\pm + \cos\gamma_1\rho^\pm ; G_1^\pm = -\cos\gamma_1\eta_1^\pm + \sin\gamma_1\rho^\pm$$

$$m_{H_1^\pm}^2 = M_{\eta_1^+\eta_1^-}^2 + M_{\rho^+\rho^-}^2 = \frac{g^2}{8}(u^2+v^2) - (X_\eta + X_\rho) - \frac{\kappa_1}{2\sqrt{2}}\left(\frac{uV}{v} + \frac{vV}{u}\right)$$

and
$$\tan 2\gamma_1 = \frac{2M_{\eta_1^+\rho^-}^2}{M_{\eta_1^+\eta_1^-}^2 - M_{\rho^+\rho^-}^2} \tag{35}$$

Similarly we have another sets of single charged massive Higgs and their physical fields also and Goldstone bosons like $(H_2^\pm, h_2^\pm)$, $(G_3^\pm, H_3^\pm)$ and $(H_4^\pm, h_4^\pm)$.

4.3. Mass spectrum for double charged scalars

The spectrum of doubly charged scalars include one doubly charged goldstone boson and three massive physical fields. The squared mass matrix in the basis $\left(\rho^{++},\ (\chi^{--})^*,\ (\rho'^{--})^*,\ \chi'^{++}\right)$

$$M_{dc}^2 = \begin{pmatrix} M^2_{\rho^{++}\rho^{--}} & M^2_{\rho^{++}\chi^{--}} & 0 & M^2_{\rho^{++}\chi'^{--}} \\ & M^2_{\chi^{++}\chi^{--}} & M^2_{\chi^{++}\rho'^{--}} & 0 \\ & & M^2_{\rho'^{++}\rho'^{--}} & M^2_{\rho'^{++}\chi'^{--}} \\ & & & M^2_{\chi'^{++}\chi'^{--}} \end{pmatrix} \quad (36)$$

We consider a vanishing determinant and non-vanishing trace of 2 x 2 matrix in the basis $\left(\rho^{++} \quad (\chi^{--})^*\right)$. The scalars iinclude Goldstone boson G^{++} and a physical charged Higgs H^{++} whose mass eigenvalue is

$$H^{++} = \sin\theta_1 \rho^{++} + \cos\theta_1 \chi^{++}; G^{++} = -\cos\theta_1 \rho^{++} + \sin\theta_1 \chi^{++}$$

$$m^2_{H^{++}} = M^2_{\rho^{++}\rho^{--}} + M^2_{\chi^{++}\chi^{--}} = \frac{g^2}{8}(V^2 + u^2) - (X_\rho + X_\chi) - \frac{\kappa_1}{2\sqrt{2}}\left(\frac{vV}{u} + \frac{uv}{V}\right)$$

and
$$\tan 2\theta_1 = \frac{2M^2_{\rho^{++}\chi^{--}}}{M^2_{\rho^{++}\rho^{--}} - M^2_{\chi^{++}\chi^{--}}} \quad (37)$$

From the 2 x 2 matrix in basis $\left(\rho'^{++} \quad , \chi'^{++}\right)$, two massive scalars $\left(H'^{++} \quad , h'^{++}\right)$ are obtained where H'^{++} is heavier than h'^{++}.

Constraints on Higgs scalar masses

In analogy with MSSM, we define $\tan\beta = u/v$ with the constraint

$$v^2 + u^2 + v'^2 + u'^2 = (246GeV)^2 \quad (38)$$

This follows from the gauge boson masses [11]. The relation for scalar mass-squared

$$m^2_{H^\pm} \rangle m^2_{A_1}; m^2_{h_1} + m^2_{H_1} \rangle m^2_{A_1}$$

5. NUMERICAL ANALYSIS

The numerical assignments for the various parameters used in obtaining the mass spectrum are as follows $f_1 = 1, f_1' = 10^{-6}$ are dimensionless parameters; $\kappa_1 = -\kappa_1' = -10GeV$; $-\mu_\eta = \mu_\rho = \mu_\chi = 1000GeV$;

The constraint on vev's, $v^2+u^2+v'^2+u'^2=(246GeV)^2$ is used along with $\tan\beta=u/v$; $u'=v'=V'=1GeV$.

The variation of the lightest scalar masses with $\tan\beta$ can be plotted for V= $1000GeV$. We find the condition $m_{H_1^\pm}\rangle m_{A_1}$ to be satisfied at all values of $\tan\beta$ along with the second condition. The value of the mass of lightest neutral Higgs is in the range of 114-128 GeV at $\tan\beta$ = 4 – 5, with V=1TeV. We can plot Higgs masses vs. V at $\tan\beta$ = 5. The value for Higgs masses at $V\le 2TeV$ are found to be within experimental ranges. Since $\kappa_1 V$ plays the role of μB in MSSM, a value of 10-15 TeV is obtained for this parameter at $\tan\beta$ = 5. The model does not allow large values for u/v within these ranges.

6. HIGGS COUPLINGS TO FERMIONS

The Yukawa Lagrangians that respects the 3-3-1 gauge symmetry are given by

$$L_Y^Q=\sum\nolimits_{i=1,2}Q_{iL}\left[\sum_{\alpha=1}^{3}(\kappa_{i\alpha}^{d}\eta\overline{d_{R\alpha}}+\kappa_{i\alpha}^{u}\rho\overline{u_{R\alpha}})+\sum_{\beta=1,2}\kappa_{i\beta}^{D}\chi\overline{D_{R\beta}}\right]$$
$$+Q_3\left[\sum_{\alpha=1}^{3}(\kappa_{\alpha}^{b}\rho'\overline{d_{R\alpha}}+\kappa_{\alpha}^{t}\eta'\overline{u_{R\alpha}})+\kappa^{T}\chi'\overline{T_R}\right]$$

$$-L_Y^l=\sum_{a=1}^{3}L_{aL}\left[\sum_{b=1}^{3}(\kappa_{ab}^{e}\rho' e_{Rb}^{c}+\kappa_{ab}^{P}\chi' P_{Rb}^{c}\right]$$

Here κ^q,κ^l are Yukawa coupling constants which are expressed in terms of fermion masses and vev's of Higgs fields as

$$\kappa_{i\alpha}^{d}=\sqrt{2}\frac{m_d}{v};\kappa_{i\alpha}^{u}=\sqrt{2}\frac{m_u}{u};\kappa_{i\alpha}^{b}=\sqrt{2}\frac{m_b}{u'};\kappa_{i\alpha}^{t}=\sqrt{2}\frac{m_t}{v'}$$
$$\kappa_{i\alpha}^{D}=\sqrt{2}\frac{m_D}{V};\kappa_{i\alpha}^{T}=\sqrt{2}\frac{m_T}{V'};\kappa_{ab}^{e}=\sqrt{2}\frac{m_e}{u'};\kappa_{ab}^{P}=\sqrt{2}\frac{m_P}{V'}$$

Unlike MSSM, the third generation quarks couple to adjoint scalars ρ',η' which give the massive physical Higgses (h_2,H_2,A_2) distinct from the lightest Higgs scalars (h_1, H_1, A_1). We can list out the Feynman rules for Higgs couplings for third generation quarks.

6.1. Higgs-boson couplings to gauge bosons

We consider the interactions of Higgs scalars with gauge bosons in this section. The gauge invariant kinetic term is

$$L_{Higgs}^{kinetic} = \left(\Delta_\mu \varphi\right)^\dagger \left(\Delta^\mu \varphi\right) + \left(\Delta_\mu \varphi'\right)^\dagger \left(\Delta^\mu \varphi'\right)$$

The covariant derivative Δ_μ in the kinetic term is defined model as [12]

$$\Delta_\mu = \partial_\mu - i\frac{g}{2}W_\mu^a T^a - ieA_\mu Q - i\frac{e}{\sin\theta_W \cos\theta_W}(T_{3L} - Q\sin^2_{\theta_W})Z_\mu$$
$$+ i\frac{g}{\cos\theta_W}\sqrt{(1-4\sin^2_{\theta_W})}\left[T_{8L} + \frac{\sqrt{3\sin^2_{\theta_W}}}{(1-4\sin^2_{\theta_W})}X\right]Z'_\mu$$

The trilinear and quartic vertices arise from the Lagrangian

$$L_{Higgs}^{kinetic} = L_{VVH} + L_{HHV} + L_{VVHH}$$

V_μ represents the 15-plet gauge bosons of $SU(3)_L$. For neutral scalars $H_i = \left(h_i^0, H_i^0\right)$, $i = 1,2,3$

- $$L_{VVH_1} = \frac{g^2}{2}V_\varphi\left[W_\mu^+ W^{-\nu} + Y_\mu^+ Y^{-\nu} + \frac{1}{2\cos^2_{\theta_W}}\left\{Z_\mu Z^\nu + \frac{1}{\sqrt{3}}Z_\mu Z'^\nu + \frac{1}{3}Z'_\mu Z'^\nu\right\}\right]g_{VVH_1}H_1$$

- $$L_{VVH_2} = \frac{g^2}{2}\left[W_\mu^+ W^{-\nu} + Y_\mu^+ Y^{-\nu} + Y_\mu^{++} Y^{\nu--} + \frac{1}{2\cos^2_{\theta_W}}\left\{Z_\mu Z^\nu + \frac{1}{\sqrt{3}}Z_\mu Z'^\nu + \frac{1}{3}Z'_\mu Z'^\nu\right\}\right]g_{VVH_2}H_2$$

- $$L_{VVH_3} = \frac{g^2}{2}\left[Y_\mu^+ Y^{-\nu} + Y_\mu^{++} Y^{\nu--} + \frac{2\sin^2_{\theta_W}}{1-4\sin^2_{\theta_W}}Z'_\mu Z'^\nu\right]g_{VVH_3}H_3$$

7. RESULT AND CONCLUSIONS

We have proposed a supersymmetric version of a 3-3-1 model with exotic charged quarks and leptons. [11], which can be embedded in $SU(4)_C \otimes SU(4)_W$ gauge symmetry[12,16]. The weak isospin group $SU(2)_L$ is a subgroup of a larger $SU(3)_L \otimes U(1)_{Y_1}$ symmetry group. The mass spectrum and couplings of the extended Higgs sector is studied. A possible set of parameters of the model are obtained which are consistent with other supersymmetric versions of 3-3-1 models[8]. The lightest neutral scalar h_1^0, pseudoscalar A_1 and single-charged scalar $H_1^{\pm}$ are found to be in agreement with MSSM predictions with a trilinear soft term introduced in 3-3-1 models as $\varepsilon\eta\rho\chi$ [8]. This is distinct from a study of this model with only bilinear soft terms which allows very light Higgs masses[13]. The exotic scalars and fermions are predicted to be at TeV energies.The spectrum for super particles will be presented elsewhere. Different graphs and enormous number of application will be done in near future. LHC had produced lots of data. We can perform a huge level of higgs phenomenology with the help of LHC data what so ever will suit to this supersymmetric version of 3-3-1 model.

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