Binary Jaya Algorithm Based Optimal Placement of Phasor Measurement Units for Power System Observability

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Abstract: Owing to the technological advancement in communication areas, presently, the security and the reliability of operating power system is very much possible on real-time platform. Phasor measurement units (PMUs) are based on these advance communication technology and are one of the most important equipment for electric utility, now a days, helpful in maintaining a healthy operating power system. It is necessary requirement to deploy PMUs in whole power system to achieve full observability but it is not economically feasible. Therefore, placement of PMUs at strategic locations is required to be obtained optimally while preserving the full observability of the system. In this paper, the solution of optimal PMUs placement problem (OPPP) is addressed using a novel binary Jaya algorithm. The complete power system observability with maximum measurement redundancy is considered as the performance index in OPPP. To test the supremacy and accuracy of the proposed algorithm, different standard test systems are examined for solving OPPP and the obtained results are compared with other state-of-art algorithms reported in the literature. The analysis of the results shows that the proposed algorithm is equally good or better in solving the problem when compared to other reported algorithms.

Keywords: Binary Jaya algorithm (BJA); measurement redundancy (MR); observability; optimal PMUs placement (OPP); Phasor measurement units (PMUs).

1. INTRODUCTION

Ever increasing power demand in today's world is threatening the security and reliability of operating power system by over stressing the power networks available in this competitive market [1]. A real-time monitoring of the system under these conditions is helpful in enhancing the security and reliability of the system. State estimation is an excellent tool to estimate the real time states of the system thus, providing a real time monitoring of the system [2]. With present advancement in communication technology, real-time monitoring is possible to a greater extent. Phasor measurement units (PMUs) are newly evolved equipment which take synchronised measurements of the power system [3]. PMUs utilize the global positioning system (GPS) to produce time synchronized measurements of voltage and current phasors of the power system networks [4].

In a power system network if a PMU is located at a bus, then, it will measure the voltage phasor of the bus along with the current phasors of all the branches connected to the bus. In this case, the bus with PMU is said to be directly observable. The voltage phasors of the adjacent bus connected to the bus with PMU can be computed since the current of the branch connecting them is known from the measurements. So, in this way, it can be said that the other bus is indirectly observable. Thus, it can be concluded that the PMU equipped bus and all its surrounding buses (connected through a line) are observable. This provides a platform for an optimization problem termed as optimal PMUs placement problem (OPPP). Its main objective is to find optimal number of PMUs to be located in a system so as to have full observability of the system for state estimation.

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Several research works, in the past, are reported in literature to solve an OPPP. Some of the important contributions in this regard are integer linear programming (ILP) approach [5, 6], Tabu search [7], iterative local search (ILS) [8], weighted least squares algorithm (WLSA) [9], non-dominated sorting genetic algorithm (NSGA) [10], adaptive clonal algorithm (ACA) [11], binary particle swarm optimization (BPSO) [12, 13], differential evolution (DE) [14, 15], biogeography based optimization (BBO) [16], information-theoritic approach (ITA) [17] and binary teacher-learner-based optimization (BTLBO) [18].

This paper presents the solution of OPPP using binary Jaya algorithm (BJA). Jaya algorithm (JA) is proposed by Rao in 2016 [19]. It is a very simple algorithm based on the movement of the solution towards best solution and away from the worst solution. At the same time, it is free from any algorithm-specific parameters (though it contains some common control parameters). Due to advantages associated with this algorithm like simplicity in its implementation and being free from algorithm-specific parameters, it has been applied to various engineering problems such as dimensional optimization of a micro-channel heat sink [20], solution of complex constrained design optimization problems [21], surface grinding process optimization [22], tea category identification [23], etc. In this work, the binary version of JA is proposed to solve OPPP. The man objective of OPPP formulated here is the minimization of the number of required PMUs in a network while preserving the full observability and at the same time maximizing the measurement redundancy. Different standard test systems are considered with or without zero-injection buses to test the robustness of the proposed algorithm. The superiority and efficacy of the algorithm is proved by comparative assessment with other algorithms presented in the literature.

2. PROBLEM FORMULATION

A. Basic Formulation

A simple OPPP is defined as obtaining the optimal number of PMUs required to be installed at appropriate locations so as to have full network observability. The decision vector X for PMU placement in system having N_{BUS} number of total buses is defined as:

$$[X]_i = x_i = \begin{cases} 1 & \text{if a PMU is installed at ith bus} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The basic objective function is:

$$\min \sum_{i}^{N_{BUS}} x_i, \quad i = 1, 2, ..., N_{BUS} \quad (2)$$

such that $F(X) \geq b$

where $b = [111...]^T$ = an unit vector of length N_{BUS}. $F(X)$ is the observability constraint vector calculated as:

$$F(X) = \begin{cases} \text{nonzero,} & \text{if the respective bus is observable} \\ & \text{w.r.t. given measurement set} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Full network observability is ensured by constraint vector $F(X)$. A binary connectivity matrix A having information of the bus connected to each other in the power system is used in the formation of constraint vector $F(X)$. The different elements of matrix A is expressed as:

$$[A]_{p,q} = a_{p,q} = \begin{cases} 1 & \text{if } p = q \\ 1 & \text{if bus } p \text{ is connected to bus } q \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Thus, the constraint vector F(X) is defined as:

$$F(X) = AX \geq b \tag{5}$$

The particular entry of vector F(X) at the ith bus of the system can be denoted as f_i and calculated as:

$$f_i = a_{i,1}x_1 + ... + a_{i,i}x_i + ... + a_{i,N_{BUS}}x_{N_{BUS}} \tag{6}$$

It can be easily understood from (6) that if $a_{i,q} = 0$; $q = 1, 2, ..., N_{BUS}$ then, the product $a_{i,q}x_q$ is equal to zero and thus, it will vanish from (6). At the same time, if any x_q appearing in f_i is nonzero along with a nonzero $a_{i,q}$ then their product will be nonzero which signifies that f_i is observable. Again, if all entries of f_i in F are such nonzero then, the system considered is fully observable.

B. Measurement Redundancy

Measurement redundancy, MR is an important aspect in solving OPPP. It is the total count of observability of a bus by PMUs either directly or indirectly. Hence, to obtain full observability of the system, MR value of each bus should be at least one. It can be said that greater the value of MR, the system is more closer to maintain its complete observability. So, the maximization of MR is included in (2) to form a modified objective function as described in [24]. The resultant objective function is expressed as:

$$\min \left(\sum_{i=1}^{N_{BUS}} x_i \right) + \omega \left(\sum_{i=1}^{N_{BUS}} MR_i^{Ideal} - \sum_{i=1}^{N_{BUS}} MR_i \right) \tag{7}$$

where $\omega \in \mathfrak{R}$ is a weighing factor which can be suitably selected to compare two parts of (7) in terms of magnitude. MR_i^{Ideal} is the total count of ith bus being ideally observed where as MR_i is the total count of ith bus being actually observed. The second component of (7) calculates the difference between ideal and actual counts of each bus in the system being observed. The minimization of this difference leads to attain higher value of MR i.e., the maximization of MR takes place with the minimization of this difference. The system constraints for (7) remains same as defined in (2), (3), (5) and (6).

C. Zero Injection Buses (ZIBs)

Zero injection buses (ZIBs) play a very important role in minimization of required PMUs for full observability of the system. The buses in the system are considered as ZIBs if they do not have any generation or load. That means, the current injection into the system at these ZIBs is equal to zero. This information is useful in reducing the required PMUs while preserving the system observability. Consider a 4-bus example as shown in Figure 1 with node 1 as ZIB. Applying KCL at bus 1 yields
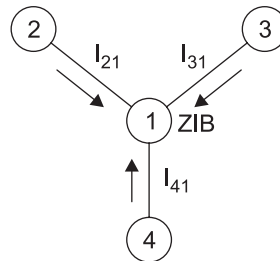
$$I_{21} + I_{31} + I_{41} = 0 \tag{8}$$

Figure 1: ZIB Model

In this paper, minimization of (7) is considered as the objective function for OPPP with or without consideration of ZIBs.

3.　BINARY JAYA ALGORITHM (BJA)

A.　Brief Discussion of Jaya Algorithm

Jaya algorithm (JA) is one of the recently proposed population based algorithm for the solution of unconstrained and constrained optimization algorithm [19]. This algorithm has a specific advantage of simplicity in its application to a problem and it is free from any algorithm-specific parameters. Although common control parameters are present in JA but the tedious task of tuning algorithm-specific parameters for different applications is completely omitted. These advantages provide a great scope for application of JA to different engineering problems. The structure of this algorithm is based on the movement of obtained solution towards best solution and away from worst solution.

Let there are N number of candidate solutions (i.e. population size, $i = 1, 2, ..., N$) and D number of decision variables (i.e., $j = 1, 2, ..., D$) for each candidate solution. The initial population is generated randomly in between their boundaries of size (N × D). At any nth iteration, the best and worst solutions obtained are denoted as x^n_{best} and x^n_{worst}, respectively. If $x^n_{i,j}$ is the jth decision variable of the ith candidate solution during nth iteration then, $x^n_{i,j}$ is updated according to:

$$X^n_{i,j} = x^n_{i,j} + \alpha^n_j\left(x^n_{j,\text{best}} - \left|x^n_{i,j}\right|\right) - \beta^n_j\left(x^n_{j,\text{worst}} - \left|x^n_{i,j}\right|\right) \tag{9}$$

where $x^n_{j,\text{best}}$ and $x^n_{j,\text{worst}}$ are the best and worst candidate solution of the jth decision variable, respectively and $X^n_{i,j}$ is the updated value of $x^n_{i,j}$. α^n_j and β^n_j are two random numbers during nth iteration in the range [0, 1]. A greedy selection approach is adopted to select the best candidate solution taking part in the next iteration. It is expressed as:

$$Y^n_i = \begin{cases} X^n_i & \text{if} \quad f(X^n_i) < f(x^n_i) \\ x^n_i & \text{if} \quad f(X^n_i) > f(x^n_i) \end{cases} \tag{10}$$

where Y^n_i is the selected candidate solution taking part in $(n+1)$th iteration. $f(X^n_i)$ and $f(x^n_i)$ are respective fitness function value. This process of updation and selection of candidate solution is continued until any termination criteria is met.

B.　Binary Jaya Algorithm (BJA)

Binary Jaya algorithm (BJA) is the binary version of JA where the updated candidate solution $X^n_{i,j}$ is represented in binary form. Each decision variable of the initial population x is randomly generated in the range (0, 1) and then, converted into binary form according to following rule:

$$x_{i,j} = \begin{cases} 1 & \text{if rand ( )} \geq 0.5 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $x_{i,j}$ is the binary form of the jth decision variable of the ith candidate solution.

The value of $x_{i,j}$ obtained in (11) is updated according to (9). The updated value $X_{i,j}$ is then transformed to a value in the range (0, 1) by the means of 'tanh' transformation. It is expressed mathematically as:

$$\tanh\left(\left|X_{i,j}\right|\right) = \frac{e^{\left(\left|2X_{i,j}\right|\right)} - 1}{e^{\left(\left|2X_{i,j}\right|\right)} + 1} \tag{12}$$

The transformed value of $X_{i,j}$ obtained using (12) is then represented in binary form according to:

$$X_{i,j} = \begin{cases} 1 & \text{if rand ( )} < \tanh\left(\left\|X_{i,j}\right\|\right) \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

A greedy selection approach described by (10) is adopted to form new updated candidate solution taking part in the next iteration. This process is continued until any termination criteria is met.

C.   Implementation of BJA to OPPP

The objective function described by (7) is considered for solving OPPP. If N is the total number of solution candidate and the total number of buses in the considered test system is N_{BUS} then, the size of initial population is $N \times N_{BUS}$. The various steps involved in the solution of OPPP are as follows:

Step 1: Generate initial population in binary form using (11) randomly.

Step 2: Evaluate objective function value described in for each candidate solution in the population.

Step 3: Find best and worst candidate solution based on objective function evaluation from step 2.

Step 4: Update each decision variable of all candidate solution using (9).

Step 5: Evaluate the value of each updated decision variable obtained in step 4 according to (12).

Step 6: Replace the value of each decision variable obtained in step 5 by a binary number according to (13).

Step 7: Obtain the candidate solution taking part in next iteration using (10).

Step 8: Go to step 2 and repeat until any stopping criteria is met.

4.   RESULTS AND DISCUSSION

The proposed algorithm is executed to solve OPPP keeping in view of meeting the objective of placing minimum number of PMUs at strategic locations so as to have full network observability and maximizing MR of the whole system. To confirm the success of the proposed algorithm, it is tested on IEEE 14-bus, 30-bus, New England 39-bus, IEEE 57-bus and 118-bus test system. The method has taken account of PMU measurements for full observability of the considered system while operating under normal conditions only. The presence of ZIBs is considered in all test systems to indicate the lesser number of PMUs requirement to attain full observability as when compared to system without consideration of ZIBs.

Table 1 shows the different test systems with total number of transmission lines and total number of ZIBs with their respective locations. Table 2 shows the minimum required PMUs to have full observability of the system and their respective locations along with maximum measurement redundancy MR under normal operating conditions without any consideration of ZIBs obtained using proposed algorithm. From this table, it is affirmed that for IEEE 14-bus test system, 4 PMUs are required to have complete observability when ZIBs are not considered. The bus locations are 2, 6, 7 and 9 and maximum measurement redundancy MR is found to 19. Likewise, the least number of PMUs required to have full observability for IEEE 30-bus, NE 39-bus, IEEE 57-bus and IEEE 118-bus test systems are 10, 13, 16 and 32 with maximum measurement redundancy MR equal to 52, 52, 72 and 184, respectively. The results obtained after solving OPPP from the proposed method is compared with other available methods reported in the literature. Table 3 shows the comparative lists of minimum required PMUs in order to preserve full observability for different systems. The results obtained from the proposed method are equal to others listed in different literature for considered systems except for IEEE 57-bus test system where the proposed method and Ref. [25] are better than others. The optimal required PMUs for this test system is 16.

Simulation results for optimal placement of PMUs and their corresponding locations while considering the presence of ZIBs are tabulated in Table 4. From the table, it can be easily observed that for IEEE 14-bus test system, the optimal required PMUs are 3 and their locations are 2, 6 and 9. The maximum measurement redundancy MR is obtained as 16 in this case. Similarly, the analysis for IEEE 30-bus, NE 39-bus, IEEE 57-bus and IEEE 118-bus test system reveals that optimal number of PMUs required are 7, 8, 11 and 28 with maximum measurement redundancy MR equal to 41, 43, 59 and 156, respectively. The effect of the presence of ZIBs in the system can easily be understood from the reduced number of optimal PMUs required for full observability. Comparative results of the proposed method considering ZIBs in the system is listed in Table 5. These results show that the proposed algorithm is capable of solving OPPP efficiently and is comparably better or equal to others reported in the literature.

The discussion from the results suggest that the proposed algorithm can easily be applied to complex problem to find the global optima. The basic reason behind the success of this algorithm is its simple structure which reduces the complexity that lies within the algorithm itself and being free from any algorithm-specific parameters which reduces the effort of tuning the parameters for its proper applicability to a problem. Thus, the binary version of the algorithm is very much capable in solving any integer decision variable problem.

Table 1
Specifications of the test system under consideration

Test system	Transmission lines	Number of ZIBs	Location of ZIBs
IEEE 14-Bus	20	1	7
IEEE 30-Bus	41	6	6, 9, 22, 25, 27, 28
NE 39-Bus	46	12	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22
IEEE 57-Bus	80	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
IEEE 118-Bus	186	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81

Table 2
Optimal PMUs placement obtained without considering ZIBs
under normal operating condition

Test system	Optimal number Of PMUs	Locations of PMUs	MR
IEEE 14-Bus	4	2, 6, 7, 9	19
IEEE 30-Bus	10	1, 5, 8, 9, 10, 12, 15, 20, 25, 29	52
NE 39-Bus	13	2, 6, 9, 10, 12, 14, 17, 19, 22, 23, 25, 29, 34	52
IEEE 57-Bus	16	1, 6, 9, 15, 19, 22, 25, 28, 32, 36, 38, 41, 47, 51, 53, 57	72
IEEE 118-Bus	32	3, 5, 9, 12, 15, 17, 21, 26, 23, 28, 30, 36, 40, 44, 46, 51, 54, 57, 62, 64, 68, 71, 75, 80, 85, 86, 91, 94, 101, 105, 110, 114	184

Table 3
Comparison of optimal PMUs placement with available algorithms reported in
literature without considering ZIBs

Method	Test system				
	IEEE 14-Bus	IEEE 30-Bus	NE 39-Bus	IEEE 57-Bus	IEEE 118-Bus
Proposed method	4	10	13	16	32
Ref. [14]	4	10	13	17	--

Method	Test system				
	IEEE 14-Bus	*IEEE 30-Bus*	*NE 39-Bus*	*IEEE 57-Bus*	*IEEE 118-Bus*
Ref. [25]	4	10	13	16	32
Ref. [26]	4	10	--	17	32
Ref. [27]	4	10	13	17	32
Ref. [28]	4	10	13	--	--
Ref. [8]	4	--	--	17	32
Ref. [12]	4	10	--	17	32

'--' indicates the unavailability of data

Table 4
Optimal PMUs placement obtained considering ZIBs
under normal operating condition

Test system	Optimal number Of PMUs	Locations of PMUs	MR
IEEE 14-Bus	3	2, 6, 9	16
IEEE 30-Bus	7	1, 7, 10, 12, 19, 24, 27	41
NE 39-Bus	8	3, 8, 13, 16, 20, 23, 25, 29	43
IEEE 57-Bus	11	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	59
IEEE 118-Bus	28	2, 8, 11, 12, 17, 21, 25, 28, 33, 34, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	156

Table 5
Comparison of optimal PMUs placement with available algorithms reported
in literature considering ZIBs

Method	Test system				
	IEEE 14-Bus	*IEEE 30-Bus*	*NE 39-Bus*	*IEEE 57-Bus*	*IEEE 118-Bus*
Proposed method	3	7	8	11	28
Ref. [27]	3	7	8	11	28
Ref. [26]	3	7	--	12	28
Ref. [1]	3	7	8	--	--
Ref. [12]	3	7	--	13	29
Ref. [5]	3	7	--	11	--
Ref. [29]	3	--	10	13	--
Ref. [13]	3	7	8	11	28

'--' indicates the unavailability of data

5.   CONCLUSION

In this paper, binary Jaya Algorithm (BJA) based optimal PMU placement to have full system observability is presented. The OPPP is a problem to place PMUs at strategic locations so as to have full observability. Being simple in structure and free from any algorithm-specific parameter, the proposed algorithm is easily implemented to the problem. The obtained solution from the proposed method fulfils the requirement of maintaining full system observability along with maximization of the measurement redundancy MR. Different standard test systems are considered for the simulation with inclusion and exclusion of ZIBs. The simulation results confirms the efficacy of the proposed algorithm in solving OPPP and proves its comparative capability of obtaining optimal solution to existing methods.

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