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A Novel Hyperchaotic Hyperjerk System with Two Nonlinearities, its Analysis, Adaptive Control and Synchronization via Backstepping Control Method

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Abstract: A hyperjerk system is a dynamical system, which is modelled by an n–th order ordinary differential equation with $n \geq 4$ describing the time evolution of a single scalar variable. Equivalently, using a chain of integrators, a hyperjerk system can be modelled as a system of n first order ordinary differential equations with $n \geq 4$. In this research work, a novel 4-D hyperchaotic hyperjerk system with two nonlinearities has been proposed, and its qualitative properties have been detailed. The Lyapunov exponents of the novel hyperjerk system are obtained as $L_1 = 0.13403$, $L_2 = 0.03849$, $L_3 = 0$ and $L_4 = -1.20579$. The Lyapunov dimension of the novel hyperjerk system is obtained as $D_L = 3.1431$. Next, an adaptive backstepping controller is designed to stabilize the novel hyperjerk chaotic system with three unknown system parameters. Furthermore, an adaptive backstepping controller is designed to achieve global hyperchaos synchronization of the identical novel hyperjerk systems with three unknown system parameters. MATLAB plots are shown to illustrate all the main results of this research work.

Keywords: Chaos, hyperchaos, chaotic systems, hyperchaotic systems, chaos control, chaos synchronization, adaptive control, backstepping control, stability.

1. INTRODUCTION

A *chaotic system* is commonly defined as a nonlinear dissipative dynamical system that is highly sensitive to even small perturbations in its initial conditions [1]. In other words, a chaotic system is a nonlinear dynamical system with at least one positive Lyapunov exponent. Some paradigms of chaotic systems can be listed as Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], Tigan system [10], etc.

In the last two decades, many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12-13], Vaidyanathan systems [14-33], Pehlivan systems [34-35], Pham systems [36-37], Jafari system [38], etc.

Hyperchaotic systems are the chaotic systems with more than one positive Lyapunov exponent. They have important applications in control and communication engineering.

Some recently discovered 4-D hyperchaotic systems are hyperchaotic Vaidyanathan systems [39-40], hyperchaotic Vaidyanathan-Azar system [41], etc. A 5-D hyperchaotic system with three positive Lyapunov exponents was also recently found [42].

Chaos theory has several applications in a variety of fields such as oscillators [43-44], chemical reactors [45-58], biology [59-80], ecology [81-82], neural networks [83-84], robotics [85-86], memristors [87-89], fuzzy systems [90-91], etc.

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The problem of control of a chaotic system is to find a state feedback control law to stabilize a chaotic system around its unstable equilibrium [92-93]. Some popular methods for chaos control are active control [94-98], adaptive control [99-100], sliding mode control [101-103], etc.

Chaos synchronization problem can be stated as follows. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The synchronization of chaotic systems has applications in secure communications [104-107], cryptosystems [108-109], encryption [110-111], etc.

The chaos synchronization problem has been paid great attention in the literature and a variety of impressive approaches have been proposed. Since the pioneering work by Pecora and Carroll [112-113] for the chaos synchronization problem, many different methods have been proposed in the control literature such as active control method [114-142], adaptive control method [143-149], sampled-data feedback control method [150-151], time-delay feedback approach [152], backstepping method [153-164], sliding mode control method [165-173], etc.

In mechanics, if the scalar $x(t)$ represents the *position* of a moving object at time t, then the first derivative $\dot{x}(t)$ represents the *velocity*, the second derivative $\ddot{x}(t)$ represents the *acceleration* and the third derivative $\dddot{x}(t)$ represents the *jerk* or *jolt*. In mechanics, a *jerk system* is described an explicit third order ODE describing the time evolution of a single scalar variable x according to the dynamics

$$\frac{d^3x}{dt^3} = f\left(\frac{d^2x}{dt^2}, \frac{dx}{dt}, x\right) \tag{1}$$

A particularly simple example of a jerk system is the famous Coullet system [174] given by

$$\frac{d^3x}{dt^3} + a\frac{d^2x}{dt^2} + \frac{dx}{dt} = g(x) \tag{2}$$

where $g(x)$ is a nonlinear function such as $g(x) = b(x^2 - 1)$, which exhibits chaos for $a = 0.6$ and $b = 0.58$.

A generalization of the jerk dynamics (1) is given by the dynamics

$$\frac{d^{(n)}x}{dt^n} = f\left(\frac{d^{(n-1)}x}{dt^{n-1}}, \ldots, \frac{dx}{dt}, x\right), \quad (n \geq 4) \tag{3}$$

An ordinary differential equation of the form (3) is called a *hyperjerk* system since it involves time derivatives of a jerk function [175].

In this research work, we propose a novel 4-D hyperchaotic hyperjerk system by adding a quadratic nonlinearity to the Chlouverakis-Sprott hyperjerk system [176]. Our novel hyperjerk system thus consists of two nonlinearities. First, we detail the qualitative properties of the novel hyperchaotic hyperjerk system. Then we obtain the Lyapunov exponents of the novel hyperjerk system as $L_1 = 0.13403$, $L_2 = 0.03849$, $L_3 = 0$ and $L_4 = -1.20579$. The presence of two positive Lyapunov exponents demonstrates that the novel hyperjerk system is hyperchaotic. The Lyapunov dimension of the novel hyperjerk system is obtained as $D_L = 3.1431$.

Next, this paper derives an adaptive backstepping control law that stabilizes the novel hyperjerk system when the system parameters are unknown. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems. This paper also derives an adaptive backstepping control law that achieves global chaos synchronization of the identical 4-D novel hyperjerk systems with unknown parameters.

All the main adaptive control results derived in this paper are established using Lyapunov stability theory [177]. MATLAB simulations are depicted to illustrate the phase portraits of the novel hyperjerk system, adaptive stabilization and synchronization of the novel hyperjerk system with unknown parameters. This paper concludes with a summary of the main results for the novel 4-D hyperchaotic hyperjerk system with two nonlinearities.

2. A NOVEL 4-D HYPERCHAOTIC HYPERJERK SYSTEM

In [176], Chlouverakis and Sprott discovered a simple hyperchaotic hyperjerk system given by the dynamics

$$\frac{d^4x}{dt^4}+\frac{d^3x}{dt^3}x^4+A\frac{d^2x}{dt^2}+\frac{dx}{dt}+x=0 \tag{4}$$

In system form, the differential equation (4) can be expressed as

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = x_3\\ \dot{x}_3 = x_4\\ \dot{x}_4 = -x_1 - x_2 - Ax_3 - x_1^4 x_4\end{cases} \tag{5}$$

When A = 3.6, the Chlouverakis-Sprott hyperjerk system (5) exhibits hyperchaos with Lyapunov exponents $L_1 = 0.132$, $L_2 = 0.035$, $L_3 = 0$ and $L_4 = -1.25$.

The Lyapunov dimension of the Chlouverakis-Sprott hyperjerk system (5) is calculated as

$$D_L = 3+\frac{L_1+L_2+L_3}{|L_4|} = 3.1336 \tag{6}$$

In this research work, we propose a novel hyperjerk system by adding a quadratic nonlinearity to the Chlouverakis-Sprott hyperjerk system (5) and with a different set of values for the system parameters.

Our novel hyperjerk system is given in system form as

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = x_3\\ \dot{x}_3 = x_4\\ \dot{x}_4 = -x_1 - x_2 - ax_3 - bx_2^2 - cx_1^4 x_4\end{cases} \tag{7}$$

where a, b, c are constant, positive, parameters.

In this research work, we shall show that the hyperjerk system (7) is *hyperchaotic* for the parameter values

$$a = 3.7,\quad b = 0.05,\quad c = 1.3 \tag{8}$$

For the parameter values in (8), the Lyapunov exponents of the novel hyperjerk system (7) are obtained as

$$L_1 = 0.13403,\quad L_2 = 0.03849,\quad L_3 = 0,\quad L_4 = -1.20579 \tag{9}$$

From the LE spectrum given in (9), it is easily seen that the hyperjerk system (7) is hyperchaotic since it has two positive exponents. Also, the maximal Lyapunov exponent (MLE) of our novel hyperjerk system (7) is $L_1 = 0.13403$, which is greater than the MLE of the Chlouverakis-Sprott hyperjerk system (5).

Also, the Lyapunov dimension of the novel hyperjerk system (7) is calculated as

$$D_L = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1431 \tag{10}$$

We observe that the Lyapunov dimension of the novel hyperjerk system (7) is greater than the Lyapunov dimension of the Chlouverakis-Sprott hyperjerk system (5). This shows that the novel hyperjerk system (7) exhibits more complex behaviour than the Chlouverakis-Sprott hyperjerk system (5).

For numerical simulations, we take the initial values of the novel hyperjerk system (7) as

$$x_1(0) = 0.5, \quad x_2(0) = 0.5, \quad x_3(0) = 0.5, \quad x_4(0) = 0.5 \tag{11}$$

Figures 1-4 depict the 3-D projections of the 4-D novel hyperjerk system (7) on (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

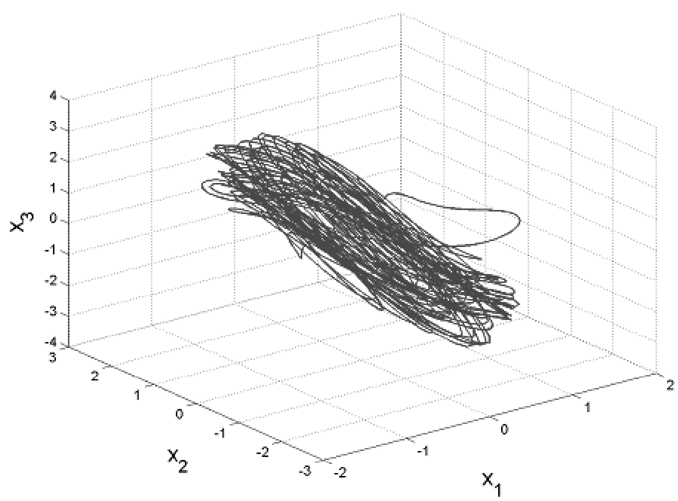

Figure 1: 3-D projection of the 4-D novel hyperjerk system on the (x_1, x_2, x_3) space

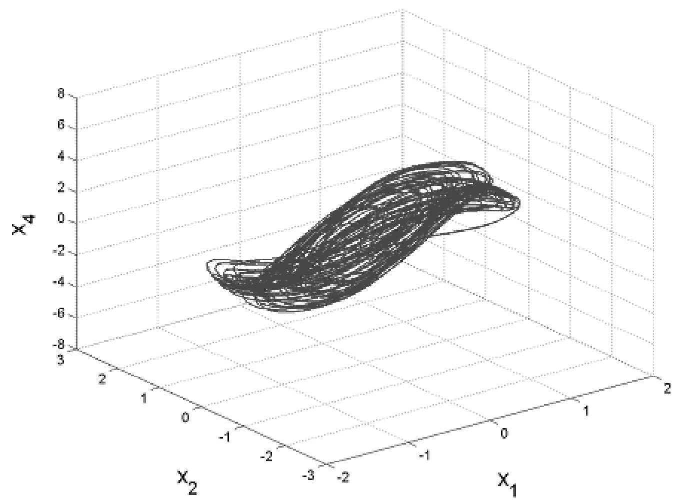

Figure 2: 3-D projection of the 4-D novel hyperjerk system on the (x_1, x_2, x_4) space

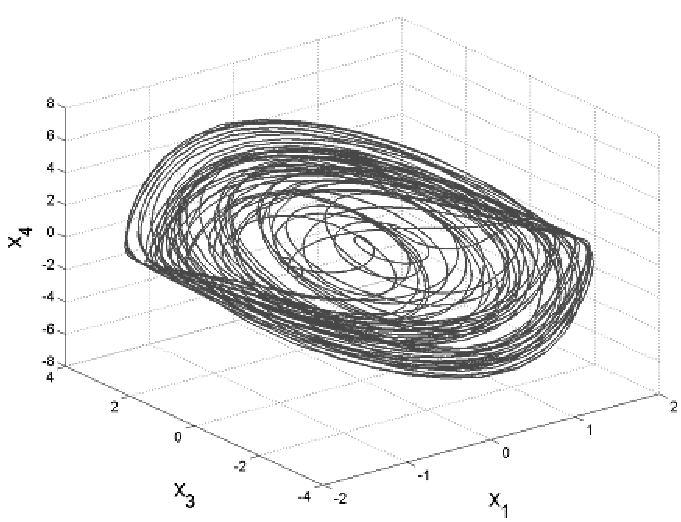

Figure 3: 3-D projection of the 4-D novel hyperjerk system on the (x_1, x_3, x_4) space

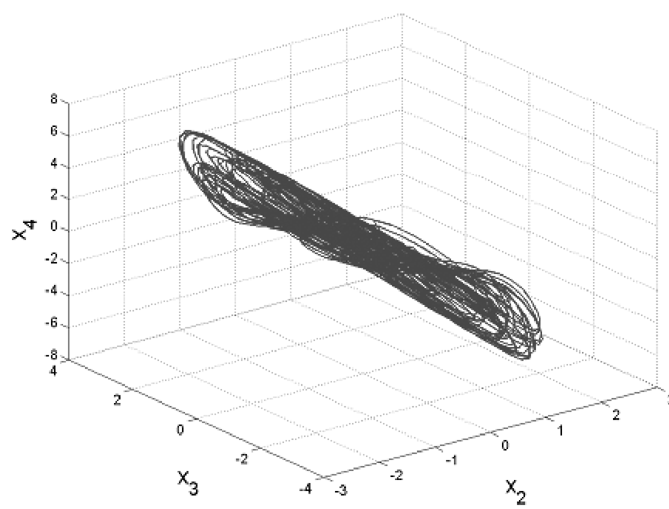

Figure 4: 3-D projection of the 4-D novel hyperjerk system on the (x_2, x_3, x_4) space

3. PROPERTIES OF THE NOVEL 4-D HYPERJERK SYSTEM

In this section, we detail the qualitative properties of the novel 4-D hyperchaotic hyperjerk system (7), which is described in Section 2.

3.1. Equilibrium Points

The equilibrium points of the novel 4-D hyperjerk system (7) are obtained by solving the following system of equations

$$\begin{cases} x_2 & = & 0 \\ x_3 & = & 0 \\ x_4 & = & 0 \\ -x_1 - x_2 - ax_3 - bx_2^2 - cx_1^4 x_4 & = & 0 \end{cases} \tag{12}$$

We take the parameter values as in the hyperchaotic case, *viz.*

$$a = 3.7, \quad b = 0.05, \quad c = 1.3 \tag{13}$$

Solving the equations (12) using the values (13), we obtain the unique equilibrium point

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

The Jacobian matrix of the novel hyperjerk system (7) at E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -3.7 & 0 \end{bmatrix} \tag{15}$$

The eigenvalues of J_0 are numerically obtained as

$$\lambda_{1,2} = 0.1550 \pm 1.8674\text{i}, \quad \lambda_{3,4} = -0.1550 \pm 0.5107i \tag{16}$$

This shows that the equilibrium E_0 is a saddle-focus, which is unstable.

3.2. Lyapunov Exponents

We take the parameter values of the novel hyperjerk system (7) as

$$a = 3.7, \quad b = 0.05, \quad c = 1.3 \tag{17}$$

We take the initial conditions of the novel hyperjerk system (7) as

$$x_1(0) = 0.5, \quad x_2(0) = 0.5, \quad x_3(0) = 0.5, \quad x_4(0) = 0.5 \tag{18}$$

The Lyapunov exponents of the system (7) are numerically obtained with MATLAB as

$$L_1 = 0.13403, \quad L_2 = 0.03489, \quad L_3 = 0, \quad L_4 = -1.20579 \tag{19}$$

Thus, the hyperjerk system (7) is chaotic, since it has two positive Lyapunov exponents.

The MATLAB plot of the Lyapunov exponents of the novel chaotic system (1) is depicted in Figure 5. From this figure, we see that the maximal Lyapunov exponent (MLE) of the novel hyperjerk system (7) is obtained as $L_1 = 0.13403$.

Since $L_1 + L_2 + L_3 + L_4 = -1.0333 < 0$, the novel hyperjerk system (7) is dissipative.

3.3. Lyapunov Dimension

The Lyapunov dimension of the novel 4-D hyperjerk system (7) is determined as

$$D_L = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1431 \tag{20}$$

which is fractional.

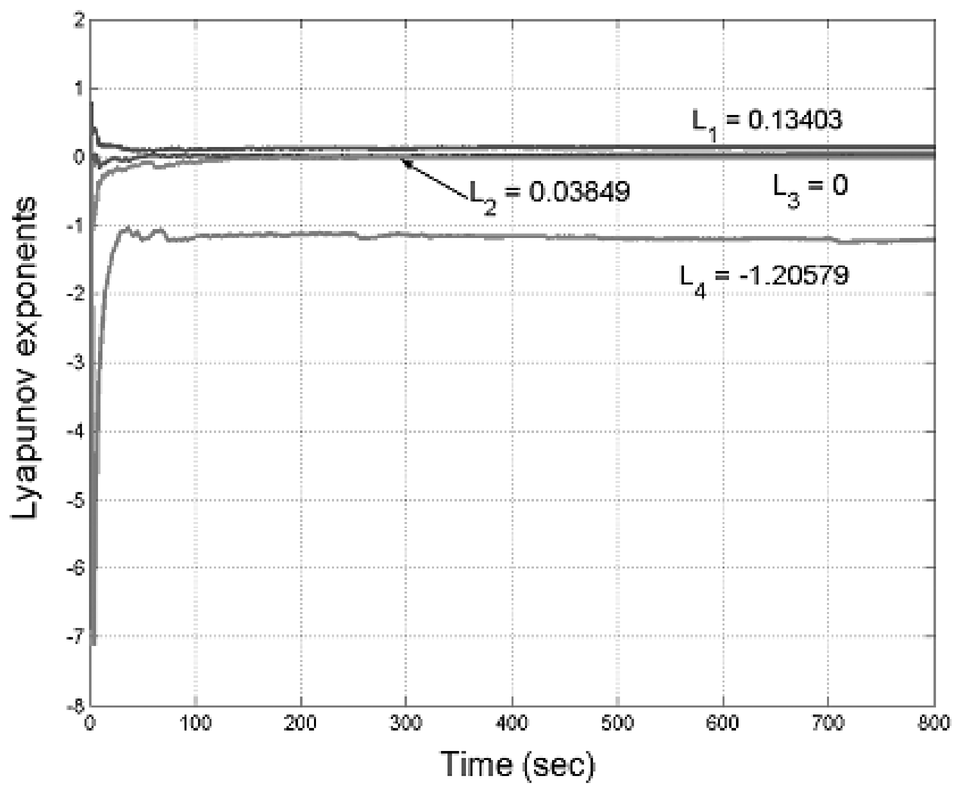

Figure 5: Lyapunov exponents of the novel 4-D hyperjerk system

4. ADAPTIVE CONTROL OF THE NOVEL 4-D HYPERJERK SYSTEM WITH UNKNOWN PARAMETERS

In this section, we design new results for the adaptive controller to stabilize the novel 4-D hyperjerk system with unknown parameters for all initial conditions.

Thus, we consider the novel 4-D hyperjerk system with a single control given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_1 - x_2 - ax_3 - bx_2^2 - cx_1^4 x_4 + u \end{cases} \tag{21}$$

where x_1, x_2, x_3, x_4 are state variables, a, b, c are constant, unknown, parameters of the system and u is a backstepping controller to be designed using estimates of the unknown system parameters.

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{22}$$

Differentiating (22) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{23}$$

Next, we shall state and prove the main result of this section.

Theorem 1. The 4-D novel hyperjerk system (21), with unknown parameters a, b and c, is globally and exponentially stabilized by the adaptive feedback control law

$$u(t) = -4x_1 - 9x_2 - [9 - \hat{a}(t)]x_3 - 4x_4 + \hat{b}(t)x_2^2 + \hat{c}(t)x_1^4 x_4 - kz_4, \tag{24}$$

where $k > 0$ is a gain constant,

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4 \tag{25}$$

and the update law for the parameter estimates $\hat{a}(t)$, $\hat{b}(t)$, $\hat{c}(t)$ is given by

$$\begin{cases} \dot{\hat{a}} = -x_3 z_4 \\ \dot{\hat{b}} = -x_2^2 z_4 \\ \dot{\hat{c}} = -x_1^4 x_4 z_4 \end{cases} \tag{26}$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory [177].

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{27}$$

where

$$z_1 = x_1 \tag{28}$$

Differentiating V_1 along the dynamics (21), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \tag{29}$$

Now, we define

$$z_2 = x_1 + x_2 \tag{30}$$

Using (30), we can simplify the equation (29) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{31}$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{32}$$

Differentiating V_2 along the dynamics (21), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \tag{33}$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \tag{34}$$

Using (34), we can simplify the equation (33) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{35}$$

Thirdly, we define a quadratic Lyapunov function

$$V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right) \tag{36}$$

Differentiating V_3 along the dynamics (21), we get

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3\left(3x_1 + 5x_2 + 3x_3 + x_4\right) \tag{37}$$

Now, we define

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4 \tag{38}$$

Using (38), we can simplify the equation (37) as

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3 z_4 \tag{39}$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, z_4, e_a, e_b, e_c) = V_3(z_1, z_2, z_3) + \frac{1}{2} z_4^2 + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2\right) \tag{40}$$

which is a positive definite function on R^7.

Differentiating V along the dynamics (21) and (23), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4\left(z_4 + z_3 + \dot{z}_4\right) - e_a\dot{\hat{a}} - e_b\dot{\hat{b}} - e_c\dot{\hat{c}} \tag{41}$$

Eq. (41) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 S - e_a\dot{\hat{a}} - e_b\dot{\hat{b}} - e_c\dot{\hat{c}} \tag{42}$$

where

$$S = z_4 + z_3 + \dot{z}_4 = z_4 + z_3 + (3\dot{x}_1 + 5\dot{x}_2 + 3\dot{x}_3 + \dot{x}_4) \tag{43}$$

A simple calculation gives

$$S = 4x_1 + 9x_2 + (9-a)x_3 + 4x_4 - bx_2^2 - cx_1^4 x_4 + u \tag{44}$$

Substituting the adaptive control law (24) into (44), we obtain

$$S = -[a-\hat{a}(t)]x_3 - [b-\hat{b}(t)]x_2^2 - [c-\hat{c}(t)]x_1^4 x_4 - kz_4 \tag{45}$$

Using the definitions (22), we can simplify the equation (45) as

$$S = -e_a x_3 - e_b x_2^2 - e_c x_1^4 x_4 - kz_4 \tag{46}$$

Substituting the value of from (46) into (42), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+k)z_4^2 + e_a\left(-x_3 z_4 - \dot{\hat{a}}\right) + e_b\left(-x_2^2 z_4 - \dot{\hat{b}}\right) + e_c\left(-x_1^4 x_4 z_4 - \dot{\hat{c}}\right) \tag{47}$$

Substituting the parameter update law (26) into (47), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+k)z_4^2, \tag{48}$$

which is a negative semi-definite function on R^7.

From (48), it follows that the vector $z(t) = (z_1(t), z_2(t), z_3(t), z_4(t))$, and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$ are globally bounded, i.e.

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & z_4(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix} \in L_\infty \tag{49}$$

Also, it follows from (48) that

$$\dot{V} \le -z_1^2 - z_2^2 - z_3^2 - z_4^2 = -\|\boldsymbol{z}\|^2 \tag{50}$$

That is,

$$\|\boldsymbol{z}\|^2 \le -\dot{V} \tag{51}$$

Integrating the inequality (51) from 0 to t, we get

$$\int_0^t \|\boldsymbol{z}(\tau)\|^2 d\tau \le V(0) - V(t) \tag{52}$$

From (52), it follows that $z(t) \in L_2$.

From (21), it can be deduced that $\dot{z}(t) \in L_\infty$.

Thus, using Barbalat's lemma [177], we conclude that $z(t) \to 0$ as $t \to \infty$ exponentially for all initial conditions $z(0) \in R^4$.

Hence, it follows that $x(t) \to 0$ as $t \to \infty$ exponentially for all initial conditions $x(0) \in R^4$.

This completes the proof. ■

4.1. Numerical Simulations

The classical fourth-order Runge-Kutta method with step-size $h = 10^{-8}$ is used to solve the systems of differential equations (21) and (26), when the adaptive control law (24) is applied.

The parameter values of the novel 4-D hyperjerk system (21) are taken as in the hyperchaotic case, *i.e.* $a = 3.7$, $b = 0.05$ and $c = 1.3$. The positive gain constant k is taken as $k = 9$.

The initial conditions of the novel 4-D hyperjerk system (21) are taken as

$$x_1(0) = -8.7,\ \ x_2(0) = 12.4,\ \ x_3(0) = -9.2,\ \ x_4(0) = 15.1 \tag{53}$$

The initial conditions of the parameter estimates are taken as

$$\hat{a}(0) = 7.9,\ \ \hat{b}(0) = 5.2,\ \ \hat{c}(0) = 10.3 \tag{54}$$

Figure 6 shows the exponential convergence of the controlled states $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$.

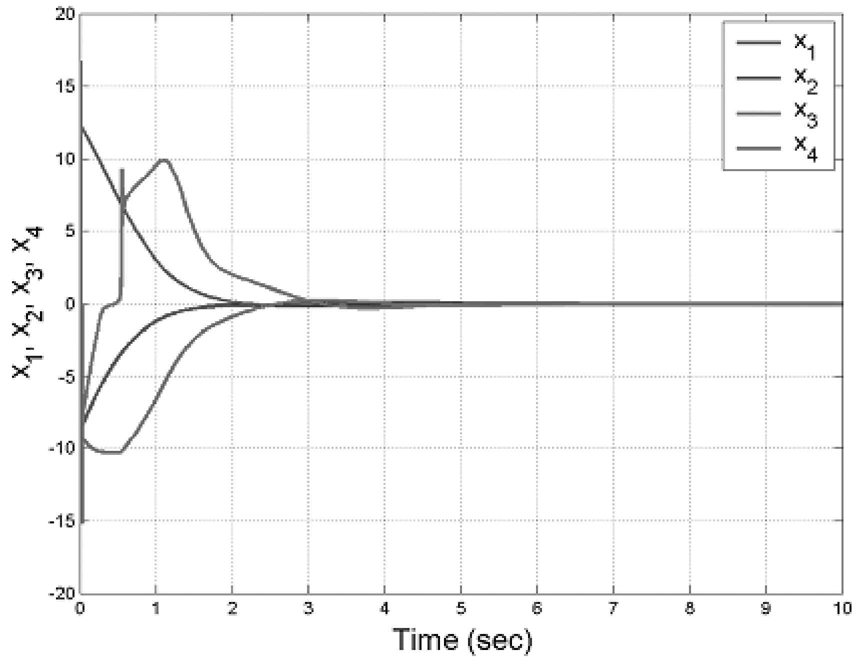

Figure 6: Time history of the controlled novel hyperjerk system

5. ADAPTIVE SYNCHRONIZATION OF THE NOVEL 4-D HYPERJERK SYSTEMS WITH UNKNOWN PARAMETERS

In this section, we use backstepping control method to derive an adaptive control law for globally and exponentially synchronizing the identical novel 4-D hyperjerk systems with unknown parameters.

As the master system, we consider the novel 4-D hyperjerk system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -x_1 - x_2 - ax_3 - bx_2^2 - cx_1^4 x_4 \end{cases} \tag{55}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c are constant, unknown, parameters.

As the slave system, we consider the novel 4-D hyperjerk system with a control given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = y_4 \\ \dot{y}_4 = -y_1 - y_2 - ay_3 - by_2^2 - cy_1^4 y_4 + u \end{cases} \tag{56}$$

where y_1, y_2, y_3, y_4 are the states and u is a backstepping control to be determined using estimates of the unknown system parameters.

We define the synchronization error between the hyperjerk systems (55) and (56) as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4 \end{cases} \tag{57}$$

Then the error dynamics is easily obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = -e_1 - e_2 - ae_3 - b(y_2^2 - x_2^2) - c(y_1^4 y_4 - x_1^4 x_4) + u \end{cases} \tag{58}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{59}$$

Differentiating (59) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{60}$$

Next, we shall state and prove the main result of this section.

Theorem 2. The 4-D novel hyperjerk systems (55) and (56) with unknown system parameters are globally and exponentially synchronized by the adaptive feedback control law

$$u(t) = -4e_1 - 9e_2 - [9 - \hat{a}(t)]e_3 - 4e_4 + \hat{b}(t)(y_2^2 - x_2^2) + \hat{c}(t)(y_1^4 y_4 - x_1^4 x_4) - kz_4 \tag{61}$$

where $k > 0$ is a gain constant,

$$z_4 = 3e_1 + 5e_2 + 3e_3 + e_4 \tag{62}$$

and the update law for the parameter estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t)$ is given by

$$\begin{cases} \dot{\hat{a}} = -e_3 z_4 \\ \dot{\hat{b}} = -(y_2^2 - x_2^2) z_4 \\ \dot{\hat{c}} = -(y_1^4 y_4 - x_1^4 x_4) z_4 \end{cases} \tag{63}$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory [177].

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{64}$$

where

$$z_1 = e_1 \tag{65}$$

Differentiating V_1 along the dynamics (58), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \tag{66}$$

Now, we define

$$z_2 = e_1 + e_2 \tag{67}$$

Using (67), we can simplify the equation (66) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \tag{68}$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{69}$$

Differentiating V_2 along the dynamics (58), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{70}$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \tag{71}$$

Using (71), we can simplify the equation (70) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \tag{72}$$

Thirdly, we define a quadratic Lyapunov function

$$V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2\right) \tag{73}$$

Differentiating V_3 along the dynamics (58), we get

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3\left(3e_1 + 5e_2 + 3e_3 + e_4\right) \tag{74}$$

Now, we define

$$z_4 = 3x_1 + 5x_2 + 3x_3 + x_4 \tag{75}$$

Using (75), we can simplify the equation (74) as

$$\dot{V}_3 = -z_1^2 - z_2^2 - z_3^2 + z_3 z_4 \tag{76}$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, z_4, e_a, e_b, e_c) = V_3(z_1, z_2, z_3) + \frac{1}{2} z_4^2 + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2\right) \tag{77}$$

which is a positive definite function on R^7.

Differentiating V along the dynamics (58), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4\left(z_4 + z_3 + \dot{z}_4\right) - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{78}$$

Eq. (78) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - z_4^2 + z_4 S - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} - e_c \dot{\hat{c}} \tag{79}$$

where

$$S = z_4 + z_3 + \dot{z}_4 = z_4 + z_3 + (3\dot{e}_1 + 5\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4) \tag{80}$$

A simple calculation gives

$$S = 4e_1 + 9e_2 + (9-a)e_3 + 4e_4 - b(y_2^2 - x_2^2) - c(y_1^4 y_4 - x_1^4 x_4) + u \tag{81}$$

Substituting the adaptive control law (61) into (81), we obtain

$$S = -[a - \hat{a}(t)]e_3 - [b - \hat{b}(t)](y_2^2 - x_2^2) - [c - \hat{c}(t)](y_1^4 y_4 - x_1^4 x_4) - kz_4 \tag{82}$$

Using the definitions (59), we can simplify the equation (82) as

$$S = -e_a e_3 - e_b (y_2^2 - x_2^2) - e_c (y_1^4 y_4 - x_1^4 x_4) - kz_4 \tag{83}$$

Substituting the value of from (83) into (79), we obtain

$$\begin{aligned} \dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+k)z_4^2 + e_a\left[-e_3 z_4 - \dot{\hat{a}}\right] + e_b\left[-\left(y_2^2 - x_2^2\right)z_4 - \dot{\hat{b}}\right] \\ + e_c\left[-(y_1^4 y_4 - x_1^4 x_4)z_4 - \dot{\hat{c}}\right] \end{aligned} \tag{84}$$

Substituting the parameter update law (63) into (84), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - (1+k)z_4^2, \tag{85}$$

which is a negative semi-definite function on R^7.

From (85), it follows that the vector $z(t) = (z_1(t), z_2(t), z_3(t), z_4(t))$, and the parameter estimation error $(e_a(t), e_b(t), e_c(t))$, are globally bounded, i.e.

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & z_4(t) & e_a(t) & e_b(t) & e_c(t) \end{bmatrix} \in L_\infty \tag{86}$$

Also, it follows from (85) that

$$\dot{V} \le -z_1^2 - z_2^2 - z_3^2 - z_4^2 = -\|\mathbf{z}\|^2 \tag{87}$$

That is,

$$\|\mathbf{z}\|^2 \le -\dot{V} \tag{88}$$

Integrating the inequality (88) from 0 to t, we get

$$\int_0^t \|\mathbf{z}(\tau)\|^2 d\tau \le V(0) - V(t) \tag{83}$$

From (83), it follows that $z(t) \in L_2$. From (58), it can be deduced that $\dot{z}(t) \in L_\infty$.

Thus, using Barbalat's lemma [177], we conclude that $z(t) \to 0$ as $t \to \infty$ exponentially for all initial conditions $z(0) \in R^4$. Hence, it follows that $e(t) \to 0$ as $t \to \infty$ exponentially for all initial conditions This completes the proof. ■

5.1. Numerical Simulations

The parameter values of the novel 4-D hyperjerk systems (55) and (56) are taken as in the hyperchaotic case, *i.e.* $a = 3.7$, $b = 0.05$ and $c = 1.3$.

The positive gain constant k is taken as $k = 9$.

The initial conditions of the novel hyperjerk system (55) are taken as

$$x_1(0) = 3.2, \quad x_2(0) = -2.4, \quad x_3(0) = -1.8, \quad x_4(0) = 0.1 \tag{84}$$

The initial conditions of the novel hyperjerk system (56) are taken as

$$y_1(0) = -2.7, \quad y_2(0) = 1.3, \quad y_3(0) = -5.4, \quad y_4(0) = 2.9 \tag{85}$$

The initial conditions of the parameter estimates are taken as

$$\hat{a}(0) = 6.8, \quad \hat{b}(0) = 5.4, \quad \hat{c}(0) = 9.2 \tag{86}$$

Figures 7-10 show the complete synchronization of the novel 4-D hyperjerk systems (55) and (56).

Figure 11 shows the time-history of the synchronization errors e_1, e_2, e_3, e_4.

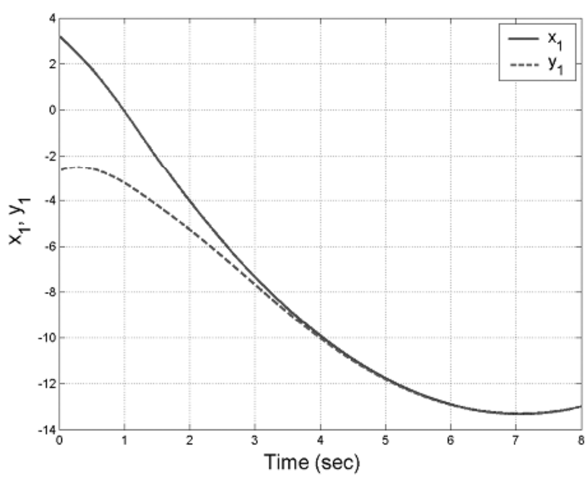

Figure 7: Complete synchronization of the states x_1 and y_1

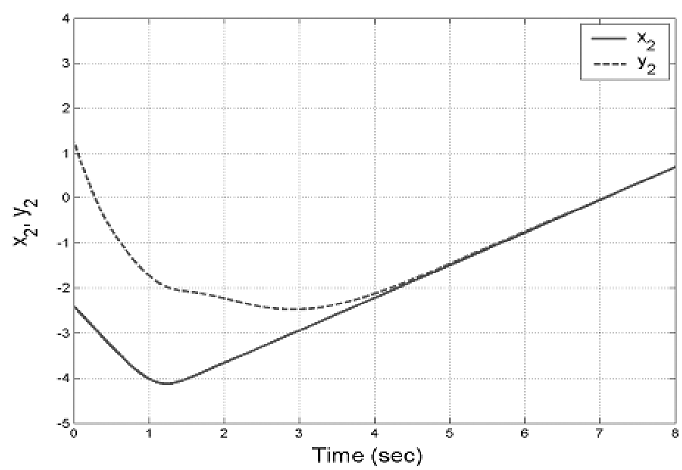

Figure 8: Complete synchronization of the states x_2 and y_2

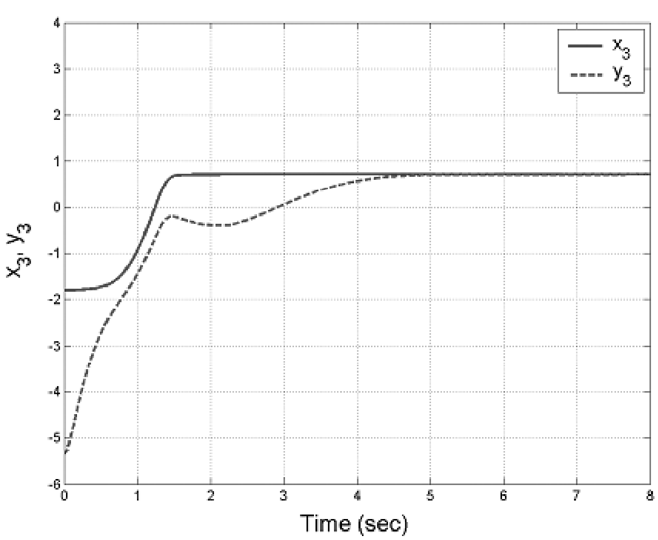

Figure 9: Complete synchronization of the states x_3 and y_3

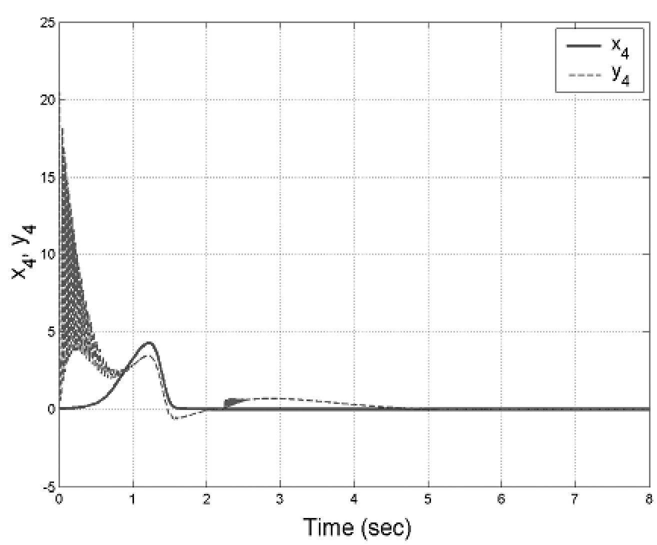

Figure 10: Complete synchronization of the states x_4 and y_4

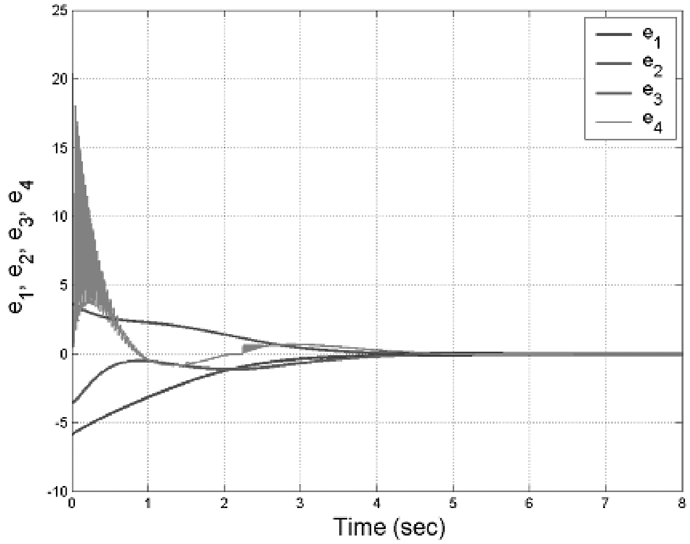

Figure 11: Time history of the chaos synchronization errors e_1, e_2, e_3, e_4

6. CONCLUSIONS

In this paper, a novel 4-D hyperchaotic hyperjerk system with two nonlinearities has been proposed, and its qualitative properties have been detailed. Next, an adaptive backstepping controller was designed to stabilize the novel hyperjerk chaotic system with three unknown system parameters. Furthermore, an adaptive backstepping controller was designed to achieve global hyperchaos synchronization of the identical novel hyperjerk systems with three unknown system parameters. MATLAB have been shown to illustrate all the main results of this work.

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