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### Performance Evaluation of Cubature and Unscented Kalman Filters For Target Tracking

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**Abstract:** Cubature Kalman filter (CKF) and Unscented Kalman filter (UKF) use Gaussian assumed density approximations. Simo Sarkka has shown that UKF is a generalized one of CKF. The results for substantial performance analysis of UKF and CKF in Monte-Carlo simulation is carried out for Bearings-only Tracking (BOT) problem. It is observed that UKF is better than that of CKF for BOT problem. **Keywords:** Target tracking, Target motion analysis, Estimation Theory, Sonar

#### 1. INTRODUCTION

Target motion analysis in 2D plane sea waters is carried out by using only bearing measurements. The target radiates noisy sonar bearing which is monitored by an observer in a passive listening mode. The acoustic signal emitted from the target is received by hull mounted sonar in the observer platform. For range observability, S-manuever performed by observer is shown in Fig. 1. The range values are not obtained in BOT and the target states related to bearing measurements are non-linear. The target is considered at constant velocity in the present work [1].

The Maximum Likelihood Estimator [1-3] is used in batch processing. As MLE requires some initial estimate alternatively under take some arbitrary initial estimate where the results in pseudo linear estimator (PLE) are evaluated for the same. PLE does not require any initial estimate. PLE generates a reasonably accurate estimate for initialization of MLE but it offers bias in the estimates. This work is a compliment to Lindgren's [4], Aidala's [5], Song & Speyer's [6] and Grossman's [7] contributions.

In case of linear models, the conventional Kalman filter works better. Unfortunately, the usefulness of Kalman filter is limited to some extent. In present estimation, problems like target tracking which uses only bearing measurements is totally non-linear model. The Kalman filter is suitable only for linear models but for non-linear models EKF (Extended Kalman Filter) is used. In MGEKF (Modified Gain Extended Kalman Filter) [6] the gain is modified to eliminate the divergence problem which occurs in EKF. The idea behind MGEKF is

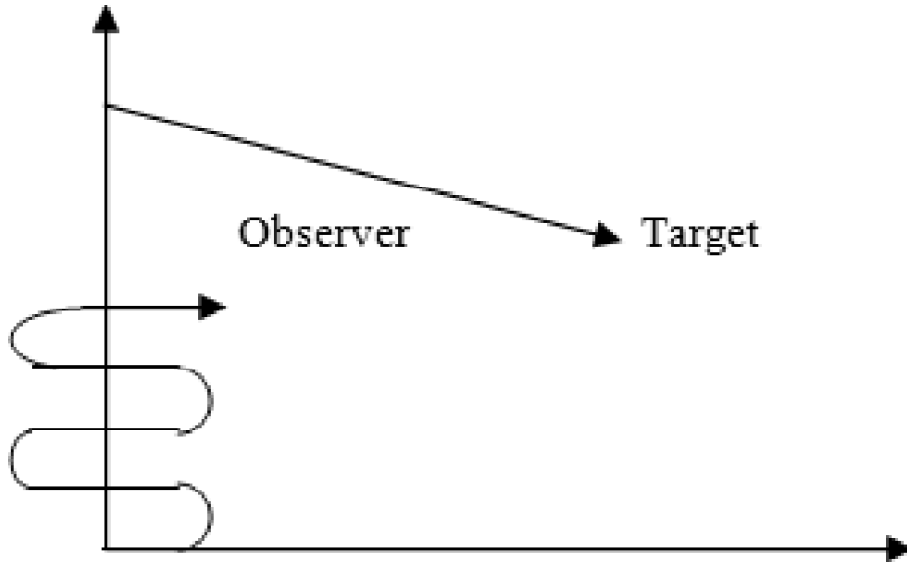


Figure 1: S-maneuver by an observer

that non-linearities are “modifiable”. By comparing PLE, MGEKF exhibits certain similarities. The gain in the PLE depends on past and previous measurements. The gain function is modified in a simple version is available in reference [8]. The algorithm of bearing-only measurement is extended for underwater application by considering both bearing and altitude measurements [9].

When the noise in the measurements has Gaussian probability density functions then Unscented Kalman Filter (UKF) works better. For target tracking in non-linear problem, UKF can be used only if the measurements are Gaussian [12-14]. At true state the PDF (Probability Density Function) predicts accurately when the particles approach infinite number and produce the best estimation of state for non-Gaussian noise and non-linear applications like BOT which increases computational effort. In this field many number of researchers from Target Motion Analysis (TMA) community are contributing their work with effectively and accurately.

In this paper, the main aim is to choose a better algorithm for BOT (Bearings-Only Tracking) from existing algorithm. The Assumption made in this paper is that the measurements in bearings are contaminated with the white noise. In highly sophisticated signal processing algorithms which are equipped with latest sonar's the assumption is valid. For these applications the usage of PF is unnecessary where EKF or UKF is enough to use. As compared with EKF and UKF perform so well which is used here safely. By using Gaussian assumed density approximations the non-linear filtering will be solved. UKF is a generalized form of Cubature Kalman Filter (CKF) which is demonstrated by Jouni, et.al.,[15-17]. The authors in this paper would like to carry out the performance evaluation of UKF with that of CKF for Bearings-Only Tracking [18-20]. The performance evaluation is considered by taking several tactical scenarios.

It is considered that observer as a submarine or ship. When the observer approaches target at i) low ( $0^{\circ}$ - $30^{\circ}$ ), ii) medium ( $31^{\circ}$ - $40^{\circ}$ ) and iii) high ( $41^{\circ}$ - $90^{\circ}$ ) ATBs the algorithm is evaluated. The observer is not interested when the target range is opening. In this research paper, the performance evaluation of CKF and UKF for different scenarios at various ATBs is implemented.

The Mathematical modeling in section II and III cover the measurements, the target state and the algorithms of UKF and CKF. In section IV performance evaluation of different algorithms are described. Section V covers summary and conclusion.

## 2. MATHEMATICAL MODELLING

Let equation of the target state be

$$X_s(k) = \begin{bmatrix} \dot{x}(k) & \dot{y}(k) & R_x(k) & R_y(k) \end{bmatrix}^T \quad (1)$$

Measurement in bearing is modeled as

$$B_m(k) = \tan^{-1} \left( \frac{R_x(k)}{R_y(k)} \right) + \zeta(k) \quad (2)$$

where  $\zeta(k)$  is zero mean with variance  $\sigma_b^2$ . The non-linearity in equation (2) is linearized. The measurement matrix is Range values are not known, so in equation (3) the estimated values are used.

$$H(k+1) = \begin{bmatrix} 0 & 0 & \hat{R}_y(k+1|k)/R^2(k+1|k) & \hat{R}_x(k+1|k)/R^2(k+1|k) \end{bmatrix} \quad (3)$$

State dynamic equation of target is

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma w(k) \quad (4)$$

where  $\phi$ , transition state, is

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (5)$$

$$b(k+1) = \begin{bmatrix} 0 & 0 & -(x_0(k+1) - x_0(k)) & -(y_0(k+1) - y_0(k)) \end{bmatrix} \quad (6)$$

$$\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \quad (7)$$

$w(k)$  indicates mean zero noise.

$$E[\Gamma(k)w(k)w^T(k)\Gamma^T(k)] = Q\delta_{ij} \quad (8)$$

where

$$\delta_{ij} = \sigma_w^2 \text{ if } i=j \quad (9)$$

= 0 otherwise

$$Q = \begin{bmatrix} t_s^2 & 0 & t_s^3/2 & 0 \\ 0 & t_s^2 & 0 & t_s^3/2 \\ t_s^3/2 & 0 & t_s^4/8 & 0 \\ 0 & t_s^3/2 & 0 & t_s^4/8 \end{bmatrix} \quad (10)$$

Assume measurement and plant noises are uncorrelated.

### (A) Unscented Transformation

The simple method where the statistical properties of RV is calculated through a nonlinear transformation is known as Unscented transformation. The Random Variable  $x$  having mean  $\bar{x}$  its covariance  $p_x$  with dimension  $L$ . The function  $y = g(x)$  is a non-linear.  $\chi_i$  is formed as in [10].

$$\begin{aligned} \chi_0 &= \bar{x} \\ \chi_i &= \bar{x} + (\sqrt{(L + \lambda)P_x})_i, i=1, \dots, L \\ \chi_i &= \bar{x} - (\sqrt{(L + \lambda)P_x})_{i-L}, i=L+1, \dots, 2L \\ W_0^{(m)} &= \lambda / (L + \lambda) \\ W_0^{(c)} &= \lambda / (L + \lambda) + (1 - \alpha^2 + \beta) \\ W_i^{(m)} &= W_i^{(c)} / \{2(L + \lambda)\}, i=1, \dots, 2L \end{aligned} \quad (11)$$

Where

$$\lambda = \alpha^2(L + k) - L$$

The vectors  $\chi_i$  are distribute as follows,

$$y_i = g(\chi_i), i=1, \dots, 2L \quad (12)$$

Given mean and covariance of  $y$

$$\bar{y} \approx \sum_{i=0}^{2L} W_i^{(m)} y_i \quad (13)$$

$$p_y \approx \sum_{i=0}^{2L} W_i^{(c)} \{y_i - \bar{y}\} \{y_i - \bar{y}\}^T \quad (14)$$

Unscented Kalman Filter is expansion of Unscented Transform to the recursive estimation. The implementation of standard UKF is shown below [10].

### (B) Unscented Kalman Filter Algorithm

At initial conditions the state vector sigma point is calculated as shown in equation (9)

Transformed sigma points by using equation (2)

$$X(k) = [X_s(k) \quad X_s(k) + \sqrt{(n + \lambda)p(k)} \quad X_s(k) - \sqrt{(n + \lambda)p(k)}] \quad (15)$$

Prediction in state vector is

$$X_s(k + 1 | k) = \sum_{i=0}^{2n} W_i^{(m)} X_s(i, k + 1 | k) \quad (16)$$

Prediction in covariance matrix is

$$P(k+1|k) = \sum_{i=0}^{2n} W_i^{(c)} [X_S(i, k+1|k) - X_S(k+1|k)] [X_S(i, k+1|k) - X_S(k+1|k)]^T + Q(k) \quad (17)$$

In state vectors the sigma points are updated as

$$X(k+1|k) = [X_S(k+1|k) \quad X_S(k+1|k) + \sqrt{(n+\lambda)p(k+1|k)} \quad X_S(k+1|k) - \sqrt{(n+\lambda)p(k+1|k)}] \quad (18)$$

The measurement model of points in state vector is predicted and changed. So the values which is predicted given as

$$y(k+1|k) = \sum_{i=0}^{2n} W_i^{(m)} Y(k+1|k) \quad (19)$$

Noise in Measurement is independent. Cross-covariance matrix and Innovation matrix are determined as follows

$$P_{yy} = \sum_{i=0}^{2n} W_i^{(c)} [Y(i, k+1|k) - y(k+1|k)] [Y(i, k+1|k) - y(k+1|k)]^T + R(k) \quad (20)$$

$$P_{xy} = \sum_{i=0}^{2n} W_i^{(c)} [X(i, k+1|k) - X(k+1|k)] [Y(i, k+1|k) - y(k+1|k)]^T \quad (21)$$

Computational Kalman gain is

$$K(k+1) = P_{xy} P_{yy}^{-1} \quad (22)$$

State estimation and error covariance are

$$X(k+1|k+1) = X(k+1|k) + K(k+1)(y(k+1|k+1) - y(k+1|k)) \quad (23)$$

$$P(k+1|k+1) = P(k+1|k) - K(k+1)P_{yy}K(k+1)^T \quad (24)$$

### (C) Cubature Kalman Filter Algorithm

Computational sigma points are

$$\xi^{(i)} = \begin{cases} \sqrt{n}e_i, & i = 1, \dots, 2n \\ -\sqrt{n}e_{i-n}, & i = n+1, \dots, 2n. \end{cases} \quad (25)$$

where the unit vector  $e_i$  in the coordinate axis  $i$ .

Integral approximation is given as

$$\int g(x)N(x|m, P)dx \approx \frac{1}{n} \sum_{i=1}^{2n} g(m + \sqrt{P}\xi^{(i)}) \quad (26)$$

In above case the approximation of unscented transform parameters are

$$\alpha = 1, \beta = 0 \text{ and } \kappa = 0$$

Unscented transform is effective for 2n-point approximation and weight of the mean is zero. The CKF [19] algorithm of the spherical cubature integration rule for the third order as follows

**Prediction:** The calculated sigma points are

$$X_{k-1}^{(i)} = m_{k-1} + \sqrt{P_{k-1}} \xi^{(i)} \quad i=1, \dots, 2n \quad (27)$$

The defined sigma points is

$$\xi^{(i)} = \begin{cases} \sqrt{n} e_i, & i = 1, \dots, 2n \\ -\sqrt{n} e_{i-n}, & i = n + 1, \dots, 2n. \end{cases} \quad (28)$$

Through the dynamic model the propagated sigma points

$$\hat{X}_k^{(i)} = f(X_{k-1}^{(i)}), \quad i = 1, \dots, 2n$$

The predicted mean  $m_k^-$  and the predicted covariance  $P_k^-$  are calculated as

$$m_k^- = \frac{1}{2n} \sum_{i=1}^{2n} \hat{X}_k^{(i)}, \quad (29)$$

$$P_k^- = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{X}_k^{(i)} - m_k^-)(\hat{X}_k^{(i)} - m_k^-)^T + Q_{k-1}. \quad (30)$$

**Update:** The formed sigma points are

$$X_k^{-(i)} = m_k^- + \sqrt{P_k^-} \xi^{(i)}, \quad i = 1, \dots, 2n, \quad (31)$$

where sigma points unit are defined as in Equation. (27).

The propagated sigma point in the measurements model is

$$\hat{y}_k^{(i)} = h(X_k^{-(i)}), \quad i = 1 \dots 2n \quad (32)$$

The predicted mean, the predicted covariance of the measurement, and the cross-covariance of the state and the measurement are calculated as

$$\mu_k = \frac{1}{2n} \sum_{i=1}^{2n} \hat{y}_k^{(i)}, \quad (33)$$

$$S_k = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{y}_k^{(i)} - \mu_k)(\hat{y}_k^{(i)} - \mu_k)^T + R_k, \quad (34)$$

$$C_k = \frac{1}{2n} \sum_{i=1}^{2n} (X_k^{-(i)} - m_k^-)(\hat{y}_k^{(i)} - \mu_k)^T. \quad (35)$$

$$K_k = C_k S_k^{-1}, \quad (36)$$

$$m_k = m_k^- + K_k [y_k - \mu_k], \quad (37)$$

$$P_k = P_k^- - K_k S_k K_k^T. \quad (38)$$

### 3. INITIALIZATION OF ALGORITHM

For initial estimation state vector of the target is given as follows. Now a days, the availability of software packages for calculation of sonar range of the day (SRD) is used for various parameters in the sea environment like salinity, temperature etc. In this research work, the observer is assumed to be submarine. If target is considered as submarine, ship and torpedo the SRD's for these are 7km, 15km and 30km respectively. The submarine is assumed to be designed to track torpedoes at long ranges.

The target velocity component with respect to submarine is assume to be 5m/sec and for ship it is 10m/sec and for torpedo it is 20m/sec whereas for all average speeds of the target it is 8m/sec. The initialized state vector of the target is

$$X_s(0|0) = [8 \quad 8 \quad SRD \sin B_m(0) \quad SRD \cos B_m(0)] \quad (39)$$

wherein initial bearing measurement is  $B_m(0)$ .

The covariance matrix is initialised as

$$P(0|0) = \begin{bmatrix} \frac{4\dot{x}(0|0)}{12} & 0 & 0 & 0 \\ 0 & \frac{4\dot{y}(0|0)}{12} & 0 & 0 \\ 0 & 0 & \frac{4R_x^2(0|0)}{12} & 0 \\ 0 & 0 & 0 & \frac{4R_y^2(0|0)}{12} \end{bmatrix} \quad (40)$$

In both algorithms the initial state vector of the target and covariance matrix are taken.

### 4. PERFORMANCE EVALUATION OF THE ALGORITHM

In this paper the main aim is to assess the performance evaluation of algorithm for CKF and UKF at the time of implementation where the measurements for every second are continuously available. In MATLAB, the algorithm is implemented. In bearing measurement the available noise is white Gaussian with SD of  $0.5^0$ . At every second the available bearing measurement is added with noise of standard deviation  $0.5^0$ .

In Fig.1 the observer carry out S-maneuver on LOS (Line Of Sight) with  $0.5^0$ /sec turn rate. In first leg at  $90^0$  course the observer travels for 2 minutes and turn towards  $270^0$  course. Similarly in second leg at  $90^0$  course the observer travel for 4 minutes. Whereas third and fourth leg is similar to second leg except in third leg course of the observer is  $270^0$  and in fourth leg observer course is  $90^0$ . For 5 legs 4 maneuvers the total scenario covers 2520 samples for the length of each run in 42 minutes. For given scenarios in Table 1, Table 2 and Table 3 the

algorithms are evaluated. In these tables the targets are assume to be ship, torpedo and submarine. The selected range and speeds of the target are near to its realistic values. The acceptance criteria of the solution is based on weapon guidance algorithm is given as

Estimation of range error  $\leq 8\%$  of the original range

Estimation of course error  $\leq 3^\circ$ .

Estimation of speed error  $\leq 1$  m/s.

The algorithm for required solution is accepted once the solution is converged in speed, course and speed. The performance of UKF is compared with that of CKF in terms of convergence of the solution. For 100 runs, the UKF and CKF algorithms is said to be converged with time in seconds as shown in Tables.4, 5 & 6 with 100 run mode. Fig.2 shows the RMS error estimation in range, course and speed with respect to low ATBs for scenarios given in Table.4. Considering submarine as target which moves with 5 m/s at  $170^\circ$  course as described in Scenario 1 for Table.1. The range between the observer (Submarine) and the target is 5000m initially with initial bearing angle  $0^\circ$ . The speed of the observer movement in S-maneuver is 4m/s as shown in Fig. 1.

The solution is said to be converged in UKF algorithm at 410,360 and 190 seconds for the estimated target range, course and speed error and hence the convergence time of total solution is at 410 seconds. Similarly, as shown in Table 4 the total solution obtained with CKF is at 2514 seconds. Estimated RMS errors of Target Motion Parameter (TMA) with respect to time for UKF and CKF algorithms are shown in Fig. 2. Correspondingly same procedures are followed by all scenarios.

**Table 1**  
**Low ATB Scenarios**

Observer and Target scenario	Actual range (m)	Actual bearing (deg)	Speed of the target (m/s)	Speed of the observer (m/s)	Course of the target (deg)
1	5000	0	5	4	170
2	10000	0	10	4	170
3	20000	0	20	4	170

**Table 2**  
**Medium ATB Scenarios**

Observer and Target scenario	Actual range (m)	Actual bearing (deg)	Speed of the target (m/s)	Speed of the observer (m/s)	Course of the target (deg)
1	5000	0	5	4	135
2	10000	0	10	4	135
3	20000	0	20	4	135

**Table 3**  
**High ATB Scenarios**

Observer and Target scenario	Actual range (m)	Actual bearing (deg)	Speed of the target (m/s)	Speed of the observer (m/s)	Course of the target (deg)
1	5000	0	5	4	110
2	10000	0	10	4	110
3	20000	0	20	4	110



**Table 4**  
Convergence time in seconds with 100 Monte-Carlo runs for Low ATB Scenarios

Observer and Target scenario	Unscented Kalman Filter				Cubature Kalman Filter			
	Range	Course	Speed	Total solution	Range	Course	Speed	Total solution
1	410	360	190	410	2154	1559	1630	2514
2	461	473	499	499	2322	1557	2187	2322
3	424	547	630	630	NC	572	NC	NC

**Table 5**  
Convergence time in seconds with 100 Monte-Carlo runs for medium ATB scenarios

Observer and Target scenario	Unscented Kalman Filter				Cubature Kalman Filter			
	Range	Course	Speed	Total solution	Range	Course	Speed	Total solution
1	333	456	335	456	1543	814	625	1543
2	400	482	449	482	1790	713	1561	1790
3	549	548	672	672	NC	1318	NC	NC

**Table 6**  
Convergence time in seconds with 100 Monte Carlo runs for high ATB scenarios

Observer and Target scenario	Unscented Kalman Filter				Cubature Kalman Filter			
	Range	Course	Speed	Total solution	Range	Course	Speed	Total solution
1	304	513	232	513	690	691	569	691
2	299	542	249	542	NC	1091	1613	1613
3	566	588	704	704	NC	1120	NC	NC

As randomness exists in the experiment, it is not genuine to calculate the performance for single run in the algorithm. So with 100 runs the Monte-Carlo simulation is carried out. The maximum ( $3\sigma$ ) error acceptable in estimated range, course and speeds are 8%, 3° and 1m/s, the RMS (1 $\sigma$ ) error allowed for acceptance of the solution are 2.66%, 1° and 0.33m/s respectively. The convergence of solution is shown in Table 4, 5 and 6. As per Monte-Carlo simulation the obtained graphs are shown in Fig. 2. It is observed that UKF converges faster than

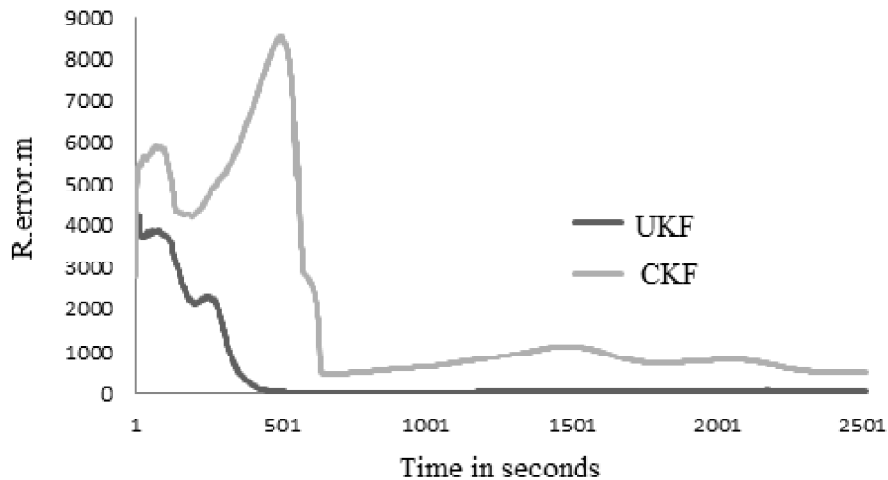


Figure 2(a): Estimation of Range error

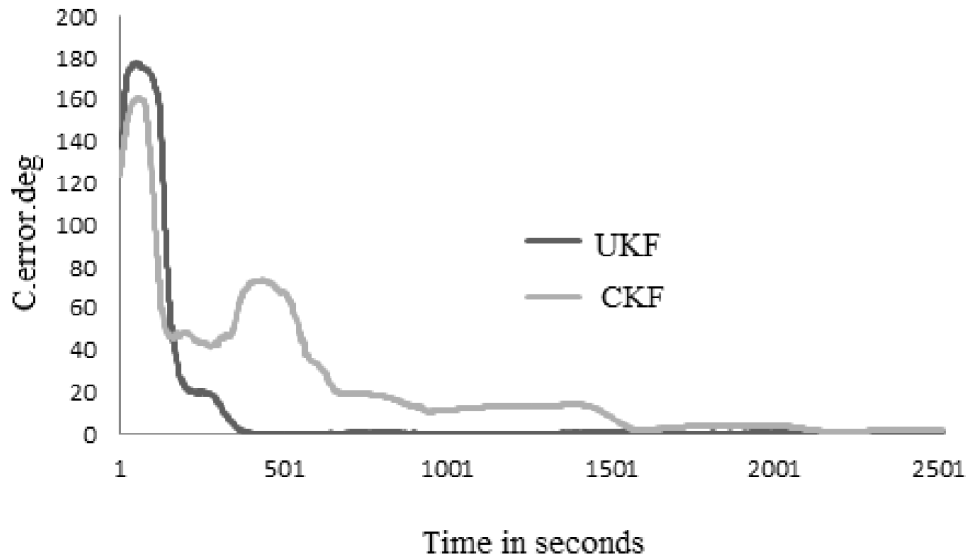


Figure 2(b): Estimation of Course error

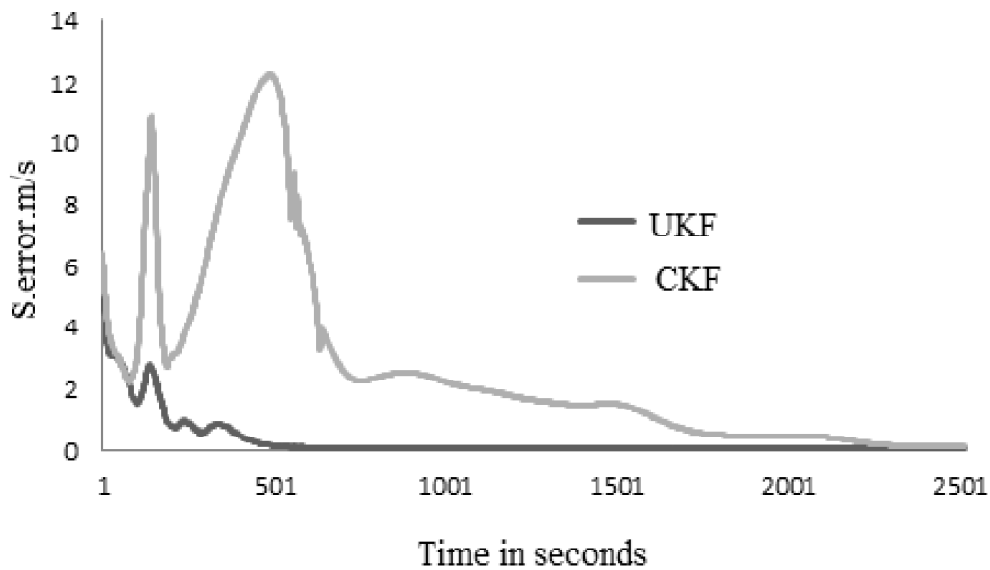


Figure 2(c): Estimation of Speed error

Figure 2: Estimated RMS errors for low ATB, scenario1

that of CKF for the most of the selected scenarios in Monte-Carlo simulation. Parameters  $\alpha$  and  $\beta$  in UKF is to modulate the solution. In UKF algorithm the initial values for  $\beta$  and  $\alpha$  is taken as 2 and 0.09 respectively. Depending upon trial and error basis  $\alpha$  and  $\beta$  parameters is still modulated. In CKF, these are chosen as zeros. UKF algorithm is recommended for passive bearings-only target tracking.

## 5. SUMMARY AND CONCLUSION

In underwater scenario, the submarine is considered as observer and the target is considered as submarine, ship or torpedo. By covering low, medium and high ATB scenarios all nine scenarios are considered. The performance evaluation of CKF and UKF algorithms are considered once the solution is converged. Various results for

different scenarios are simulated and it is confirmed that UKF generates the solution faster when compared to CKF.

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