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BER Estimation of OFDM System over Frequency Selective Fading Channels

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ABSTRACT

OFDM system is promising technology for high speed data communication in wireless media. In this paper OFDM system model is developed over flat fading channel with Additive Gaussian White Noise (AWGN). Initially BER analysis is carried out in OFDM system with the number of sub-carriers 'N' using M-ary PSK and M-ary QAM. Analysis is carried out with the number of sub-carriers (N=64) with Rayleigh and Ricean fading. Due to multipath, phase errors occur in the channel which influences distorted result at the output of the receiver. Phase errors are reduced by increasing the SNR in the system.

Keywords: AWGN, BER, OFDM, M-ary PSK, M-ary QAM

INTRODUCTION

OFDM is typically a multicarrier modulation [1,2] in which all subcarriers are orthogonal to each other. The transmitted sub channels are arbitrary with certain constraints and in practice tends to extremely numerous and close to each other[4-6]. OFDM has many advantages like provides immunity to multipath fading, better channel equalization and time synchronization [7,8]. However, it is more affected to local frequency offset and non-linearity in RF amplifiers [8,9].

Currently, OFDM plays a prominent role in modern wireless communications. In this paper, OFDM system model is developed over flat fading channel with AWGN using Matlabsimulations[10]. Initially BER analysis is carried out in OFDM system with the number of sub carriers 'N' using BPSK, QPSK, and M-ary PSK, QAM. OFDM has the advantage of averaging fades when the symbol duration is longer than the length of the fades [11]. Due to this, the OFDM symbols are partially corrupted under fading. This can be achieved by adopting a large number of sub carriers at the cost of increasing the system complexity, but it results poor peak-to-average power ratio[12]. By careful adjustment of the system parameters, such as the bit rate and the number of sub carriers under certain fading, the system performance can be improved. Analysis is done with Rayleigh and Ricean fading by considering N=64 [13-14]. Due to multipath, phase errors occur in the channel, which causes distorted result at the output of the receiver. Therefore, a need is raised to calculate BER due to phase errors using BPSK which has less complexity than other modulation techniques in OFDM system.

1.1. M-ary Signalling Scheme

This comes under the tree of variants of phase shift keying schemes. Several bandwidth efficient techniques of this group are important for practical wireless applications. As an overview of the concept of PSK modulation, the modulating symbol is formed by grouping 'm' consecutive binary bits together. So, the number of possible modulating symbols is, M = 2m and the symbol duration T = m. Tb.

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The two basic functions are similar to the QPSK, viz.,

$$\phi_1(t) = \sqrt{\frac{2}{T}}\ \cos 2\pi fct \text{ and } \phi_2(t) = \sqrt{\frac{2}{T}}\ \sin 2\pi fct;\ 0 \le t \le T \tag{1}$$

The signal points can be distinguished by their angular location:

$$\theta i = \frac{2\pi_i}{M};\ i = 0,1,........M-1 \tag{2}$$

The time-limited energy signals si(t) for modulation can be expressed in general as

$$S_i(t) = \sqrt{\frac{2E}{T}}.\cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \tag{3}$$

Considering M-ary PSK modulation schemes are narrowband-type, the general form of the modulated signal is

$$s(t) = u_I(t)\cos w_c t - u_Q(t)\sin w_c t \tag{4}$$

Without any pulse shaping, the uI(t) and uQ(t) are proportional to S1 and S2 respectively. Beside this information baseband processing unit, the M-ary PSK modulator follows the general structure of an I/Q modulator.

The in-phase and quadrature-phase correlator outputs are:

$$r_I = \sqrt{E}\cos\left(\frac{2\pi i}{M}\right) + W_I,\ i = 0,1...M-1 \tag{5}$$

$$r_Q = -\sqrt{E}\sin\left(\frac{2\pi i}{M}\right) + W_Q,\ i = 0,1...M-1 \tag{6}$$

M-QAM constellations involve inphase(I) and quadrature (Q) carriers:

$$\varphi_I(t) = \sqrt{\frac{2}{T_S}}\cos(2\pi f_c t),\ 0 \le t \le T_S \tag{7}$$

$$\varphi_Q(t) = \sqrt{\frac{2}{T_S}}\cos(2\pi f_c t),\ 0 \le t \le T_S \tag{8}$$

The ith transmitted M-QAM signal is:

$$S_i(t) = V_{I,i}\sqrt{\frac{2}{T_S}}\cos(2\pi\ f_c t) + V_{Q,i}\sqrt{\frac{2}{T_S}}\sin(2\pi\ f_c t)\, 0 \le t \le T_S \tag{9}$$

Where i = 1, 2….M

$$= \sqrt{E_i}\cos(2\pi\ f_c t - \theta_i)$$

OFDM SYSTEM MODEL

Consider an input data sequence (X_0, X_1,.......X_N) at the sub carriers with a symbol ratef_s, where N is the Number of Sub carriers. Considering cyclic prefix at the transmitter with sufficient Guard interval by padding zeros. The modulation method used in the transmission involves PSK and QAM techniques. Each sub carrier is modulated using M-ary PSK as well as QAM. The demodulated signal in the frequency domain under time variant multipath fading channel is given by [12,13]. Due to noise in the channel, the transmitted signal will result a phase error and magnitude error at the received signal which leads a bit error at the output of the system. Initially magnitude error is considered in the channel and estimates its BER over Ricean and Rayleigh fading. Later the magnitude error is constant and random phase error is considered in the channel to estimates its BER.

In flat fading environment, the received signal y in fading channel is of the form,

$$y = hx + n \tag{10}$$

where 'y' is the received symbol, 'h' is complex fading coefficient corresponding to Rayleigh or Ricean multipath channel, 'x' is the transmitted symbol and 'n' is the noise at the receiver.

The channel is randomly varying in time each transmitted symbol gets multiplied by a randomly varying complex number 'h'. Consider 'h' is modelling as Rayleigh channel or Ricean channel in the system, the real and imaginary parts are taken as zero mean Gaussian distributed and unity variance. For Ricean channel, Rice factor is taken as k=3 to estimate the BER.

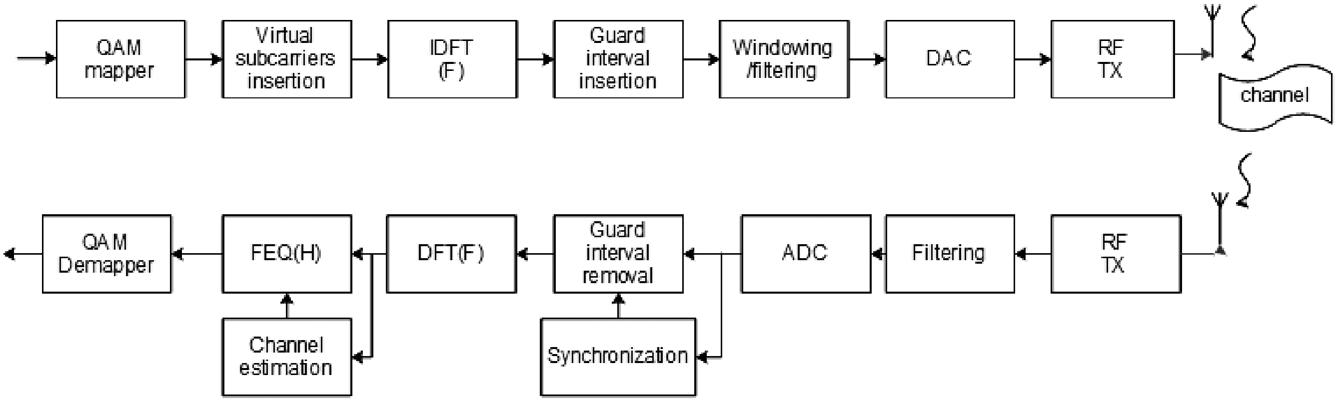

Figure 1: Block diagram of OFDM

Since OFDM contains many subcarriers, the modulated signal, can be represented a:

$$S_s(t) = \frac{1}{N}\Sigma_{n=0}^{N-1} A_n(t)\, e^{j(w_n t+\varphi_n(t))} \tag{11}$$

Where: $w_n = \omega_0 + n\Delta\omega$

It is a continuous signal, and consider the wave forms of signal components over one symbol period, then the variables $A_c(t)$ and $\varphi_c(t)$ take on fixed values, which depends on the frequency of that specific carrier, and so can be rewritten :

$$\varphi_n(t) = \varphi_n$$
$$A_n(t) = A_n$$

The resulting sampled signal is given by,

$$S_s(kT) = \frac{1}{N}\Sigma_{n=0}^{N-1} A_n e^{j[w_0+n\Delta\omega)kT+\varphi_n]} \tag{12}$$

At this point, the analyzed signal is obtained and can be written as,

$$t = NT$$

Simplifying equation (12) without a loss of generalizationand by letting $\omega_0 = 0$, then the signal becomes:

$$S_s(kT) = \frac{1}{N}\Sigma_{n=0}^{N-1} A_n e^{j\varphi_n} e^{j(n\Delta\omega)kT} \tag{13}$$

Noweqn.(13) is analogous to inverse fourier transform and can be written as,

$$g(kT) = \frac{1}{N}\Sigma_{n=0}^{N-1} G\left(\frac{n}{NT}\right)e^{j2\pi nk/N} \tag{14}$$

Ineqn. (13), the function $A_n e^{j\varphi_n}$ is composed of attenuation and phase components, where S(kT)is the tome domain representation and the eqns. (13) and (14) are equivalent if:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{1}{NT} \tag{15}$$

This is the condition that is required for orthogonality.

FADING CHANNELS

In this section, the non-coherent detection of BPSK over Rayleigh Fading channel is considered and then coherent detection is studied. For these cases, consider a simple flat fading Rayleigh channel. The channel also adds AWGN noise to the signal samples after it suffers from Rayleigh Fading.

3.1.1. Rayleigh Fading

Rayleigh Fading model is used to analyze the environments that has multiple scattering objects and NLOS components. If there are required multiple scatter components in the environment, all the reflected signals that appear at the receiver front end becomes attenuated in amplitude and distorted in phase. Eventually the I (inphase) and Q (quadrature) phase components become Gaussian random distribution and their envelope $Z = \sqrt{I^2 + R^2}$ becomes Rayleigh distributed. Here, note that distribution I2 is called Chi-Square-distribution. If I2 has zero mean, then it is called central-Chi-square distribution and if mean is non-zero it is called non-central-Chi-square distribution.

3.1. Rayleigh Distribution

Consider two Gaussian random variables X and Y with zero mean and equal variance σ^2. Then the transformation $Z = \sqrt{X^2 + Y^2}$ is Rayleigh Distributed and Z^2 is exponentially distributed.

In non-coherent detection, prior knowledge of the channel impulse response is unknown at the receiver. Consider the BPSK signaling scheme with x=+a or -a being transmitted over such a channel as described above. This signaling scheme fails completely even in the lack of noise, since the phase of the received signal y is evenly distributed between 0 and 2π regardless of whether x[m]=+a or x[m]=-a is transmitted. So the non-coherent detection of the BPSK signaling is not an appropriate method of detection especially in a Fading environment.

In coherent detection, the receiver has necessary knowledge about the channel impulse response. Methods like pilot transmissions are used to estimate the channel impulse response at the receiver, before the definite data transmission could begin. Let’s consider that the channel impulse response estimate at receiver is known and is perfect & accurate. The transmitted symbols (‘x’) can be obtained from the received signal (‘y’) by the process of equalization as given below.

$$\hat{y} = \frac{y}{h} = \frac{hx+n}{h} = x+z$$

here z is AWGN except for the scaling factor 1/h. Now the detection of x can be performed in a manner similar to the detection in AWGN channels.

3.1.2. Theoretical BER Vs. SNR for Rayleigh Fading Channel

The theoretical BER for BPSK/QPSK modulation scheme over Rayleigh fading channel (with AWGN noise) is given by

$$BER = 0.5\left(1-\sqrt{\frac{(E_b/N_0)}{1+(E_b/N_0)}}\right)$$

The theoretical BER for BPSK/QPSK modulation scheme over an AWGN channel is given here for comparison

$$BER = 0.5\,erfc\sqrt{(E_b/N_0)}$$

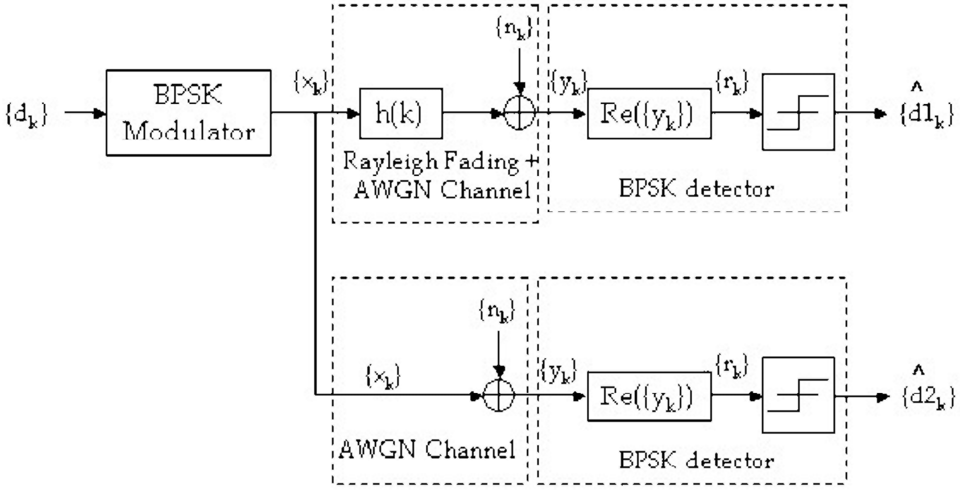

Figure 2: Effect of Rayleigh fading and AWGN noise

In the similar way, BER can be calculated for higher order modulation techniques.

3.2. Rician Fading

Rician Fading model is used to simulate environments that produces multipath components and also a prevailing Line of Sight (LOS) component. The LOS component is called “specular” component and the multipath propagating component is called “random or scatter” component. The amplitude distribution of the specular component will have non-zero mean, whereas, the random component will have zero-mean.

3.2.1. Rician Distribution

Consider two Gaussian random variables X and Y. Here X models the signal component (LOS) and Y models the random/scatter component. By definition, X has non-zero mean (m), Y has zero mean and both have equal variance σ^2. Then the transformation,

$$Z = \sqrt{X^2+Y^2}$$

is Rician Distributed. The ratio of power of LOS component to the power of random component is called Ricianfactor and it is defined as

$$k = \frac{m^2}{2\sigma^2}$$

It can be immediately ascertained that Rayleigh Fading is related to central Chi Square distribution (due to zero mean) and the Rician Fading is related to non-central Chi Square distribution (due to non-zero mean).

3.2.2. Theoretical BER Vs. SNR for Rician Fading Channel

Theoretical BER for BPSK over Rician Fading Channel with AWGN noise is given by the following expression:

$$P_b = \frac{1}{2} erfc\left[\frac{k(E_b / N_0)}{(k + E_b / N_0)} \right]$$

Simulation Model: The following model is used for the simulation of BPSK over Rician Fading channel (with AWGN noise).

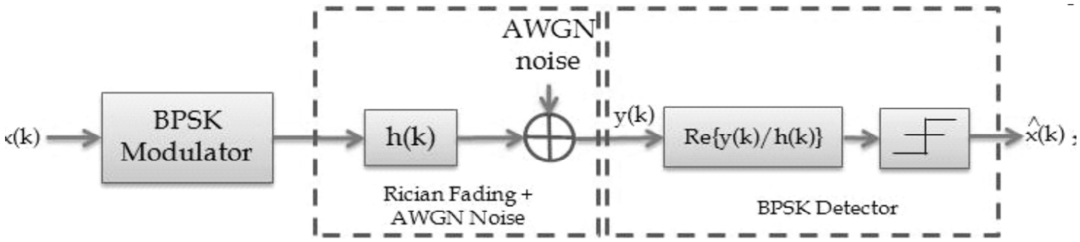

Figure 3: Effect of Rician fading and AWGN noise

In the similar way, BER can be calculated for higher order modulation techniques.

SIMULATION RESULTS

4.1. BER Analysis of OFDM Performance in AWGN channel

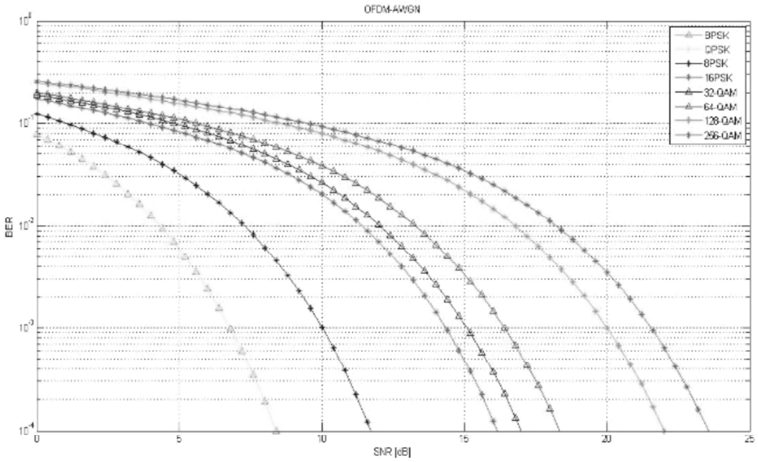

Figure 4: BER Analysis of OFDM Performance in AWGN channel distribution

From the Fig 4, it was shown that the performance of the system improves from BPSK to 256-QAM under additive noisy channel. It gives better results at extremely low SNR values.

4.2. BER analysis of OFDM performance in Ricianfading channel distribution

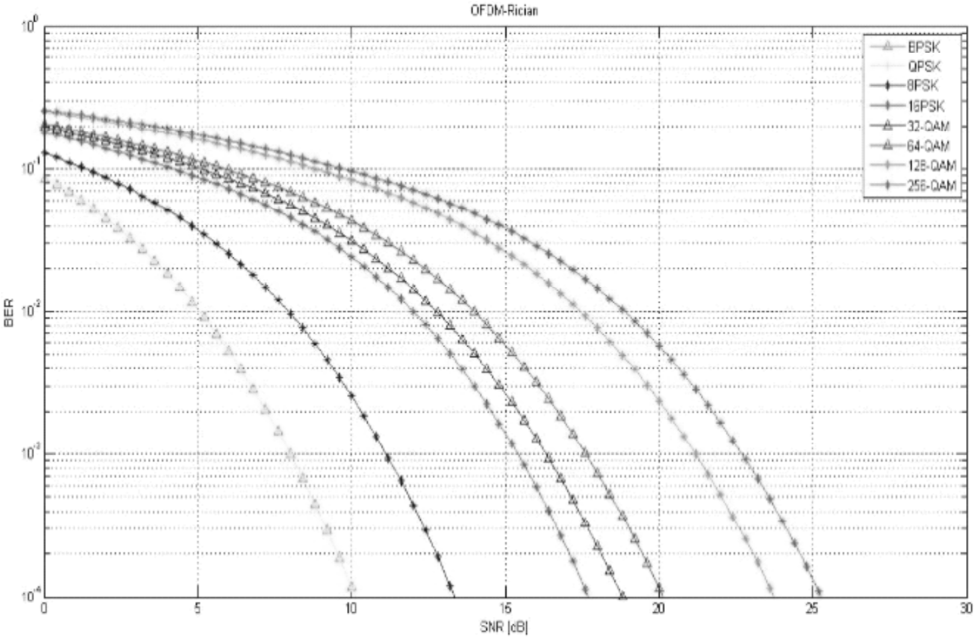

Figure 5: BER Analysis of OFDM Performance in Rician fading distribution

In Fig 5 it is clear that the performance of the system degrades for the same SNR and to further improve the BER, it is required to raise the SNR moderately.

4.3. BER Analysis of OFDM Performance in Rayleigh Fading Channel Distribution

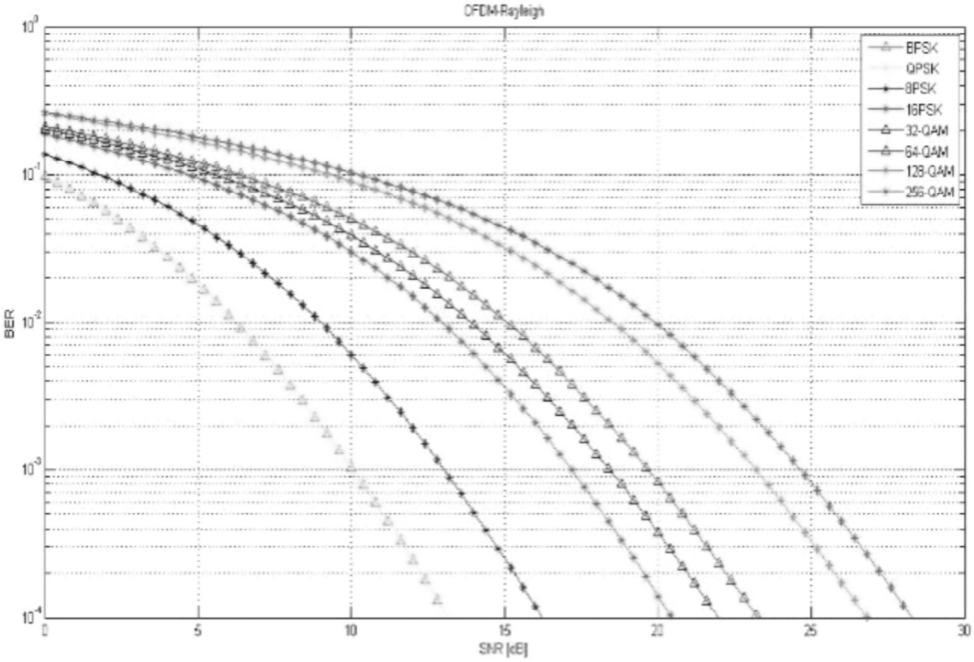

Figure 6: BER Analysis of OFDM Performance in Rayleigh fading distribution

From the Fig. 5 it is clear that inorder to maintain the constant BER it is highly required to increase the SNR values from BPSK to 256-QAM.

Table 1
Comparing of BER Vs. SNR for different modulation techniques over different channels

Modulation Technique	*AWGN*		*Rician*		*Rayleigh*	
	BER	*SNR*	*BER*	*SNR*	*BER*	*SNR*
QPSK	10^{-3}	6.8	10^{-3}	8	10^{-3}	10
8PSK	10^{-3}	10	10^{-3}	11.1	10^{-3}	12.9
16PSK	10^{-3}	14.3	10^{-3}	15.4	10^{-3}	17.2
32QAM	10^{-3}	15.1	10^{-3}	16.3	10^{-3}	18.4
64QAM	10^{-3}	16.4	10^{-3}	17.6	10^{-3}	19.6
128QAM	10^{-3}	20	10^{-3}	21.2	10^{-3}	23.2
256-QAM	10^{-3}	21.6	10^{-3}	22.7	10^{-3}	24.7

In table 1, the BER and SNR values for the different modulation schemes were compared under various channel conditions.

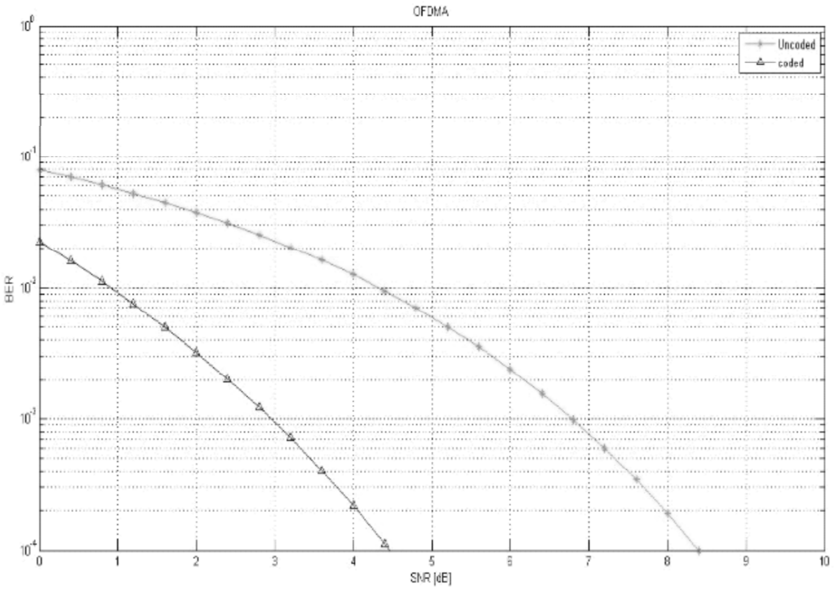

Figure 7: Coded Vs Uncoded - OFDM System for BPSK/QPSK Modulation Technique

In addition, the system performance is studied and analyzed for coded modulation schemes. This is shown in the figure 7.

CONCLUSION

In this paper, the Bit error rate (BER) performance is evaluated in OFDM system using BPSK, QPSK, 8-PSK and 16-PSK modulations with number of sub carriers N is taken as 32, 64,256,512 and 1024 and channel noise is AWGN. Graphical results show that BER is less for BPSK with N=32.The same analysis is carried out for OFDM system using Ricean and Rayliegh fading distribution with zero mean and unity variance and channel is AWGN. The BER is maintained constant with the increase in the SNR under varying channel conditions.

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