

# Application of Kalman Filter for Dunking Sonar Underwater Target Motion Analysis

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**Abstract:** In underwater, dunking sonar generates underwater target range and bearing measurements and the same information is communicated to a helicopter for further processing. The noise corrupted measurements are processed to estimate target motion parameters using online Extended Kalman Filter. These estimates are useful to find out weapon present parameters and then to release the weapon on to the target. Results obtained in simulation are presented.

**Keywords:** Dunking sonar, target motion analysis, Extended Kalman filter.

## 1. INTRODUCTION

Target motion analysis (TMA), in two dimensional scenario is generally used in underwater environment [1]. Dunking sonar is positioned in the sea from a helicopter in hovering mode to find out the path of the target submarine in sea waters. The sonar in active mode finds out target bearing and range measurements. These are communicated to the helicopter signal processing system through a cable. It is to assume that the target moves with uniform velocity and the observer is standstill. Observer estimates the target range, bearing, course and speed using the noise corrupted bearing and range measurements [2-3]. Extended Kalman filter is used to smooth the measurements and to estimate course and speed the target. Using the estimated parameters, weapon preset parameters (this topic is not dealt here) are calculated in helicopter fire control system to release weapon on the target.

Mathematical modeling of target state vector, measurements and Kalman filter in brief are described in section2. Section 3 deals with implementation of the algorithm and generation of target motion measurements in simulation environment. In section 4 results obtained in simulation are described.

## 2. MATHEMATICAL MODELING

### A. Modeling of State Vector and Measurements[4-5]

The  $X_s(k)$  be state vector is

$$X_s(k) = [\dot{x}(k)\dot{y}(k)R_x(k)R_y(k)]^T \quad (1)$$

where  $\dot{x}(k)$  and  $\dot{y}(k)$  are target velocity in  $x$  and  $y$  directions and  $R_x(k)$  and  $R_y(k)$  are target range in  $x$  and  $y$  directions. The State equation of the target is

$$X_s(k+1) = \phi(k+1/k)X_s(k) + b(k+1) + \omega(k) \quad (2)$$

where  $\omega(k)$  is noise having zero mean white Gaussian power spectral density and  $\phi(k+1/k)$  is transient matrix and it is

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$$\phi(k+1/k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 1 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (3)$$

where  $t$  is measurement interval and  $b(k+1)$  is deterministic matrix

$$b(k+1) = [0 \quad 0 \quad -[x_0(k+1) + x_0(k)] - [y_0(k+1) + y_0(k)]^T] \quad (4)$$

where  $x_0(k)$  and  $y_0(k)$  are observer position components. To reduce the mathematical complexity, all angles are measured with respect to True North.  $Z(k)$  is measurement vector

$$Z(k) = \begin{bmatrix} B_m(k) \\ R_m(k) \end{bmatrix} \quad (5)$$

where  $B_m(k)$  and  $R_m(k)$  are measurements and they are defined as

$$B_m(k) = B(k) + \gamma(k) \quad (6)$$

$$R_m(k) = R(k) + \eta(k) \quad (7)$$

where  $B(k)$  and  $R(k)$  are true bearing and range

$$B(k) = \tan^{-1} \left( \frac{R_x(k)}{R_y(k)} \right) \quad (8)$$

$$R(k) = \sqrt{R_x^2(k) + R_y^2(k)} \quad (9)$$

The noises  $\eta(k)$  and  $\gamma(k)$  are uncorrelated. Measurement equation is

$$Z(k) = H(k) X_s(k) + \xi(k) \quad (10)$$

where

$$H(k) = \begin{bmatrix} 0 & 0 & \frac{\cos \hat{B}(k)}{\hat{R}(k)} & \frac{-\sin \hat{B}(k)}{\hat{R}(k)} \\ 0 & 0 & \sin \hat{B}(k) & \cos \hat{B}(k) \end{bmatrix} \quad (11)$$

$\hat{B}(k)$  and  $\hat{R}(k)$  denotes estimated values. And

$$\xi(k) = \begin{bmatrix} \gamma(k) \\ \eta(k) \end{bmatrix} \quad (12)$$

The Extended Kalman filter [2-6] algorithm is presented in Table 1.

**Table 1**  
**Extended Kalman Filter equations**

1. To start with estimation  $X(0/0)$ ,  $P(0/0)$  which are initial state vector and its covariance matrix respectively are chosen.

2. Predicted state vector  $X_s(k+1)$  is

$$X_s(k+1) = \phi(k+1/k)X_s(k) + b(k+1) + \omega(k)$$

3. The predicted state covariance matrix is

$$P(k+1/k) = \phi(k+1/k)P(k/k)\phi^T(k+1/k) + Q(k+1) \quad (13)$$

where,  $Q(k)$  is the covariance of plant noise and it has the value  $\sigma_\omega^2$

4. Kalman gain is given as

$$G(k+1) = P(k+1/k)H^T(k+1)[r(K+1) + H(k+1)P(k+1/k)H^T(k+1)]^{-1} \quad (14)$$

where,  $r(k)$  is input measurement covariance matrix.

5. The state estimation and its error covariance are

$$X(k+1/k+1) = X(k+1/k) + G(k+1)[Z(k+1) - \hat{Z}(k+1)] \quad (15)$$

$$P(k+1/k+1) = [1 - G(k+1)H(k+1)P(k+1/k)] \quad (16)$$

6. For the next iteration

$$X(k/k) = X(k+1/k+1) \quad (17)$$

$$P(k/k) = P(k+1/k+1) \quad (18)$$

### 3. IMPLEMENTATION OF THE PROCESS

Initial of the target state vector, target velocity components are computed using first and second measurement sets of range and bearing measurements as shown in Table 2. The detailed processing of Kalman filter is shown in Figure 1.

#### A. Generation of Target Motion Measurements in Simulation Environment

A simulator is developed to generate target range and bearing measurements. This simulator accepts the inputs given and simulates the observer and target positions. It generates range and bearing measurements at each second and corrupts with white Gaussian noise.

**Table 2**  
**Extended Kalman filter Algorithm**

Initial target state vector  $X(2/2)$  is given by

$$X(2/2) = [\text{term 1} \quad \text{term 2} \quad R_m(2)\sin B_m(2)]^T \quad (19)$$

where term1 and term 2 are defined by

$$\text{term 1} = R_m(2) \sin B_m(2) - R_m(1) \sin B_m(1)/t$$

$$\text{term 2} = R_m(2) \cos B_m(2) - R_m(1) \cos B_m(1)/t \quad (20)$$

Assume that  $X(2/2)$  follows uniform distribution. Its covariance matrix is diagonal and given by

$$P_{00}(2/2) = \frac{4 \times \dot{x}^2(2/2)}{12} \quad (21)$$

$$P_{11}(2/2) = \frac{4 \times \dot{y}^2(2/2)}{12} \quad (22)$$

$$P_{22}(2/2) = \frac{4 \times R_x^2(2/2)}{12} \quad (23)$$

$$P_{33}(2/2) = \frac{4 \times R_y^2(2/2)}{12} \quad (24)$$

From the estimated state vector target motion parameters are calculated and given as

$$tcr(k) = \tan^{-1} \left( \frac{\dot{x}(x)}{\dot{y}(x)} \right) \quad (25)$$

$$v_t(k) = \sqrt{\dot{x}(x)^2 + \dot{y}(x)^2} \quad (26)$$

It is assumed that observer is at origin and stand still. The target moves with uniform speed ( $V_t$ ) and course ( $tcr$ ). Initially the observer and target are assumed to be a distance  $R$  meters. An imaginary line joining target and observer positions is called line of sight (LOS) and it makes an angle (bearing) with respect to True North /Y-axis as shown in Figure 2. It is assumed that target and observer are in the same plane. The measurements are made in active mode for every  $t$  seconds.

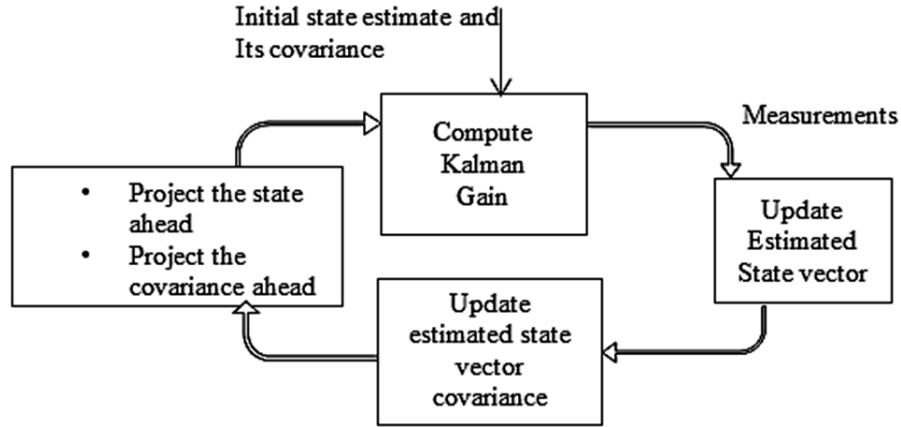


Figure 1: Extended Kalman filter process

The target position  $(x_t, y_t)$  with respect to origin is given by

$$x_t = R \times \sin(B) \quad (27)$$

$$y_t = R \times \cos(B) \quad (28)$$

After  $t$  seconds

$$dx_t = v_t \times \sin(tcr) \times t \quad (29)$$

$$dy_t = v_t \times \cos(tcr) \times t \quad (30)$$

Now the new target position after time  $t$  is given as

$$x_t = dx_t \times x_t \quad (31)$$

$$y_t = dy_t \times y_t \quad (32)$$

True bearing and range are calculated as follows

$$\text{True bearing} = \tan^{-1} \frac{x_t - x_0}{y_t - y_0} \quad (33)$$

$$\text{True range} = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2} \quad (34)$$

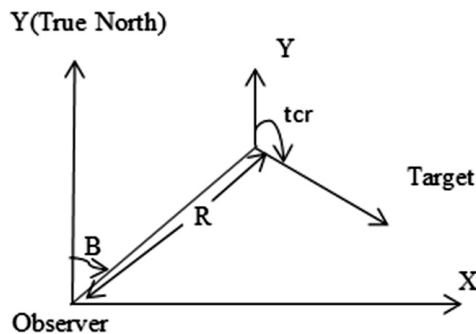


Figure 2: Target and Observer scenario

Block diagram of TMA in simulation mode is shown in Figure 3. The corrupted measurements are used to estimate target motion parameters (TMP) using EKF. The estimated TMP are compared with that of true values and the performance analysis of the algorithm is carried out against a number of scenarios.

#### 4. SIMULATION AND RESULTS

It is assumed that experiment is conducted at favorable environmental conditions and hence the range and bearing measurements are available continuously. This simulation is carried out on a personal computer using Matlab. The scenarios chosen for evaluation of algorithm are shown in Table 3. For example, scenario 1 describes a target moving at an initial range of 3000 m with course and speeds of  $255^\circ$  and 10 m/s respectively. The initial line of sight is  $45^\circ$ . The bearing and range measurements are corrupted with  $0.33^\circ(1\sigma)$  and 7 m ( $1\sigma$ ) respectively.

The velocity of sound in seawaters is 1500 m/s. As the maximum range of target is chosen as 3000m, the time taken for the transmitted pulse to reach the target and come back to observer is  $(6000/1500)$  4 seconds. Hence measurements are taken at 4 s interval. In simulation mode, estimated and actual values are available and hence the validity of the solution based on certain acceptance criterion is possible. The following acceptance criterion is chosen based on weapon control ([7], this topic is not discussed here) requirement. The solution is converged when error in course estimate  $\leq 3^\circ$  and error in speed estimate  $\leq 1$  m/s.

The estimates and true paths of target are shown in Figure 4 and 5 for scenario 1 and 2 respectively. For clarity of the concepts, the errors in estimated speed and course for scenario 1 and 2 are presented in Figure 6(a), 6(b) and 7(a) and 7(b) respectively. The solution is converged when the course and speed are converged. The convergence time (seconds) for the scenarios is given in Table 4. In simulation, it is observed that the estimated course and speed of the target are converged at 8<sup>th</sup> sample and 25<sup>th</sup> sample respectively for scenario 1. So, for scenario 1, the total solution is obtained at 25 samples (that is 100s). Similarly for scenario 2, it is observed that the estimated course and speed are converged at 10<sup>th</sup> and 22<sup>nd</sup> sample respectively. So the convergence time for scenario 2 is obtained at 22<sup>nd</sup> sample (that is 88 s).

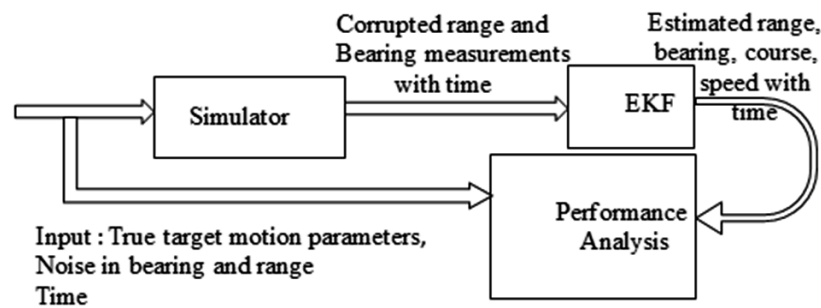


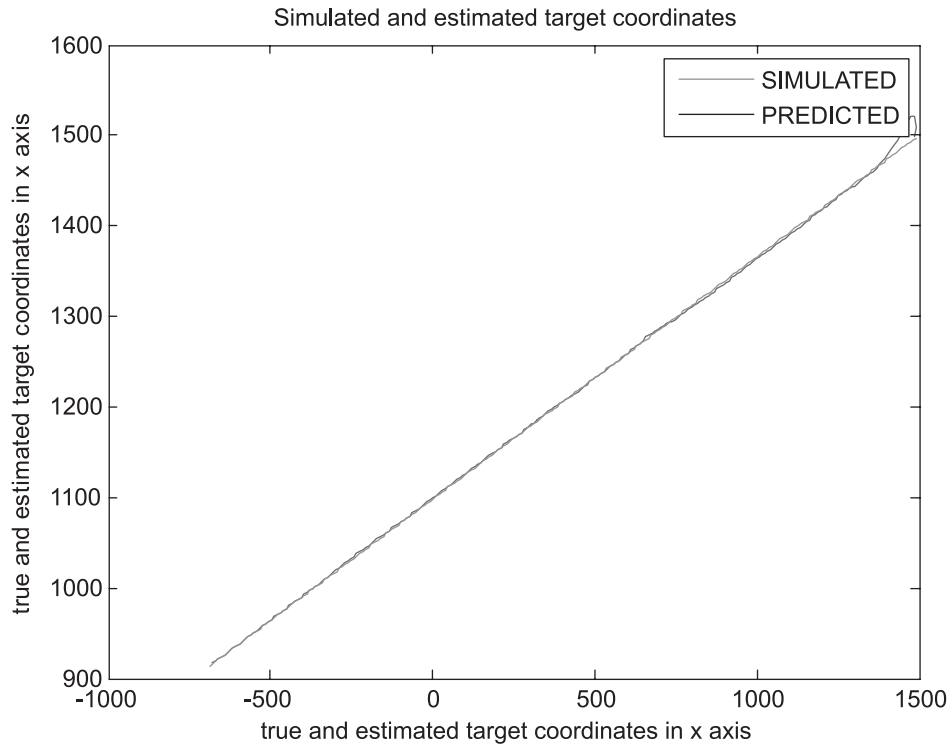
Figure 3: Block diagram of TMA in simulation mode.

Table 3  
Input scenarios chosen for the algorithm

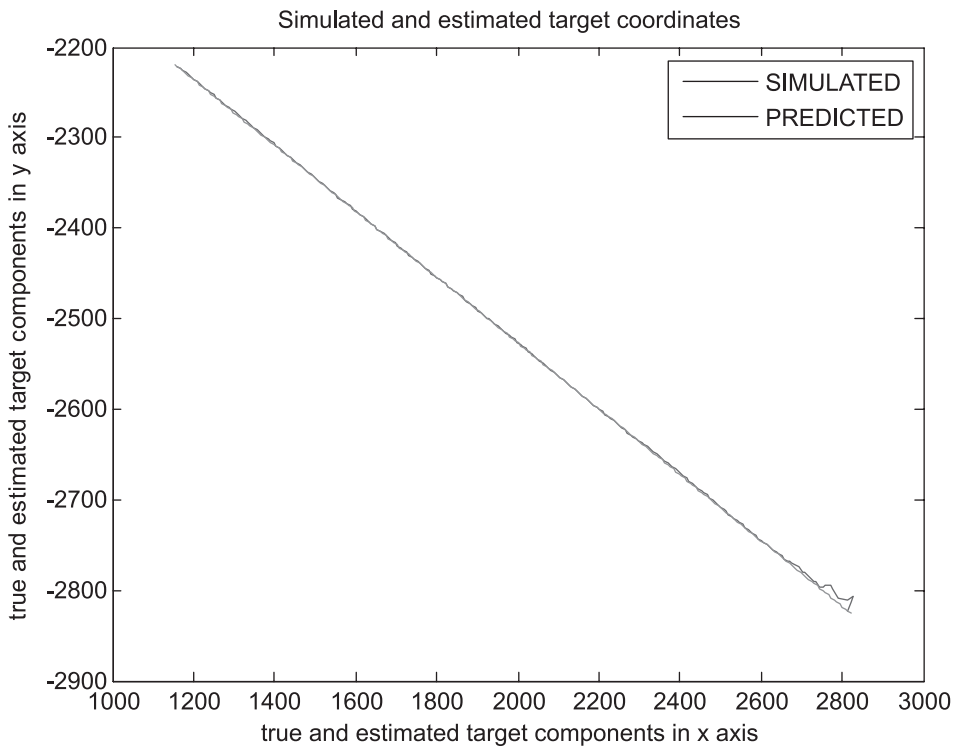
Scenario	Target range (m)	Target bearing (deg)	Target Course (deg)	Target speed (m/s)	Noise in bearing ( $1\sigma$ ) (deg)	Noise in range ( $1\sigma$ ) (m)
1	3000	45	255	10	0.33(rad)	7m
2	4000	135	315	8.5	0.33(rad)	7m

**Table 4**  
**Convergence time in samples for the chosen scenarios**

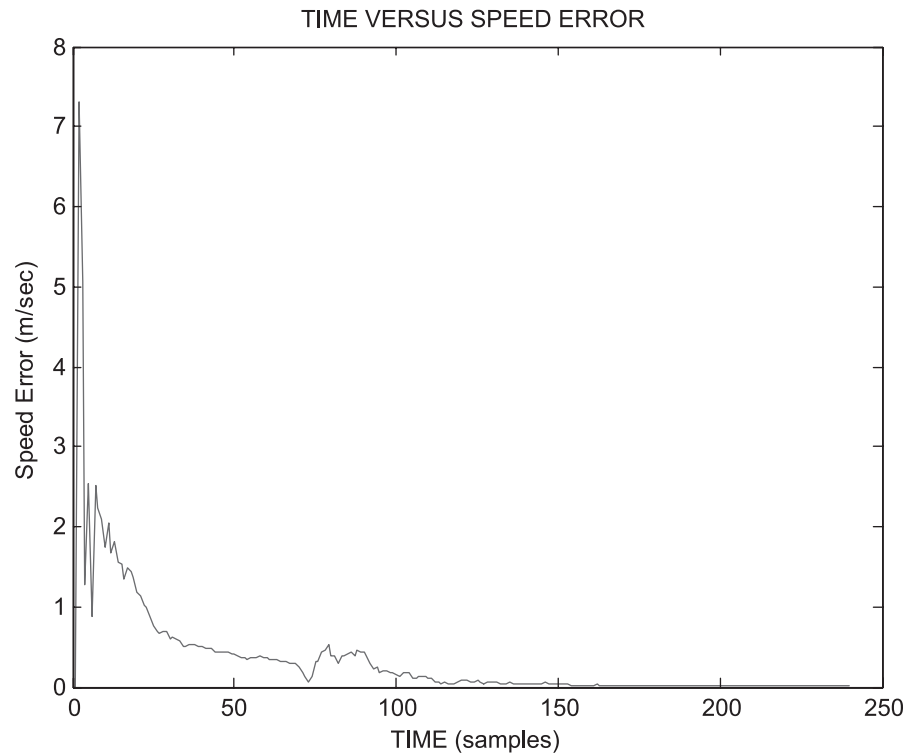
<i>Scenario1</i>	<i>Course</i>	<i>Speed</i>	<i>Total solution</i>
1	8	25	25
2	10	22	22



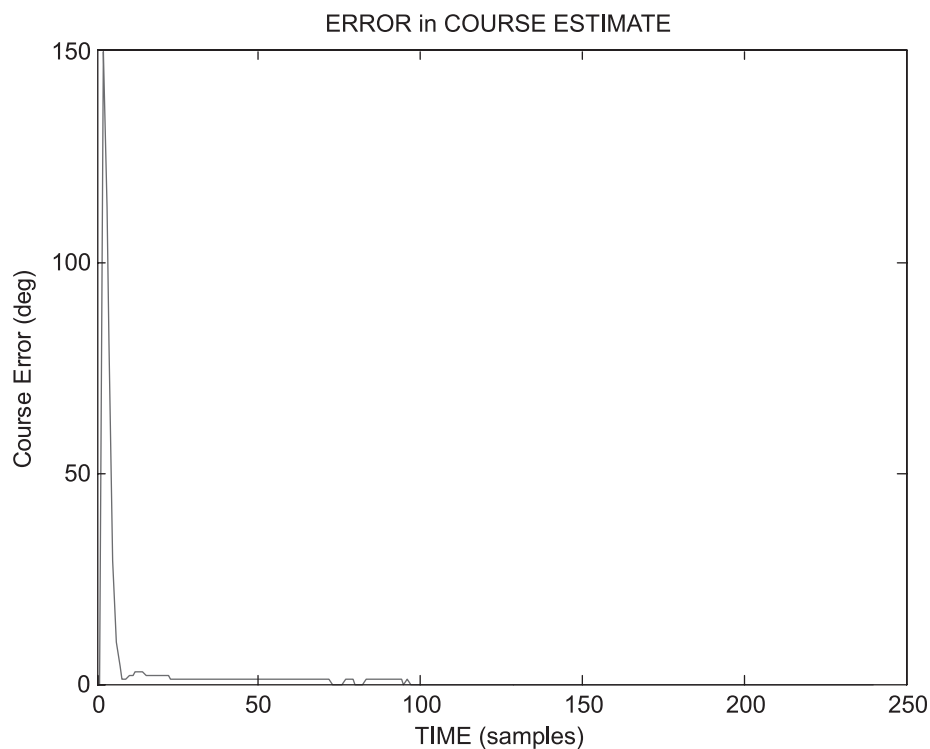
**Figure 4: Simulated and estimated target paths**



**Figure 5: Simulated and estimated target pathss**



(a) Error in speed estimate

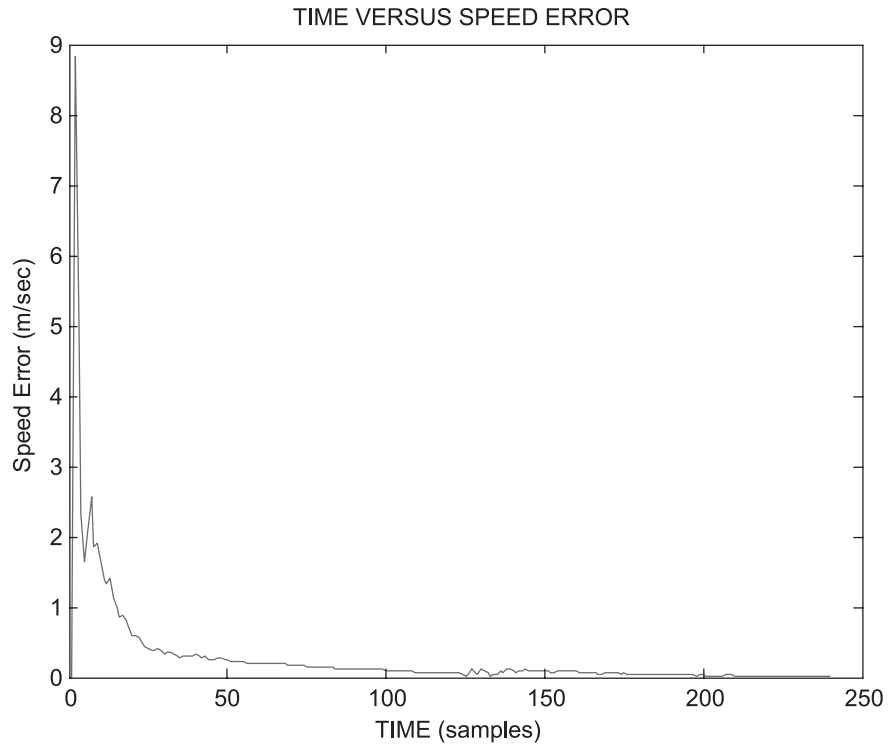


(b) Error in course estimate

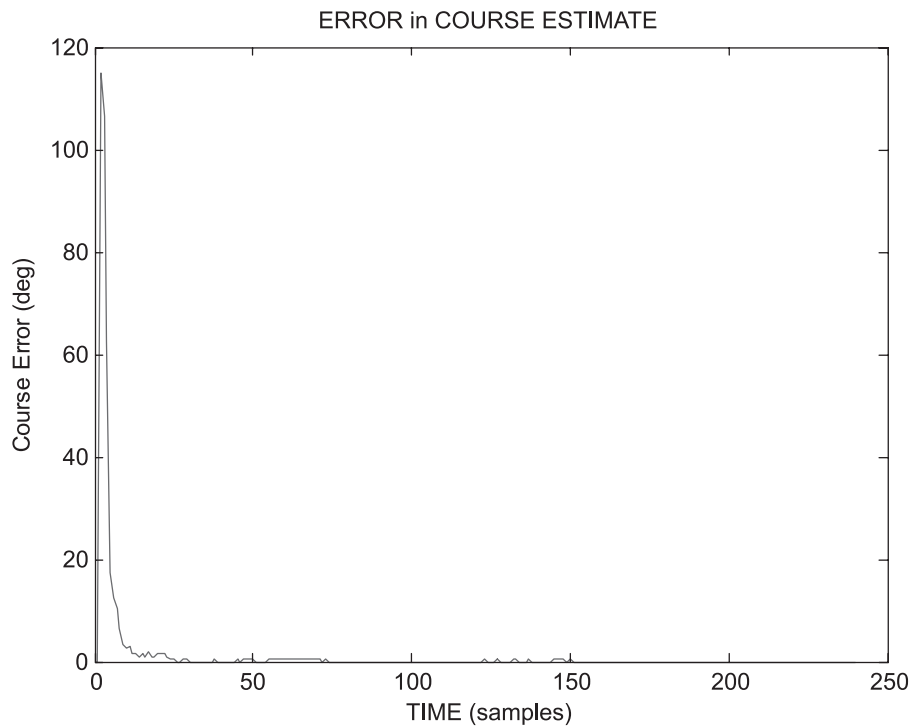
Figure 6: Errors in estimates in scenario1

## 5. CONCLUSION

Extended Kalman filter is used to estimate target course and speed in dunking Sonar system. Simulation is carried out and results are presented. Based on the results, EKF is recommended to track underwater targets using dunking sonar system.



(a) Error in speed estimate



(b) Error in course estimate

Figure 7: Errors in estimate in scenario 2

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