

ON MINIMUM COST WITH MULTI STAGE COMMUNICATIONS FOR A SUPPLY CHAIN NETWORK

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Abstract: The supply chain management becomes crucial in a world that demands fast response with an adequate and excellent service system. It works to minimize inventory costs located at different locations in the system while providing satisfactory service to an end-customer. There are challenges involved in finding out solution of a nonlinear constrained optimization problem that revolves round the complex production inventory system. The objective of this study is to find the maximum output with the minimum cost based on the given supply chain network. The paper is focusing on developing a dynamic programming model for multi-stage communication with minimum cost.

Key Words: Supply chain management, multi-stage graph, dynamic programming, minimum cost

1. INTRODUCTION

The management and control of all materials and information in the logic process is interpreted like supply chain management (SCM). It starts by the purchase of raw materials and ends with the delivery to the end consumer. Various industries like electronics, computer, automobile and several additional business unit, in which fabrication/assembly locations, distribution centers and consumer locations, through which materials, components, products and information flow from a manufacturing unit and supply system generally takes the form of a complex network of suppliers [1]. There are unlike sources of uncertainties associated with supplies, process and demands all over the network. The SCM is like the name involves "Abilities by a network facilities to transform material to intermediate and to end products, and performs the procurement of material, and supply of end product to consumer ", SCM is by no means of simple trade. Basis of this, SCM becomes a major problem for any trade in this highly competitive global market, because it does for several investigator in the area of trade. The service level in terms of fill rate or supply lead time, which by means affect influences the underneath of an activity in today's competitive location. These uncertainties and other factors affect the performance of a system. For any supply chain to apply, may have two kinds of studies which is still lack of general mathematical model. When most of the

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midsize or small-sized trade would perhaps not have the funds to put up their own mathematical models by the problem lies with lack of a general model. It would be difficult for the researchers to discuss and to discover certain problems in the management of supply chain. With the aid of maximum flow in multistage communication problem seen in network flow models, the researcher tried to answer the above call along with a general model of a supply chain [2]. Moreover, with the help of the model, makes the most use of the chain and a manager can find the maximum flow in his supply chain. Therefore, the model designed with some modification, so as to provides a fast and feasible, if not optimal, solution for any SC.

The organization of the rest of the paper is as follows. In Section II we did the literature survey. The model formulation is described in Section III. In Section IV computational method is described, section V describes the complexity analysis. Finally, Section VI concludes the paper.

2. RELATED WORK

There are two well-known approaches for constructing multistage trees with minimum costs. The first approach is based on the shortest paths, and the second uses minimum cost flow algorithms. The algorithm TM due to Takahashi and Matsuyama [3] is a shortest path based algorithm and works on asymmetric directed networks. Also it was further studied and generalized by Ramanathan [4]. The [3] algorithm is very similar to the shortest path based Prim's minimum spanning tree algorithm [5]. The researchers work on inventory models for multistage systems with uncertain demand. The literature is applicable to defined supply chains (SC). The survey articles of Axsater (1993) [6], Federgruen (1993) [7], Inderfurth (1994) [8], van Houtum et al. (1996) [9] and Diks et al. (1996) [10] are referred to the reader for inventory model in Supply chain. The joint papers by LEE and Zipkin [11, 12] are closely related to our work. The researcher study allocate manufacture systems through exponential dealing out times and supply chain at each stage. A multistage model studied by Svoronos and Zipkin [13], where the authors consider successful manufacture lead time is equal to the order wait at the manufacture capacity, they change the manufacture system into a multistage Supply chain.

3. MODEL FORMULATION

The multistage model in a supply chain is considered as network where arc denotes an upstream stage supplies a downstream stage and nodes are stages in the SC. The major processing function in supply chain is represented by stages and a stage is considered as acquisition of raw material, manufacture of a congregation, the assembly and test of a complete good, the production of component, or the transportation of a final product from a central delivery centre to a regional warehouse. The proposed algorithm is a minimum cost multistage graph (MCMG), a regular order to move from one stage (i.e RFID read point) to the next stage that explains the maximum production output with the lowest cost consideration in a SC frame-work. It works as follows. The graph can be considered as a multi-stage network $G = (V, E)$ where the vertices are partitioned into $k \geq 2$ disjoint sets and the graph is being a directed graph in which the each V_i belongs to a stage k . Let s be the source vertex and t be the sink vertices present in the stages 1 and k respectively. Let $C(i,j)$ be the cost matrix of each edge (i,j) . The cost of a path from source s to sink t is the sum of the costs of the edges on the path. The proposed method finds a minimum-cost path from source s to sink t . Figure 1 shows a multi-stage graph with 6 stages. The cost matrix of the figure 1 is depicted in figure 2.

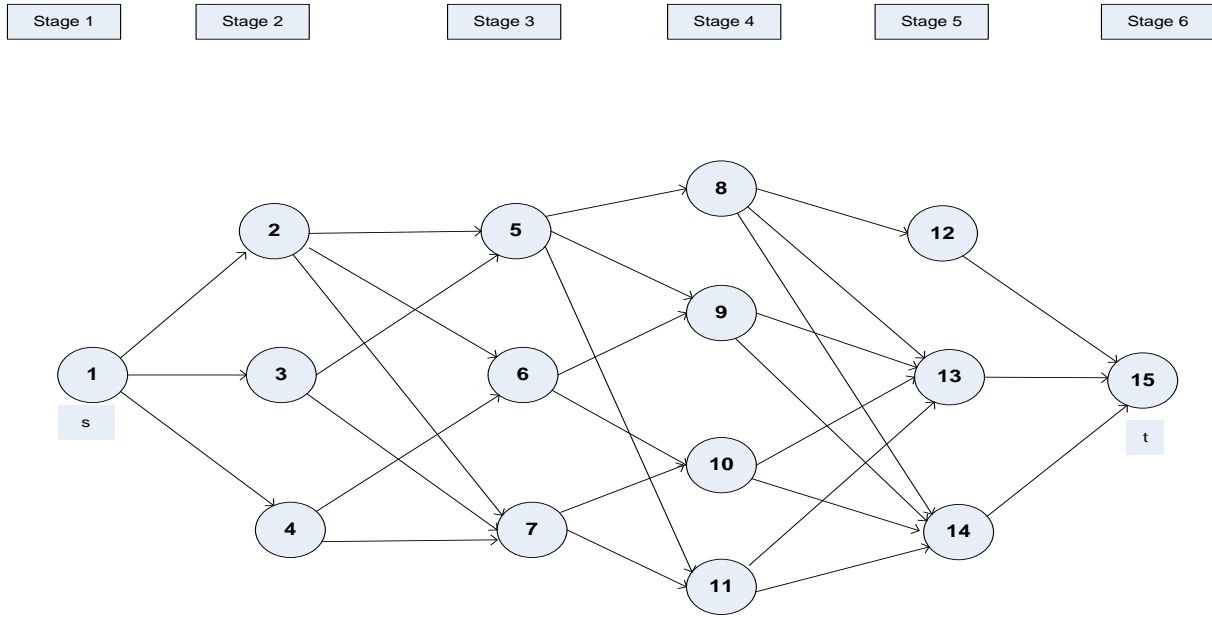


Figure 1 : Six-stage network

| Stages | I | | | II | | | III | | | IV | | | V | | | VI |
|--------|---|----|----|----|----|----|-----|----|----|----|----|----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
| 1 | | 10 | 20 | 30 | | | | | | | | | | | | |
| 2 | | | | | 10 | 20 | 30 | | | | | | | | | |
| 3 | | | | | 40 | | 50 | | | | | | | | | |
| 4 | | | | | | 40 | 30 | | | | | | | | | |
| 5 | | | | | | | | 10 | 20 | | 40 | | | | | |
| 6 | | | | | | | | | 20 | 30 | | | | | | |
| 7 | | | | | | | | | | 30 | 20 | | | | | |
| 8 | | | | | | | | | | | | 10 | 20 | 30 | | |
| 9 | | | | | | | | | | | | | 20 | 10 | | |
| 10 | | | | | | | | | | | | | 10 | 20 | | |
| 11 | | | | | | | | | | | | | 10 | 30 | | |
| 12 | | | | | | | | | | | | | | | 20 | |
| 13 | | | | | | | | | | | | | | | 10 | |
| 14 | | | | | | | | | | | | | | | 30 | |
| 15 | | | | | | | | | | | | | | | | |

Figure 2 : The cost matrix of the above network

4. COMPUTATIONAL PROCEDURE

To obtain the product progress information, we assume that RFID readers as positioned at distinct stages in supply chain. so that the supply chain manager can take the decision for shortest path with minimum cost in the states of the order (i.e Physical location) is observed through RFID at each stage. The idea is to use product progress information to rush through shortest path in each stage order, emergency order may come from a separate supplier, or the from the same supplier using an alternate transportation or replenishment mode in lowest cost.

Applying the dynamic programming approach, the problem is divided into sub problems, called stages. Figure 1 has six stages, stage 1 with vertex {1}, stage 2 with vertices {2,3,4}, stage 3 with vertices {5,6,7}, stage 4 with vertices {8,9,10,11}, stage 5 with vertices {12,13,14}, and finally stage 6 with vertex {15}. Dynamic programming starts with a small problem. A smaller problem is a nearly completed journey with just one more stage to go from the current state to the

destination. Therefore, initially, the shortest path from stage 5 to stage 6 is computed. Then the problem is enlarged by adding one more stage to the current problem. In this case, it is adding one more stage to the stages that are yet to be completed to reach the destination. It can be observed that there is no need for recomputation, as the already-stored stage cost can be reused. Proceeding in this manner, we can find the shortest path from 1 to 15. The shortest path can be obtained using both the forward and backward computation procedure. In the forward computation procedure, the decision x_i is made in terms of the optimal decisions $(x_{n-1}, x_{n-2}, \dots, x_1)$, while in the backward computation procedure, the decision x_i is made in terms of the optimal decisions $(x_1, x_2, x_3, \dots, x_{i-1})$.

i) Forward Computation Procedure

The informal algorithm for forward computational procedure for multistage graph is given as follows :

Step 1 : Read the directed graph $G = \langle V, E \rangle$ with k stages.

Step 2 : Let the number of nodes in the graph be n and $\text{dist}[1..k]$ is the cost array.

Step 3 : Set initial cost to zero.

Step 4: Loop index i from $(n-1)$ to 1.

4 a: Find a vertex v from the next stage such that
the edge connecting the current stage and the next
stage is minimum, that is $(j, v) + \text{cost}(v)$ is minimum.

4 b. Update cost and store v in another array $\text{dist}[]$

Step 5 : Return cost.

Step 6 : End

Let the shortest-path cost from point i to point j is S_{ij} . Then the objective is to find S_{1-15} , the shortest path between the source and the sink nodes, i.e., nodes 1 and 15 respectively. This is a multistage problem where decisions need to be taken at various stages. It can be observed that there are six stages. There are only one node $\{1\}$ at stage 1, four nodes $\{2,3,4\}$ in stage 2, three nodes $\{5,6,7\}$ in stage 3, four nodes $\{8,9,10,11\}$ in stage 4, three nodes $\{12,13,14\}$ in stage 5 and finally only one node $\{15\}$ in stage 6. It can be observed that one can move forward only by one stage. Thus the shortest-path problem is to minimize the sum :

$$S_{1-15} = S_{1i} + S_{ij} + S_{jk} + S_{kl} + S_{1-15}$$

where i may be 2,3,4 ; j may be 5,6,7; k may be 8,9,10,11,

l may be 12,13,14. In short i, j, k, l represent the stages. In the forward method, the decision x_i is made in terms of the optimal decisions as $x_{i+1}, x_{i+2}, \dots, x_n$. The problem starts with stage 1. Then every iteration involves enlargement of the problem, i.e., one more stage is added to the given problem.

```

Algorithm FGraph(G,k,n,p)
//The input is a k-stage graph  $G=\langle V,E \rangle$  with n vertices indexed
indexed in order of stages. E is the set of edges and  $C[i,j]$  is the
cost of (i,j). P[1:k] is a minimum cost path
{
  Cost[n]=0
  for j= (n-1) to 1 step -1 do
  {
    // Compute Cost[j]
    Let r be the vertex such that  $\langle j,r \rangle$  is an edge of G and
 $C[j,r]+ Cost[r]$  is minimum
    Cost[j]=  $C[j,r]+ Cost[r]$ 
    d[j]= r
  }
  //Find the minimum cost path
  p[1]=1
  p[k]=n
  for j= 2 to k-1 do
    p[j]= d[p[j-1]]
}

```

Simulating the Solution by Forward Costs Method

Cost(stage ,vertex)= minimum cost to travel from the vertex in stage to the target

Minimum Cost Path

p[1]=1 , p[6] =15 (Initialization)

j=2 p[2]=d[p[1]]=d[1]=2

j=3 p[3]=d[p[2]]=d[2]=5

j=4 p[4]=d[p[3]]=d[5]=8

j=5 p[5]=d[p[4]]=d[8]=13

So the minimum cost path is

1-2-5-8-13-15 and the cost will be 60.

ii) Backward Computation Procedure

The distance computation in backward method is from the source.

The informal algorithm for forward computational procedure for multistage graph is given as follows :

Step 1 : Read the directed graph $G=\langle V,E \rangle$ with k stages.

Step 2 : Let the number of nodes in the graph be n and dist[1...k] is the cost array.

Step 3 : Set initial cost to zero.

Step 4: Loop index i from 1 to (n-1).

4 a: Find a vertex v from the next stage such that
the edge connecting the current stage and the next
stage is minimum, that is $(j,v) + cost(v)$ is minimum.

4 b. Update cost and store v in another array dist[]

Step 5 : Return cost.

Step 6 :End

```

Algorithm BGraph(G,k,n,p)
//The input is a k-stage graph  $G=<V,E>$  with n
vertices indexed indexed in order of stages. E is
the set of edges and  $C[i,j]$  is the cost of (i,j).
P[1:k] is a minimum cost path
{
  bCost[1]=0
  for j= 2 to n do
  {
    // Compute bCost[j]
    Let r be the vertex such that  $<r,j>$  is an
edge of G and  $bCost[r]+C[r,j]$  is minimum
    bCost[j]= bCost[r] + C[r,j]
    d[j]= r
  }
  //Find the minimum cost path
  p[1]=1
  p[k]=n
  for j= k-1 to 2 do
    p[j]= d[p[j+1]]
  }

```

Simulating the Solution by Backward Costs Method

Cost(stage, vertex)= minimum cost to travel to the vertex in stage from the source (vertex,1)

Minimum Cost Path

p[1]= 1, p[6]=15 (Initialization)

j=5 p[5]=d[p[6]]=d[15]=12 or 13

j=4 p[4]=d[p[5]]=d[12] or d[13]=8

j=3 p[3]=d[p[4]]=d[8]=5

The required min cost path is either

1-2-5-8-12-15

or 1-2-5-8-13-15

having a cost of equal to 60.

5. COMPLEXITY ANALYSIS

The given graph $G=<V,E>$ is given in the adjacency list. Let $n=|V|$ and $m=|E|$. There is only one for-loop that gets executed $n-1$ times. The path tracking requires $\Theta(k)$ times, where k is the number of stages. Therefore, the complexity of the algorithm is $\Theta(m+n)$.

6. CONCLUSION

In an RFID enabled Supply chain environment that is characterized by near - complete visibility of the items in the system. The algorithm implemented in this study is a tread to resolve the issue concerning a general model of a SCM. The reslt of algorithm would help the manager to reach a decision to save the most from the process of supply chain while fulfilling the demand of

the market. Keeping fluctuating nature of market in view, a manager is expected to take quick decision. The algorithm structured in this chapter will help the manager to take prompt decision. It can specially be adopted by medium to small size industries, those who may not have enough resources to develop their own specific models while they still need to evaluate their supply chain based on a mathematical way. If the factor of uncertainty can be incorporated into the model, it would unfold a new dimension in the case of SCM. Recently, the SC have entered into much complex spheres those bring in tough challenges for a single business to tackle the issue. Therefore, the need of the hour to develop a better model for supply chain management, which would be helpful to facilitate the system to face upcoming challenges with minimum cost. We will keep this issue for our future research.

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